Simulating Butterfly Pitch Angle Distributions in the Inner Zone: Sensitivity to Wave Models

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My 2016 GRL: 2D diffusion simulations at L=2, to see if "butterfly" PADs reported by Zhao et al. [GRL, 2014] were reproduced.

I found that they were, using just the now-standard waves: hiss + LGW + TX (Abel and Thorne, 1998).
$D_{hiss+LGW}$ was provided by BAS [Glauert et al., JGR 2014],
D_{TX} from elaborate modeling at AFRL.

J. Li et al. (same issue of GRL), using different wave models and codes, at L=2.4, found that MS waves were also needed.

Several observational studies since then have found that strong butterfly PADs are correlated with MS waves.

Note: I did find that they “contribute to the size and promptness” of the butterflies. (They were also included in my “efficient approximations” paper [2008]. I have nothing against MS waves.)

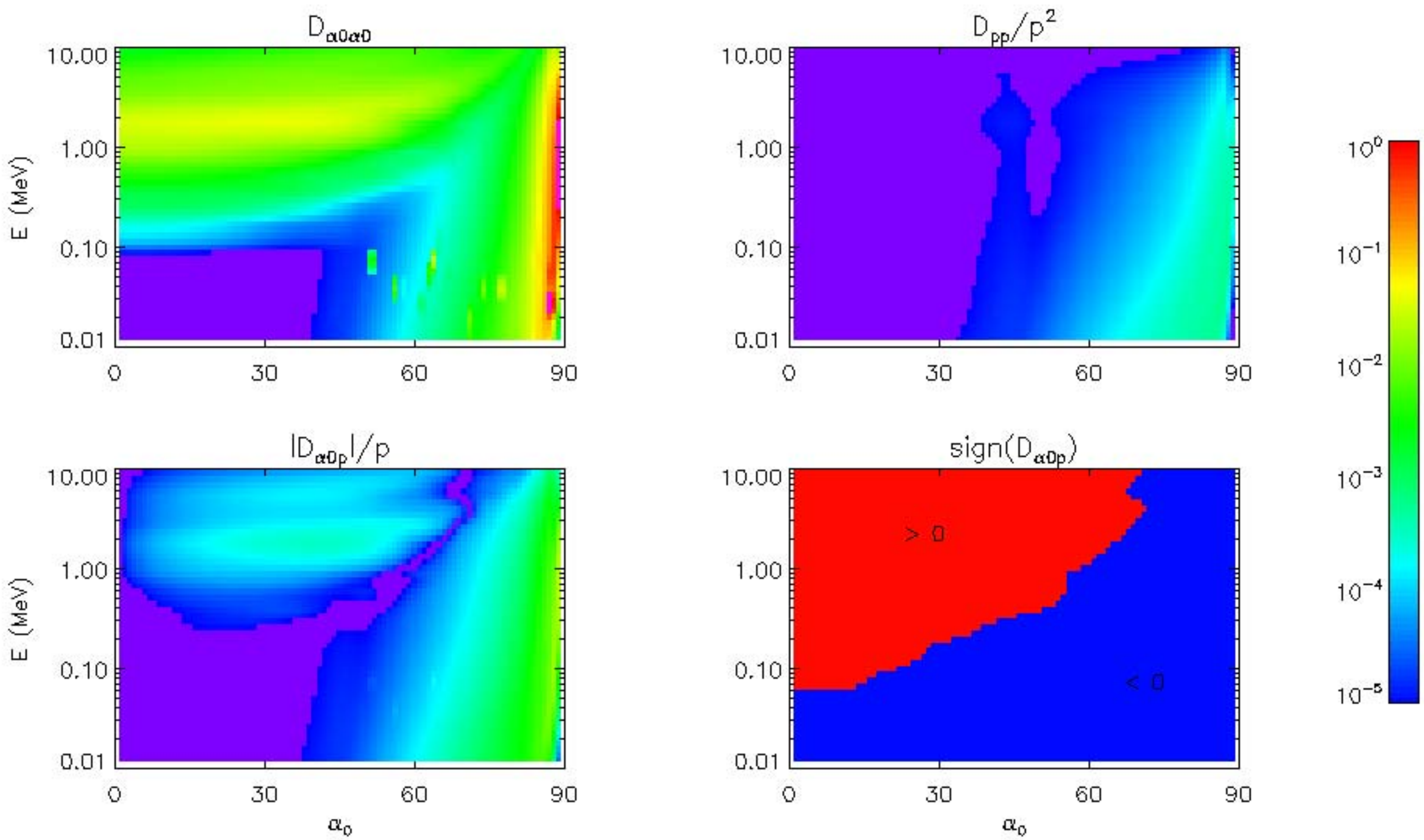
Still, I found butterflies without them, and by the same mechanism: D_{pp} at large α_0, where only n=0 occurs.

So: how robust is this? Also, how to account for PADs peaked at 90^o?

BAS $D_{hiss+LGW}$: based on CRRES data and HOTRAY ray tracing.
(see Glauert et al., JGR 2013.)

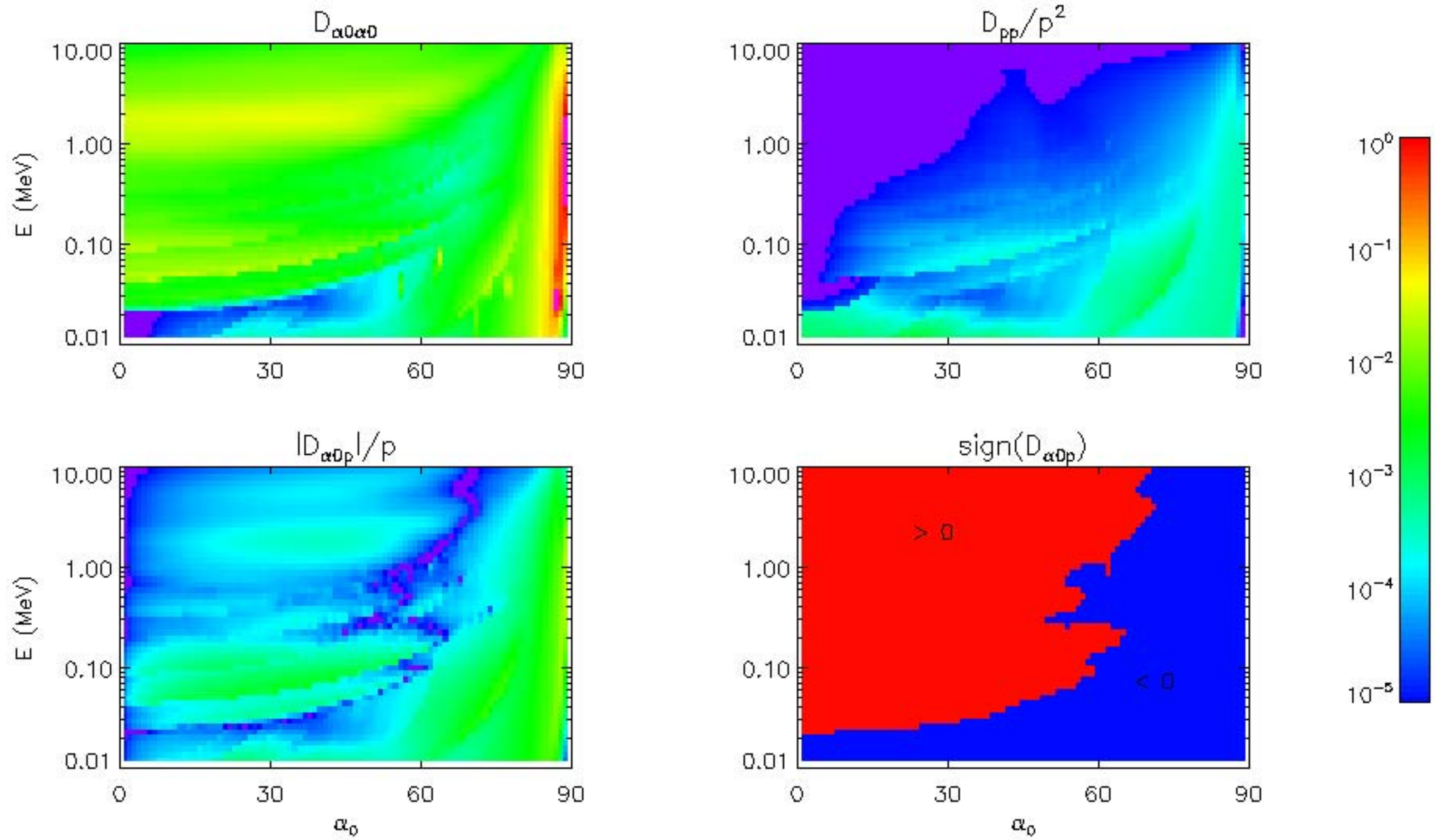
AFRL D_{TX}: modeled 9 biggest Navy transmitters in detail, using a full-wave code for trans-iono propagation, then 3D ray-and-power tracing; calculated 1-wave D's, averaged over drift shell, then over day/night and season.
(see Starks et al., 2008; Cohen et al., 2012; Albert et al., 2016.)

BAS hiss+LGW



Dα0α0
Dpp/p²
|Dα0p|/p
sign(Dα0p)
E (MeV)
α0
> 0
< 0
10.00
1.00
0.10
0.01
0
30
60
90
10⁰
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵

[BAS hiss+LGW] + [AFRL TX]



The 2D diffusion equation is

$$\frac{\partial f}{\partial t} = \frac{1}{G}\frac{\partial}{\partial \alpha_0} G\left(\frac{D_{\alpha_0\alpha_0}}{p^2}\frac{\partial f}{\partial \alpha_0} + \frac{D_{\alpha_0 p}}{p}\frac{\partial f}{\partial p}\right) + \frac{1}{G}\frac{\partial}{\partial p} G\left(\frac{D_{\alpha_0 p}}{p}\frac{\partial f}{\partial \alpha_0} + D_{pp}\frac{\partial f}{\partial p}\right)$$

Integrating across the α_0=90º boundary in the (α_0,p) plane gives the BC $\frac{D_{\alpha_0\alpha_0}}{p^2}\frac{\partial f}{\partial \alpha_0} + \frac{D_{\alpha_0 p}}{p}\frac{\partial f}{\partial p} = 0$, not $\frac{\partial f}{\partial \alpha_0} = 0$

To avoid numerical problems, I solve the equivalent

$$\frac{\partial f}{\partial t} = \frac{1}{\Gamma}\left(\frac{\partial}{\partial Q_1}\Gamma D_1 \frac{\partial f}{\partial Q_1} + \frac{\partial}{\partial Q_2}\Gamma D_2 \frac{\partial f}{\partial Q_2}\right)$$ with $\frac{\partial f}{\partial Q_1} = 0$

Interestingly, the BC alone suggests butterfly PADs:

$$\frac{\partial f}{\partial \alpha_0} \sim - D_{\alpha_0 p} \frac{\partial f}{\partial p}$$

since (1) typically $\partial f/\partial p < 0$, and
(2) resonance at large α_0 requires n=0, for which $D_{\alpha 0p} < 0$

(recall $$\frac{D^n_{\alpha p}}{D^n_{\alpha\alpha}} = \frac{p \sin\alpha \cos\alpha}{-\sin^2\alpha + \Omega_n/\omega}, \quad \frac{D^n_{pp}}{D^n_{\alpha\alpha}} = \left(\frac{D^n_{\alpha p}}{D^n_{\alpha\alpha}}\right)^2$$

where $\Omega_n = -n|\Omega_e|/\gamma$).

Also, IC as in the 2016 paper:

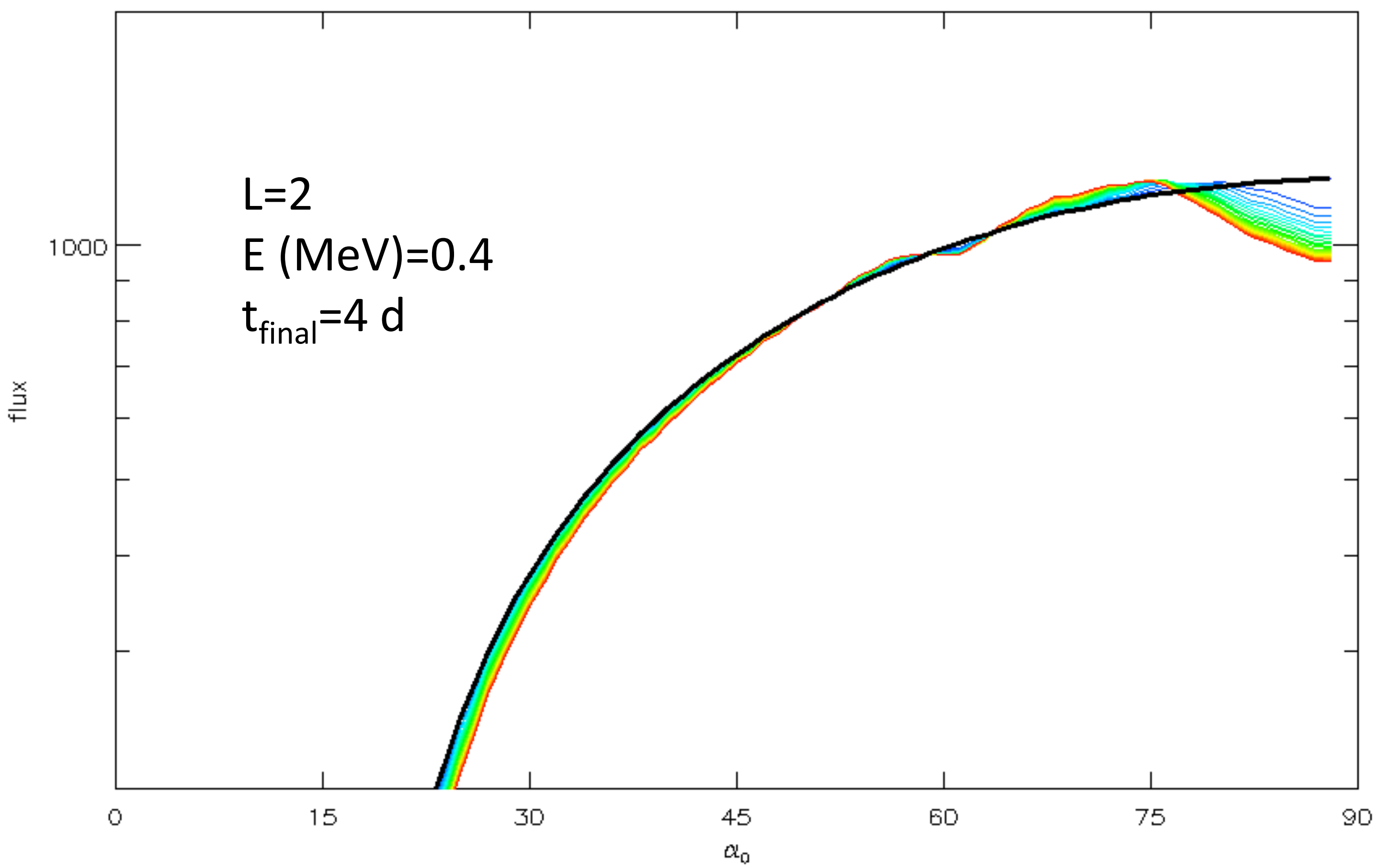
$j = j_0 \, e^{-E/E0} \, (\sin\alpha_0 - \sin\alpha_{LC})$,

with

$j_0 = 2.5 \times 10^5$ $cm^{-2}\ s^{-1}\ sr^{-1}\ keV^{-1}$, $E_0 = 80$ keV

held fixed at $E_{min} = 200$ keV.

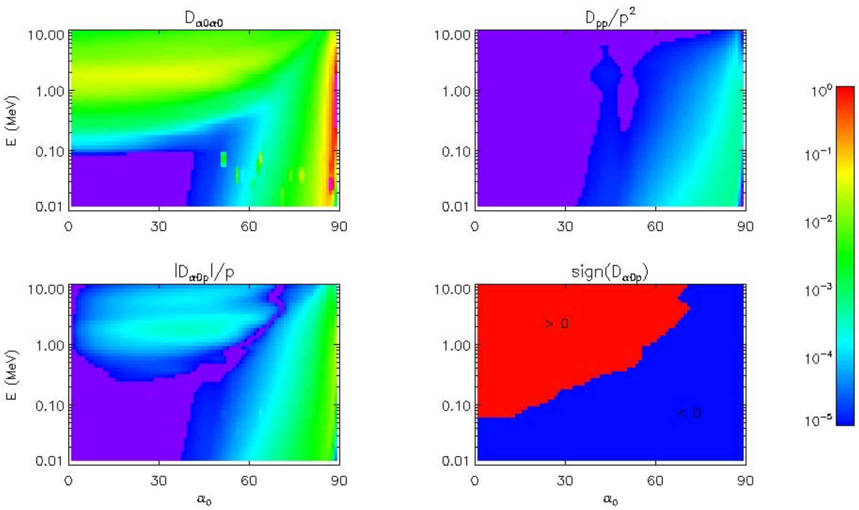
[BAS hiss+LGW] + [AFRL TX]



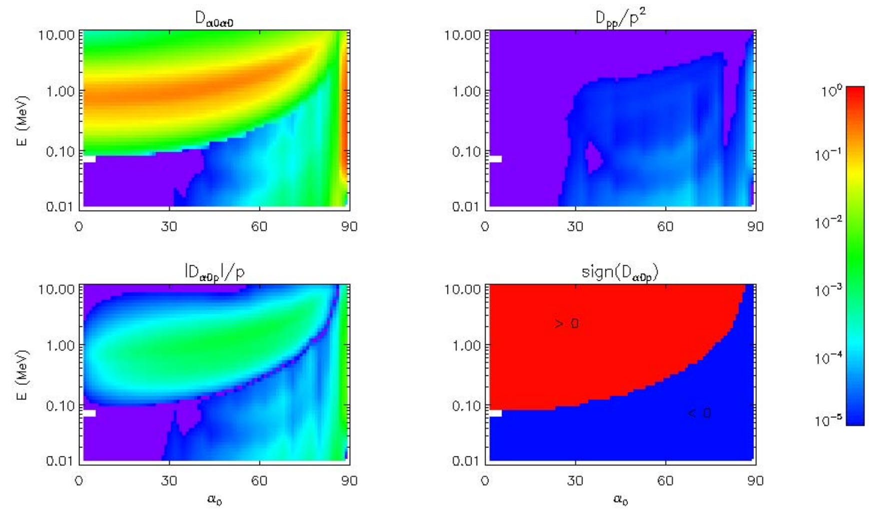
For the GEM challenge, Q. Ma provided another version of $D_{hiss+LGW}$. I tried using these in place of the BAS values.

We didn’t call the FG “quantitative assessment of radiation belt modeling” for nothing.

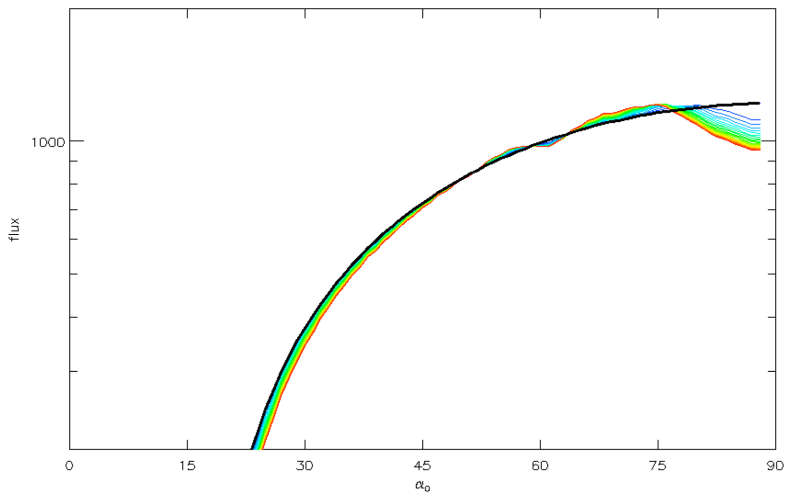
BAS hiss+LGW



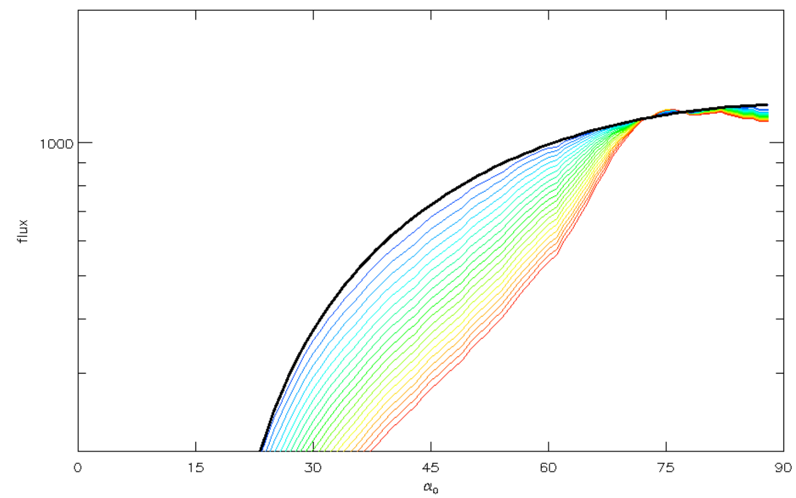
GEM hiss+LGW



[BAS hiss+LGW] + [AFRL TX]



[GEM hiss+LGW] + [AFRL TX]



These “GEM D’s” cite several UCLA papers:

$B^2(\omega)$ (VAP statistical, not Gaussian)

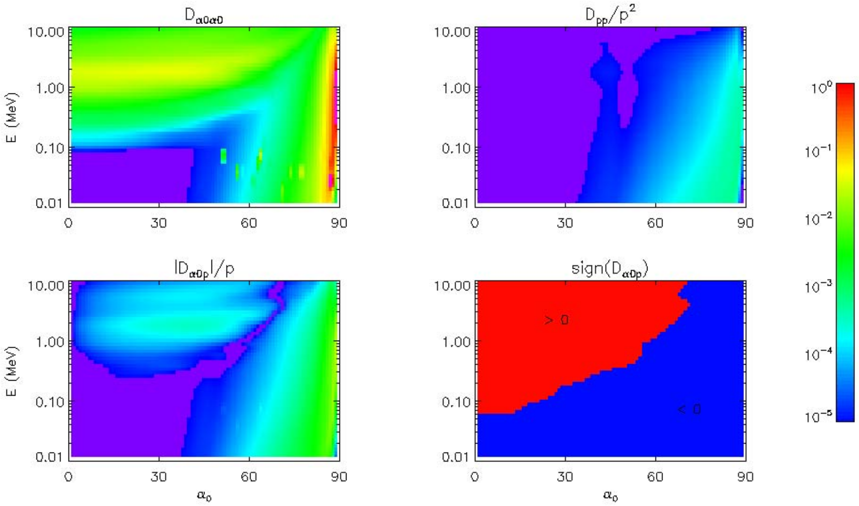
WN model from ray tracing (9 different latitude ranges)

B_{wave} from VAP statistics

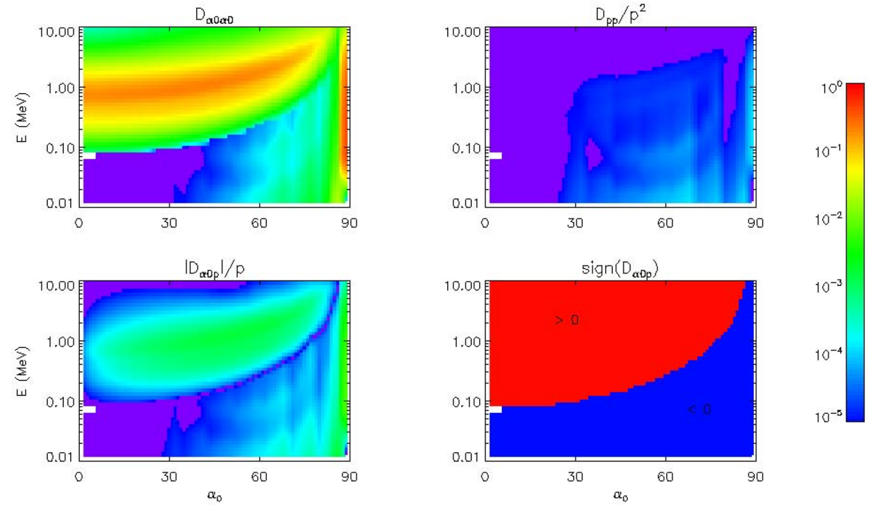
and n_e from Sheeley.

From these papers, one can actually recalculate the D’s.
(The GEM D’s are MLT-dependent, which I averaged;
to reproduce I used MLT-averaged inputs.)

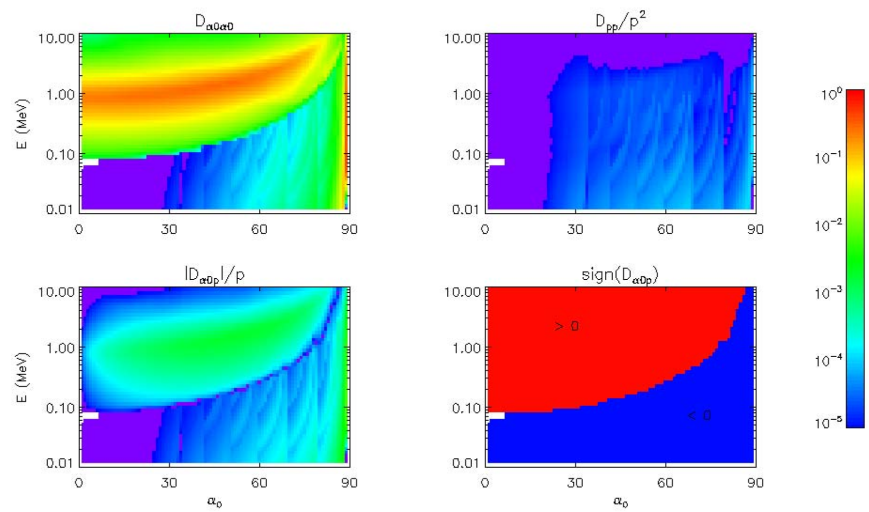
BAS hiss+LGW



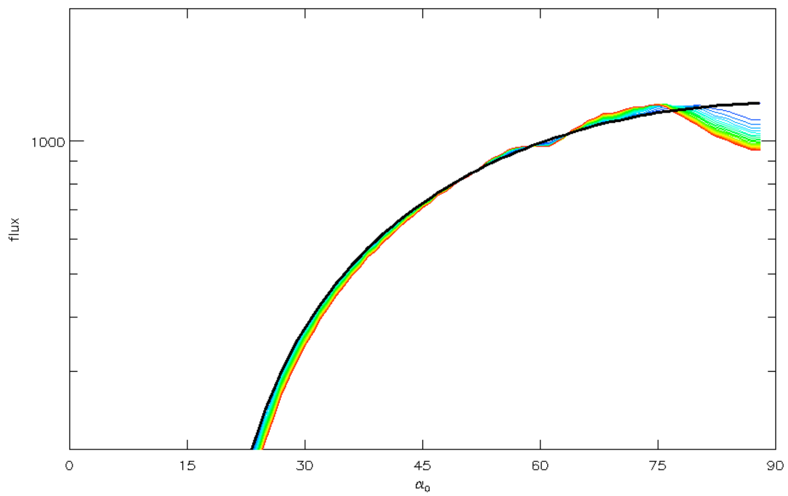
GEM hiss+LGW



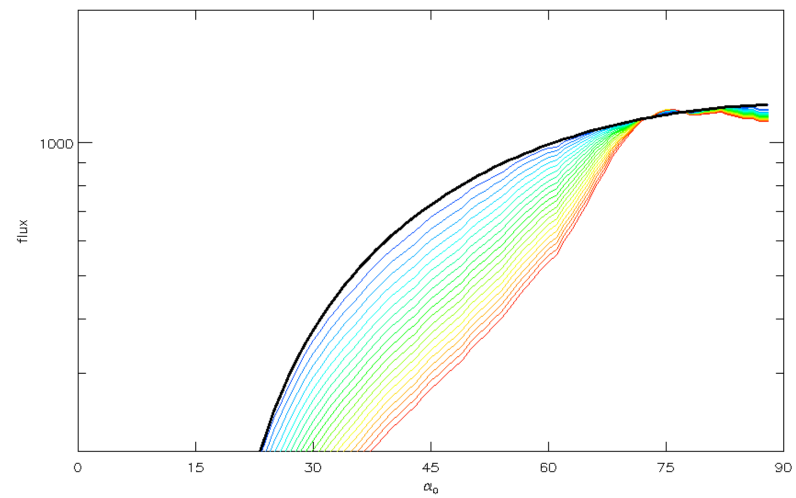
UCLA hiss+LGW (Sheeley)



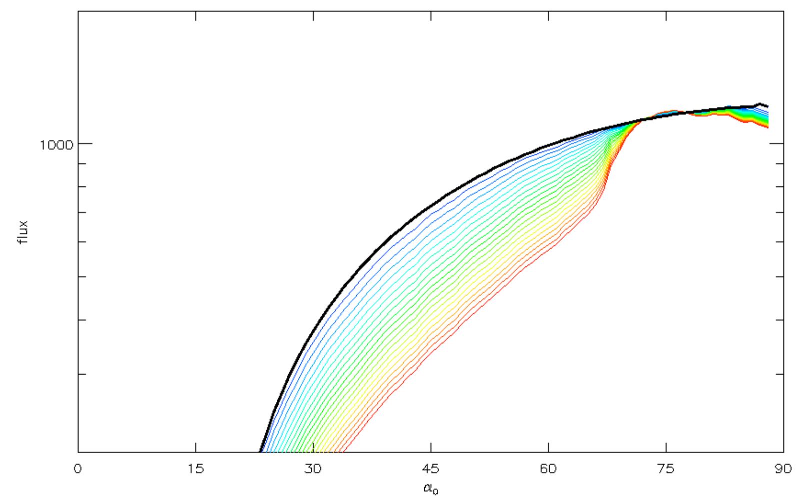
[BAS hiss+LGW] + [AFRL TX]



[GEM hiss+LGW] + [AFRL TX]

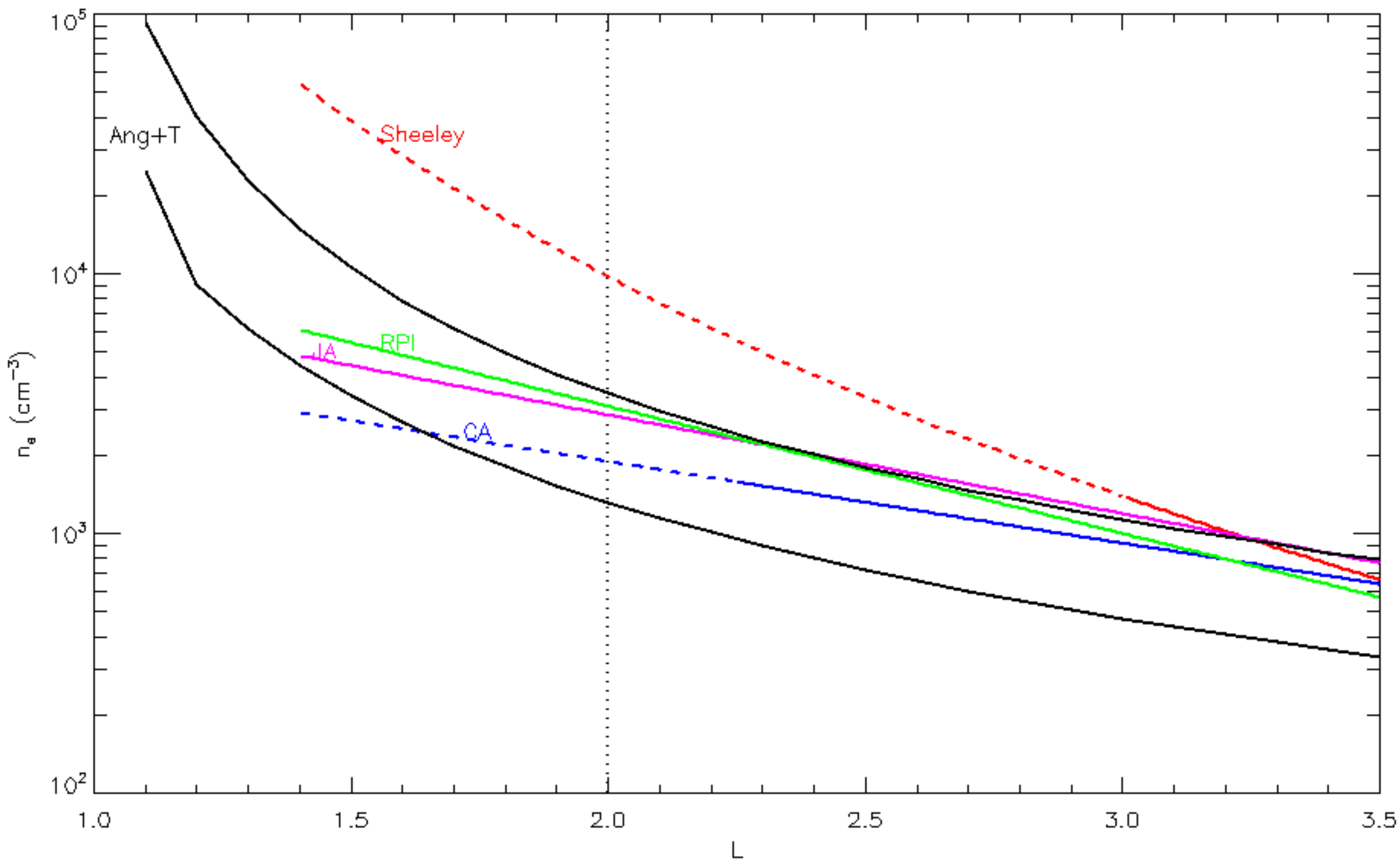


[UCLA (Sheeley)] + [AFRL TX]



My recalculated D's are pretty close to the GEM values, and the resulting flux evolution is also pretty close, i.e., different from the BAS values.

The difference is not some subtle feature of $B^2(\omega)$ or the WN profiles; it's mostly just the n_e model.



Ang+T
Sheeley
RPI
JA
CA
n_e (cm^{-3})
L

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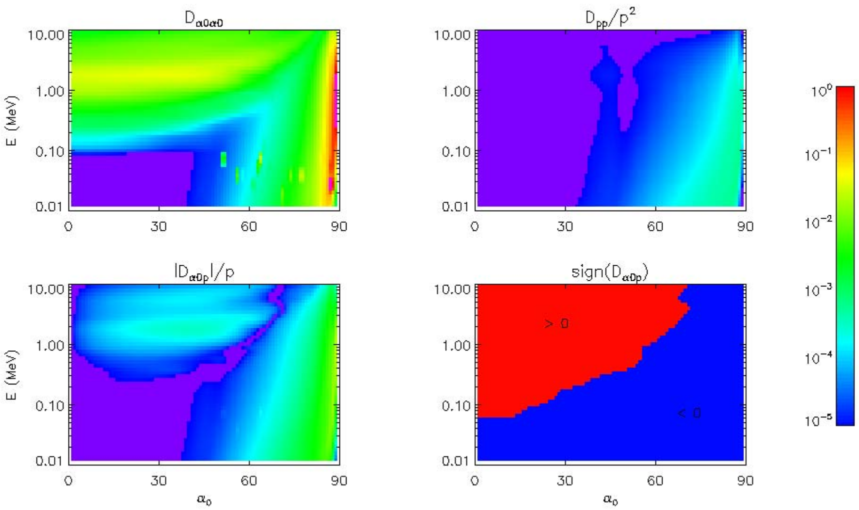
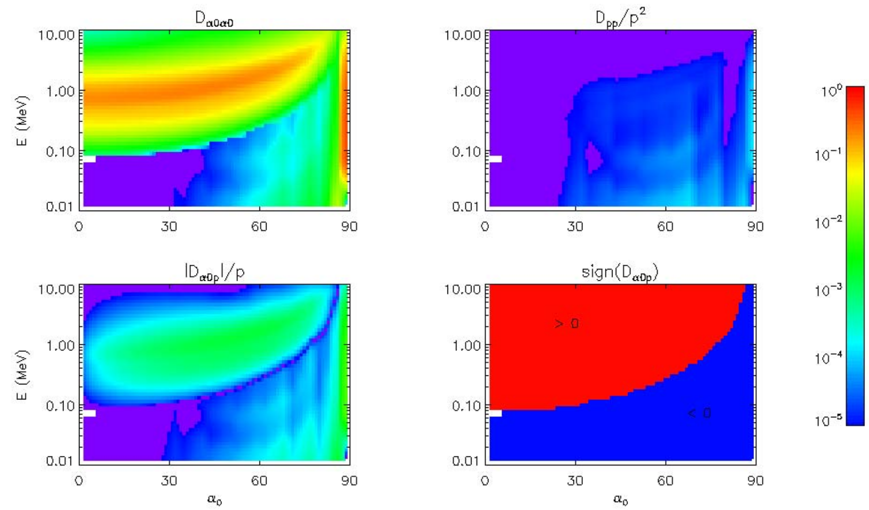
Changing from Sheeley to a more reasonable value at L=2 gives better agreement between the BAS and UCLA results.

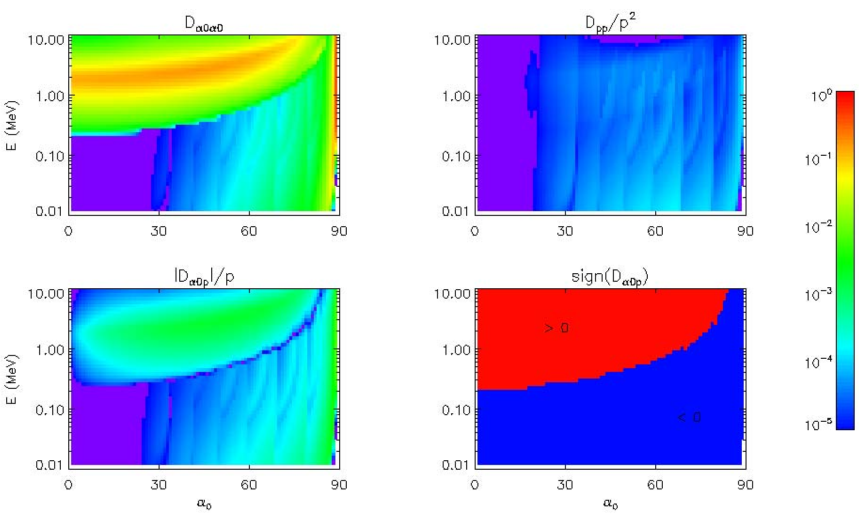
BAS hiss+LGW



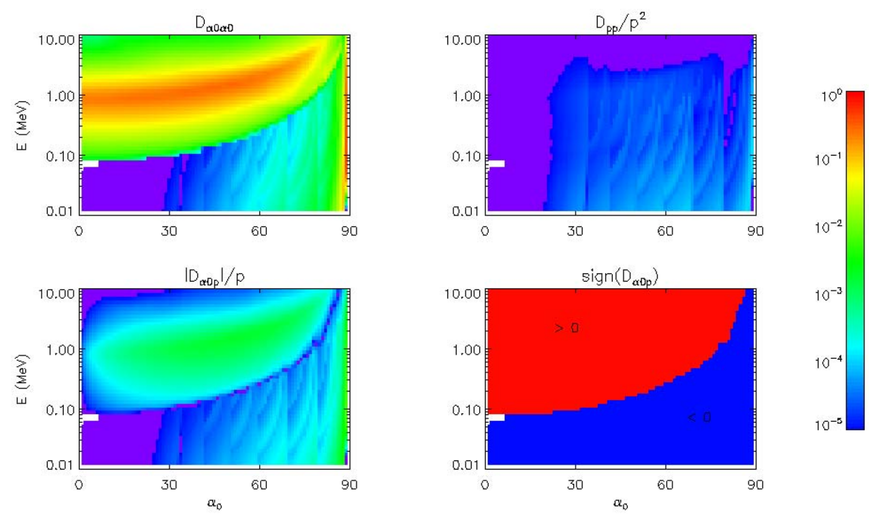
GEM hiss+LGW



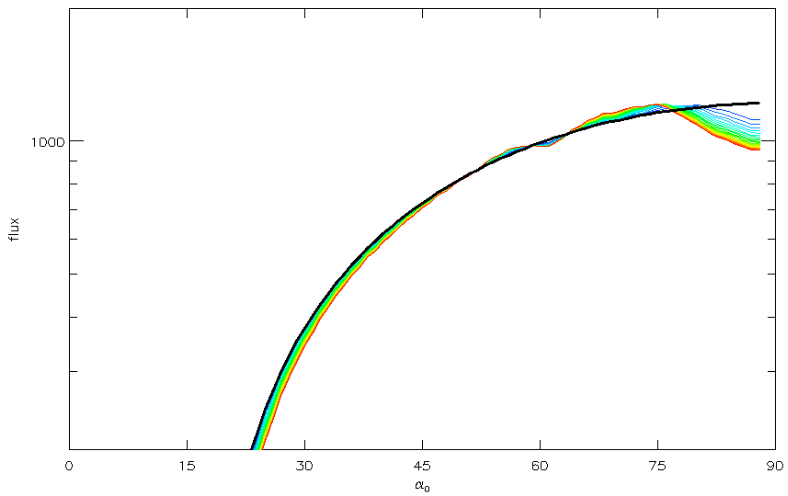
UCLA/JA hiss+LGW



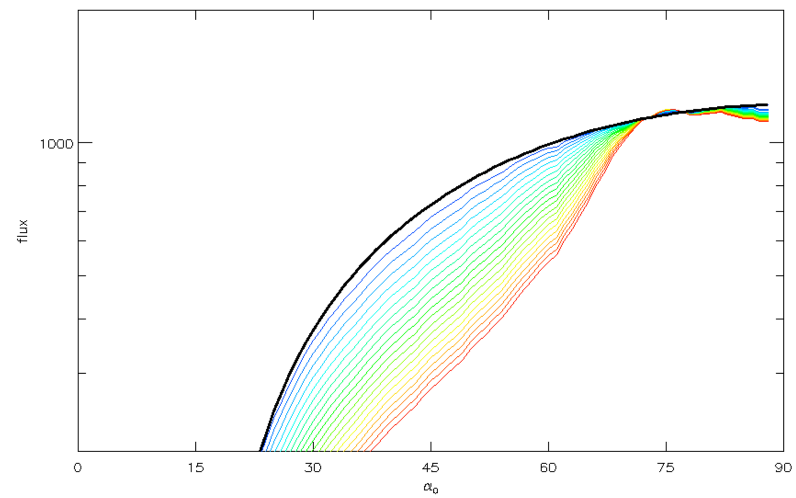
UCLA hiss+LGW (Sheeley)



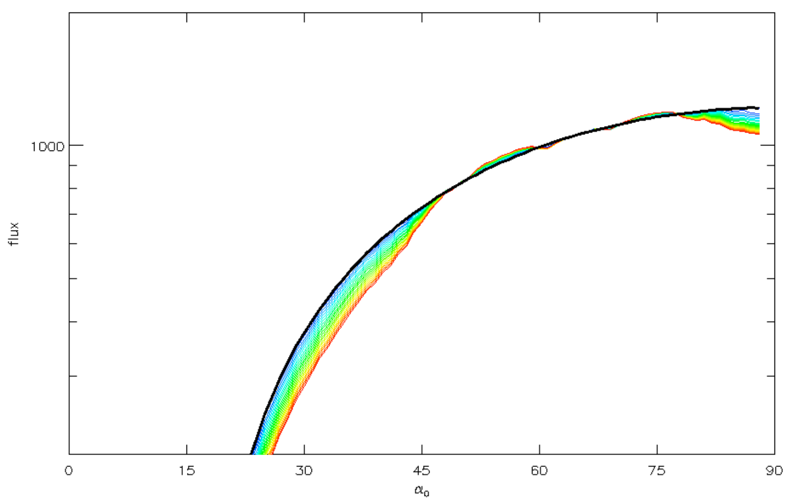
[BAS hiss+LGW] + [AFRL TX]



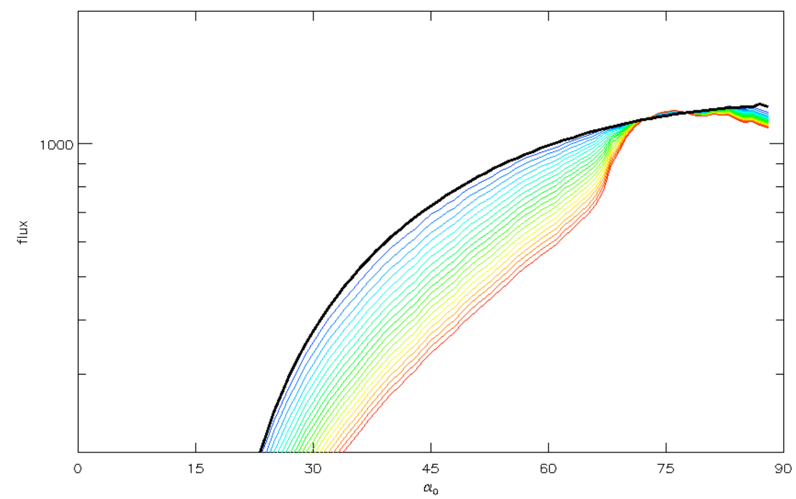
[GEM hiss+LGW] + [AFRL TX]



[UCLA/JA hiss+LGW] + [AFRL TX]



[UCLA hiss+LGW] + [AFRL TX]

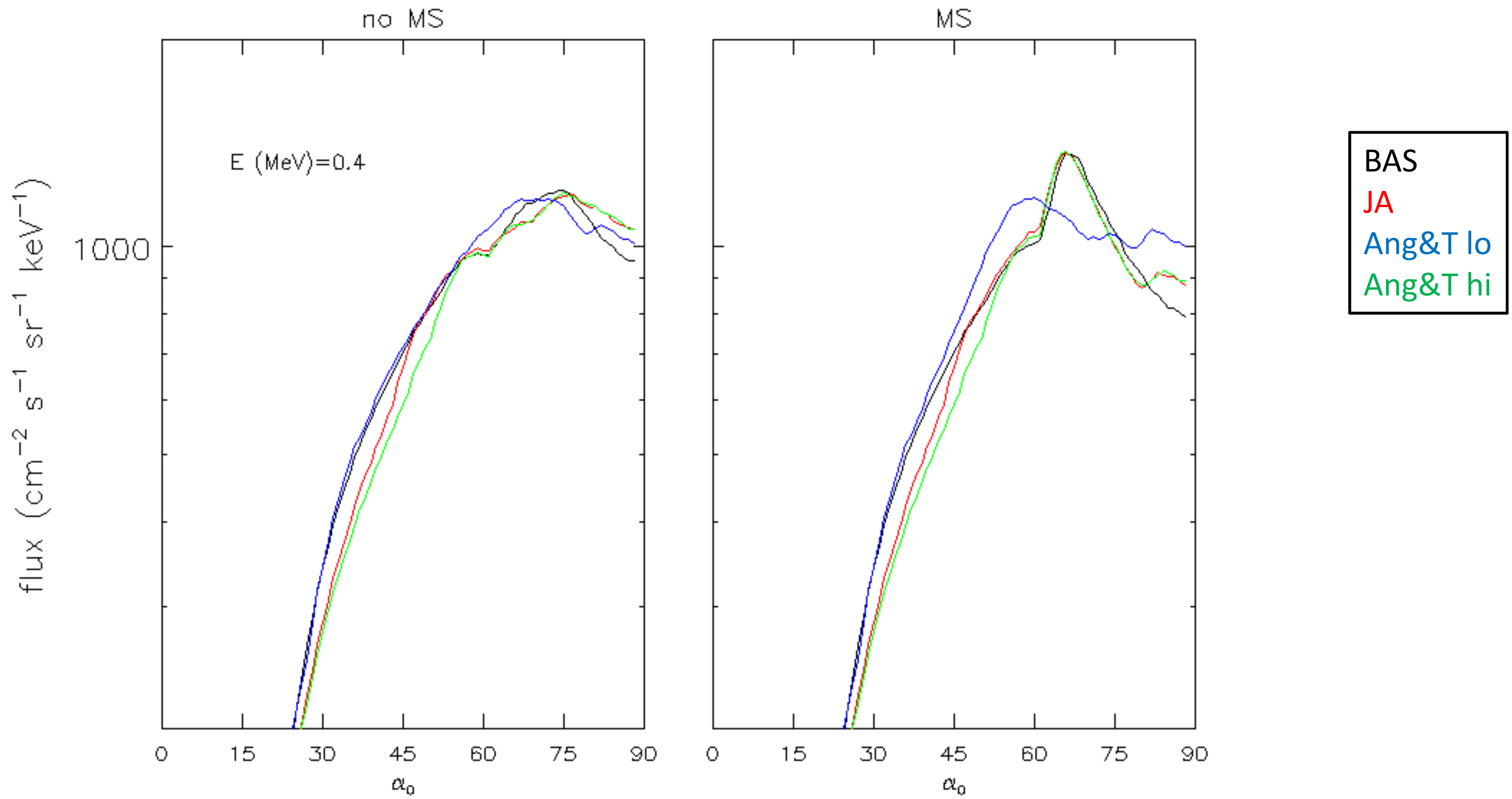


I also tried our in-house density models, namely “high” and “low” versions of the Angerami and Thomas [1964] DE model.

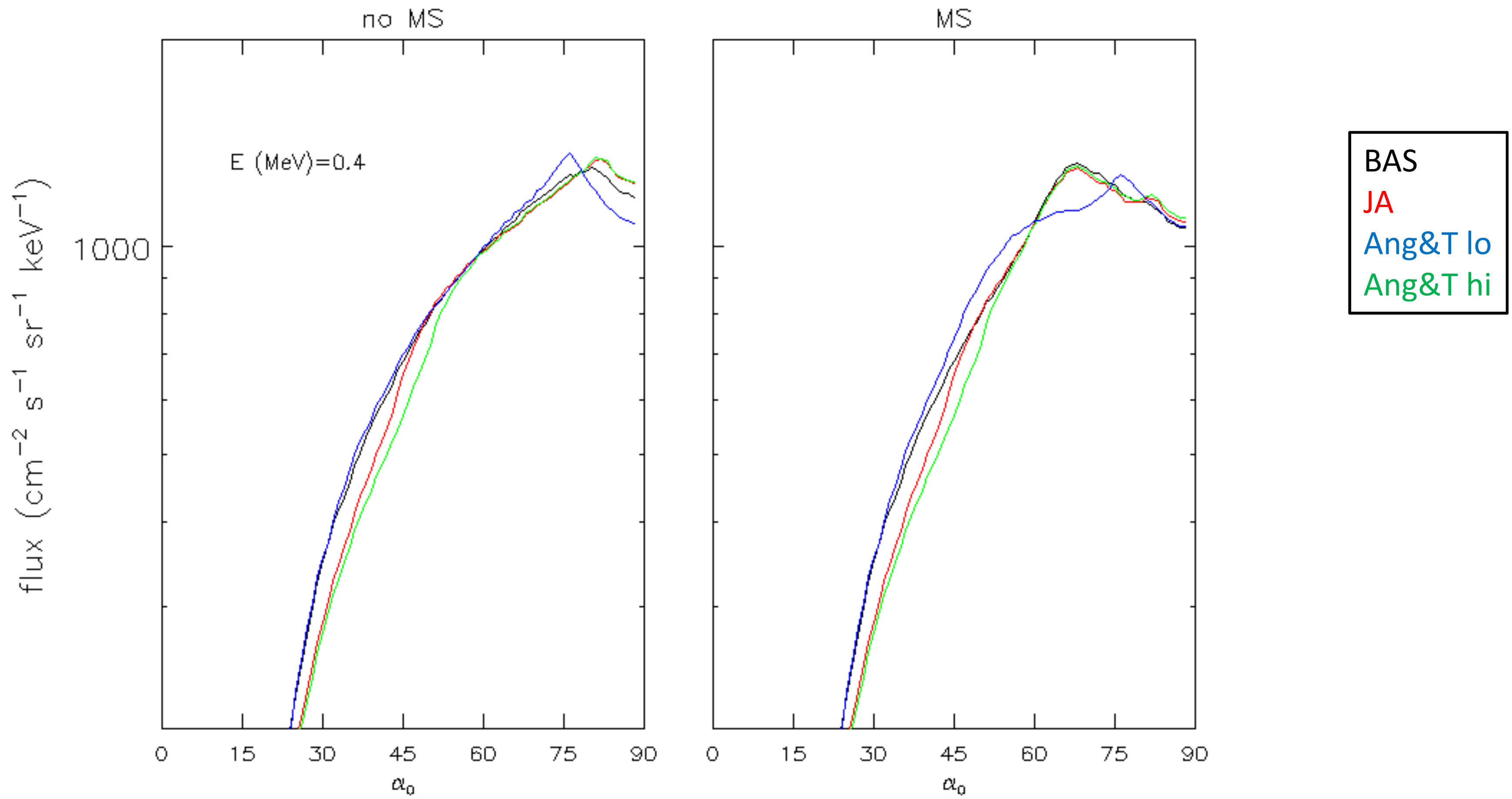
To include MS waves, I mostly followed my 2016 study:
B_{wave}=75 pT, occurrence rate=30%,
ω parameters from Horne et al. [2007],
WN peaked at 89^o.

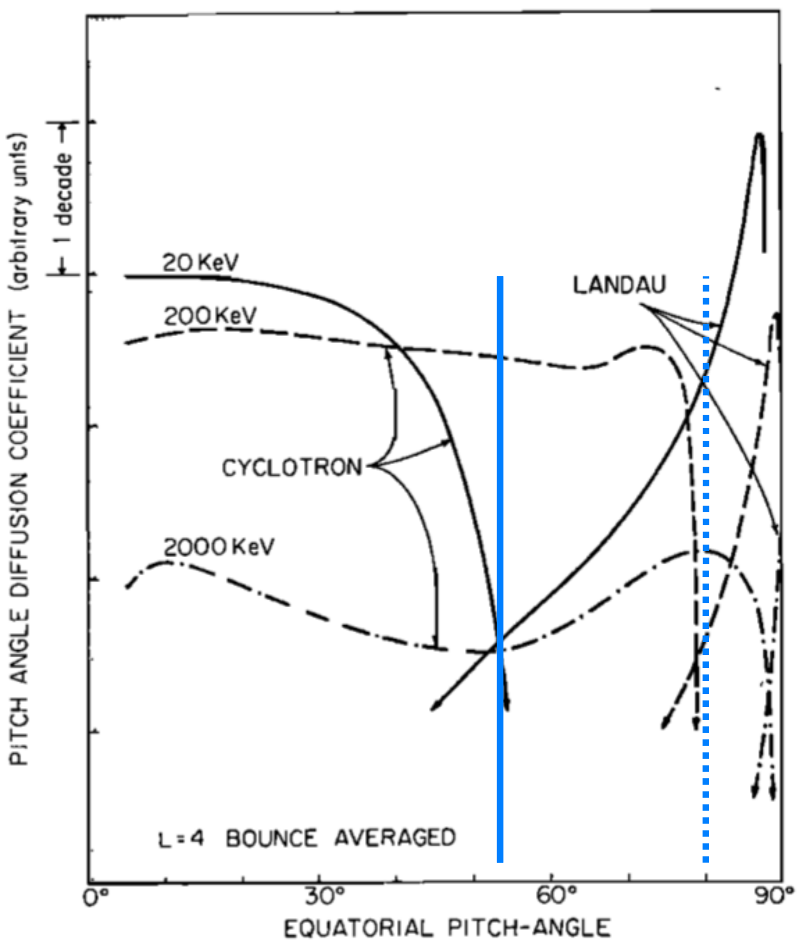
Also: the AFRL TX models have ducted/unducted versions.

hiss + LGW + TX(unducted)



hiss + LGW + TX(ducted)





$$\cos\alpha_0 \geq \frac{\Omega_{eq}}{\omega_{pe}} \frac{c}{v\gamma} \sqrt{\frac{\Omega_{eq}}{\omega_{UC}} \sec\theta_{\min}}, \quad n \neq 0$$

Low n_e => n>0 stops at lower α_0

=> n=0 takes over

=> heating

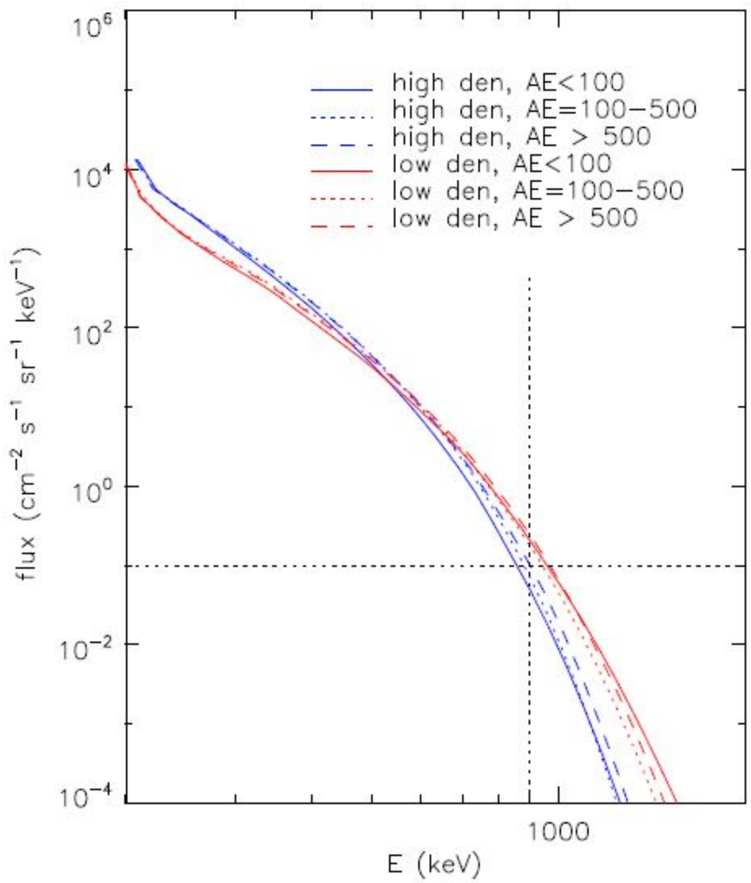
=> BFs

Finally, are the energy profiles from combined wave heating and loss consistent with observations, especially very low levels above 1 MeV?

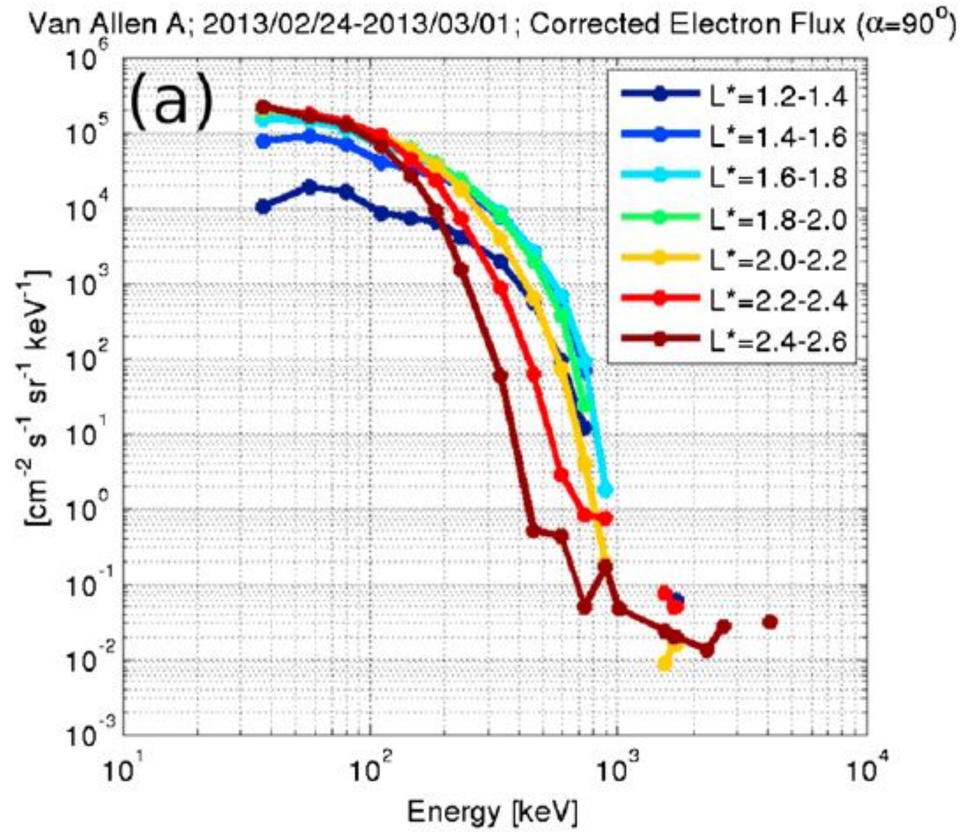
In some cases, yes.

The energy distribution is also reasonable.

L=2, α_0=85^o
(steady state)

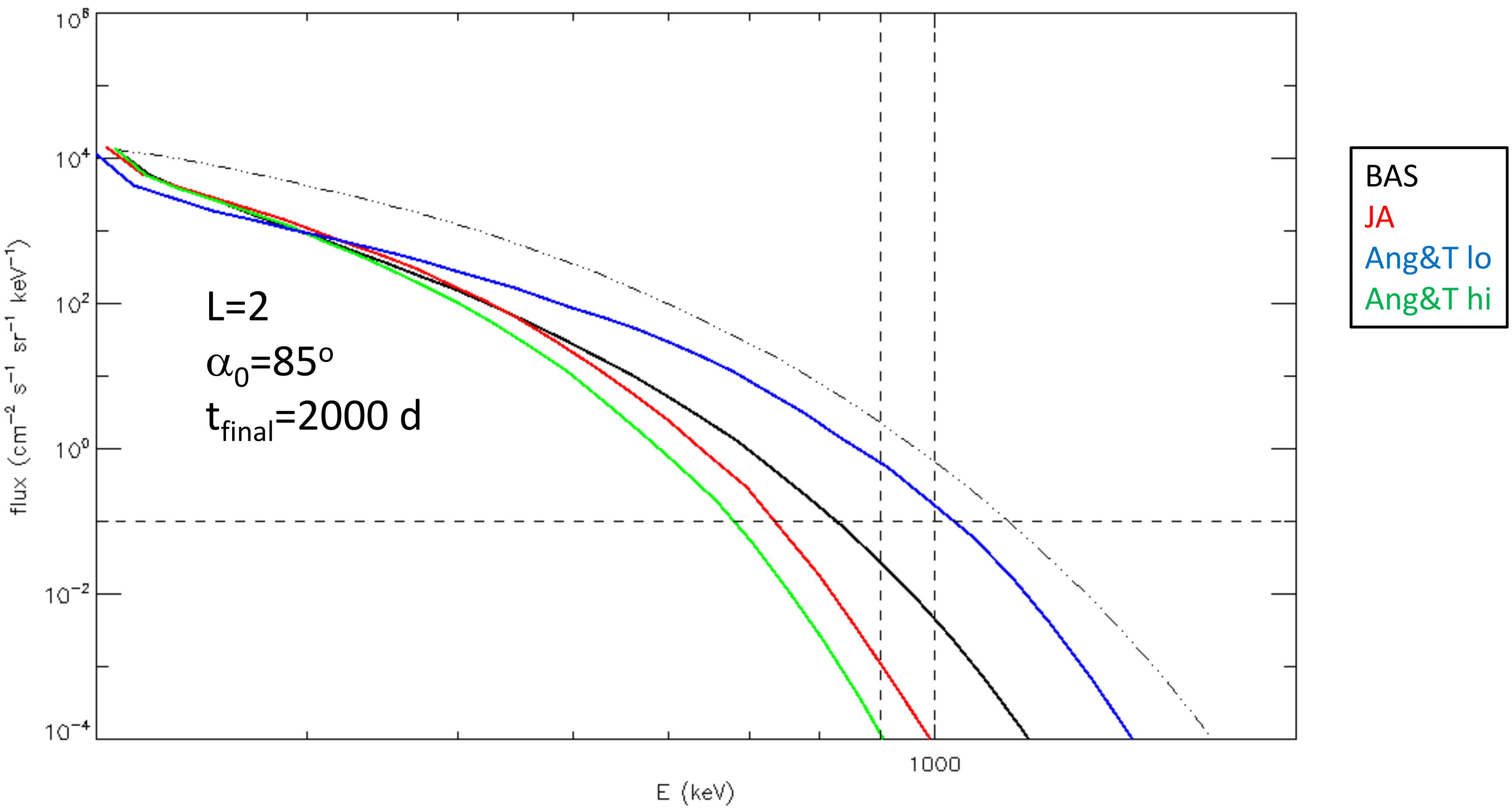


(Fennell et al., 2015).

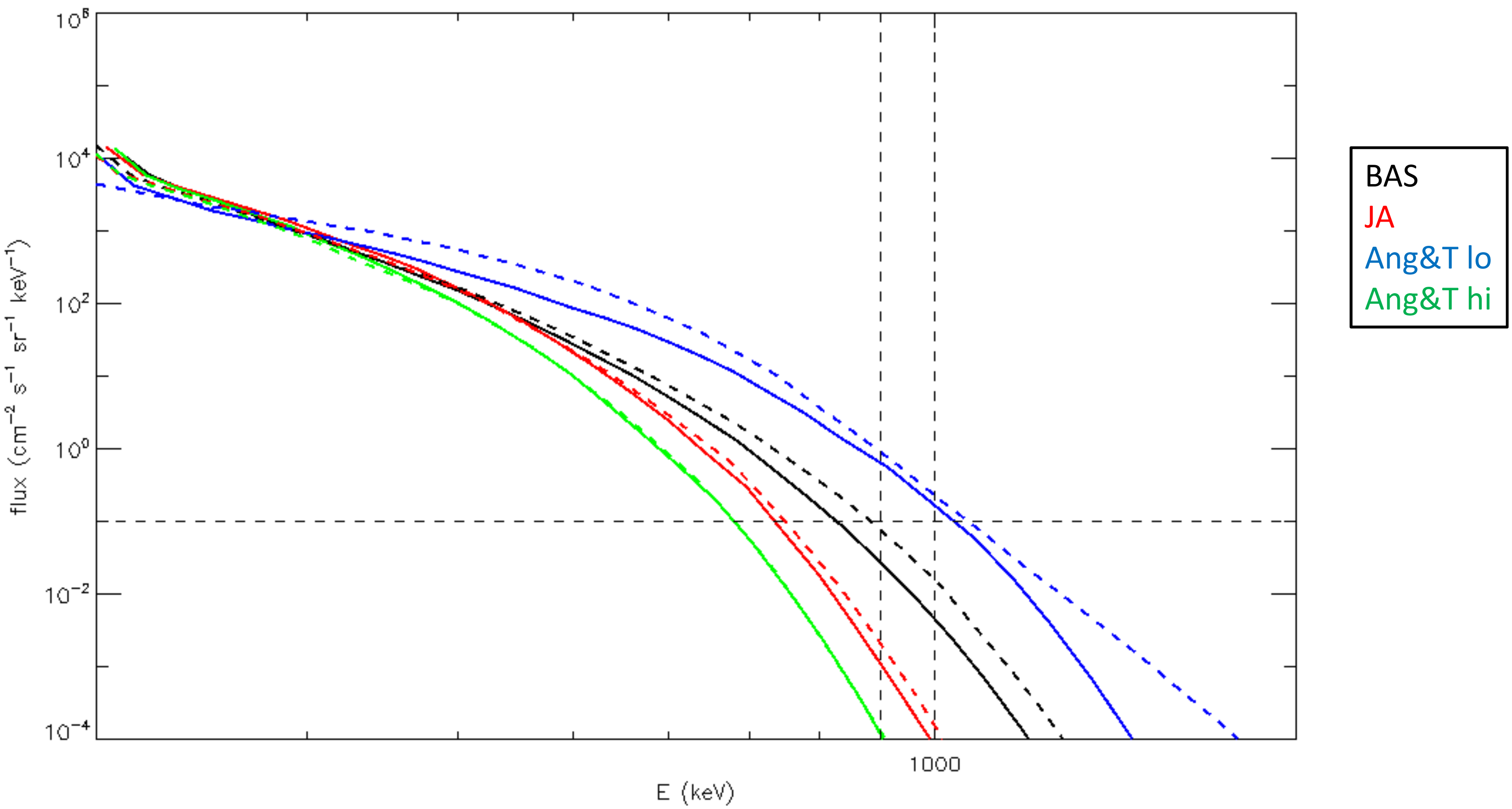


C.f. j = 0.1 at 900 keV

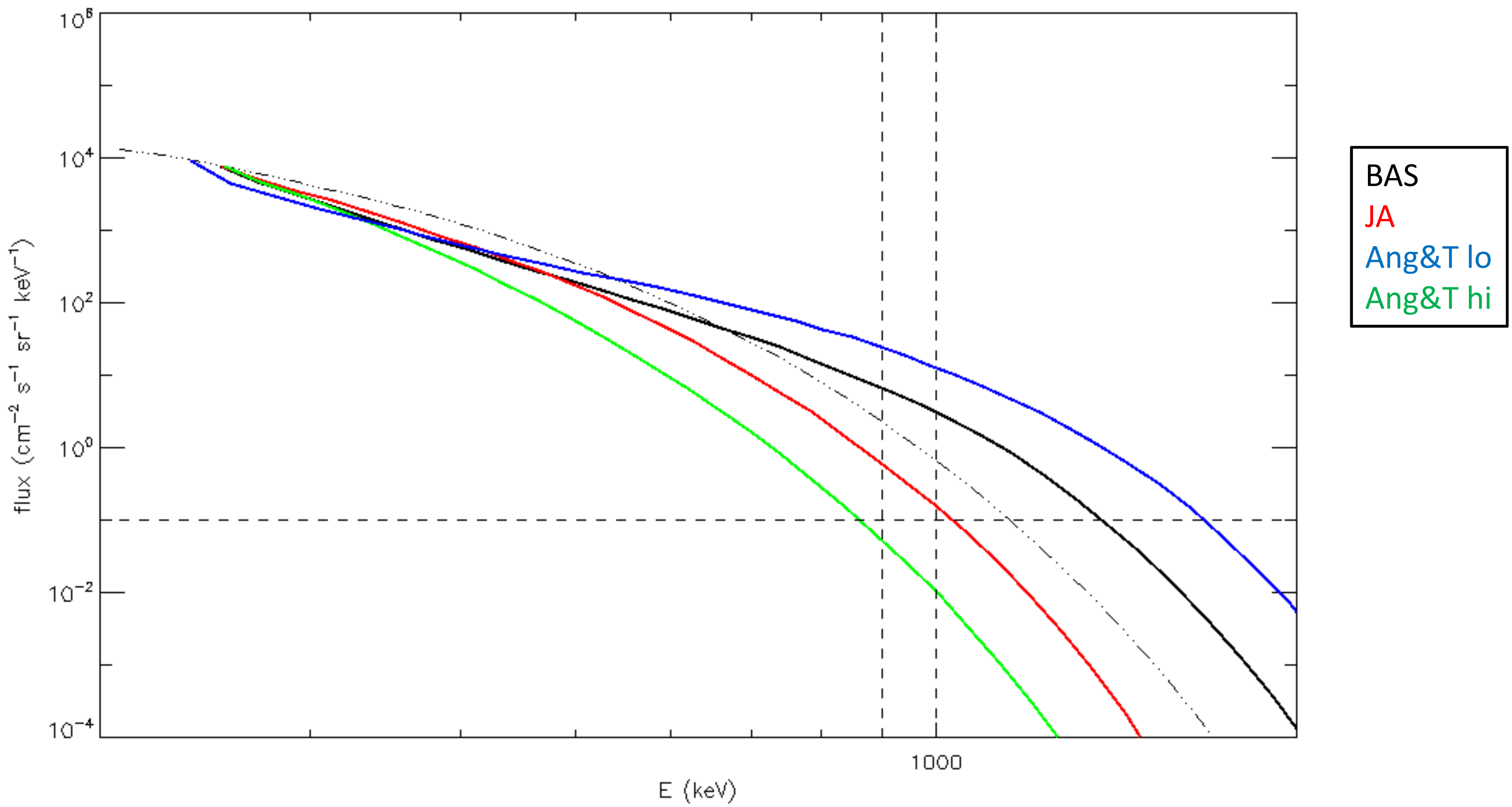
TX unducted, no MS



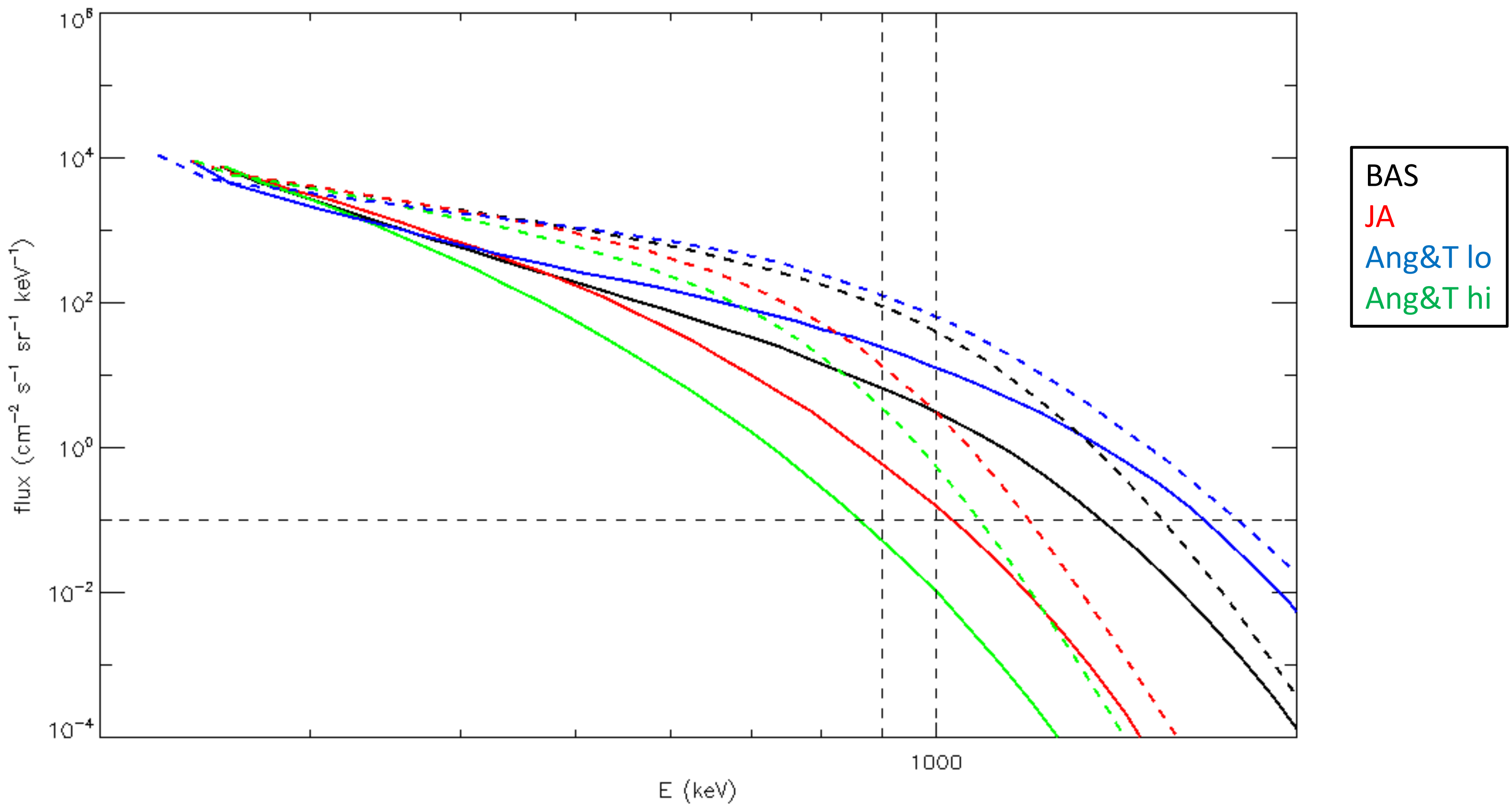
TX unducted + MS (--)



TX ducted, no MS



TX ducted + MS (--)



10^6
10^4
10^2
10^0
10^{-2}
10^{-4}
flux (cm^{-2} s^{-1} sr^{-1} keV^{-1})
1000
E (keV)
BAS
JA
Ang&T lo
Ang&T hi

Conclusions

After changing the n_e model, the BAS and UCLA hiss models, with a model for D_{TX}, give similar results at L=2.

Butterfly PADs tend to develop for all models tested, except very high n_e (Sheeley). MS waves make them stronger and faster. With or w/o MS, the mechanism is heating at large α_0 due to n=0.

With unducted TX waves and high or moderate n_e, the 2D steady state gives reasonable fluxes both < and > 1 MeV. With ducted TX, need high n_e (or weaker wave heating), especially with MS.

Main point: even at L=2, wave heating (and not just by MS) is non-negligible. It goes w/o saying that L transport should be included too.