A solar wind-parameterised, probabilistic model of ULF waves in Earth's magnetosphere

S. N. Bentley[1],C. E. J. Watt[1], M. J. Owens[1] and I. J. Rae[2]

s.bentley@pgr.reading.ac.uk

1 Department of Meteorology, University of Reading

2 Mullard Space Science Laboratory, UCL

Goals

Current goal:

An empirical, probabilistic model of ULF wave power, with

- Azimuthal (magnetic local time) resolution
- Dependence on solar wind conditions, rather than *Kp*

Such a model could be used to investigate the physics and to improve radiation belt modelling with more precise radial diffusion coefficients.

- Including uncertainty in components of D_{LL} would allow us to identify model areas requiring improvements
  - e.g. magnetic field model, power in equatorial electric and magnetic fields

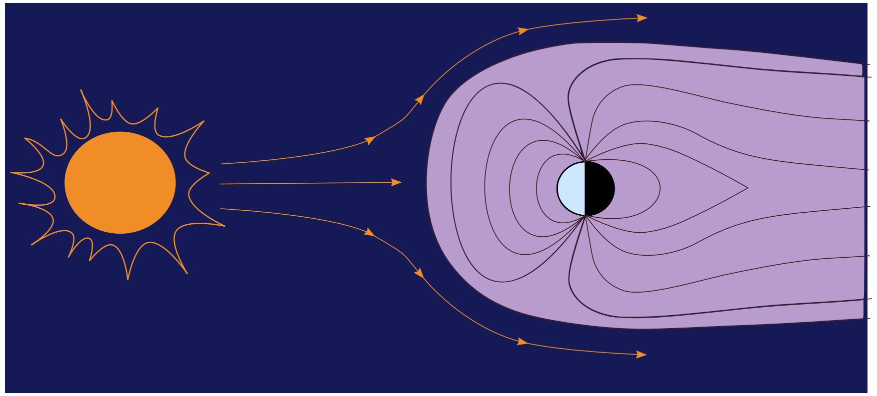
Data

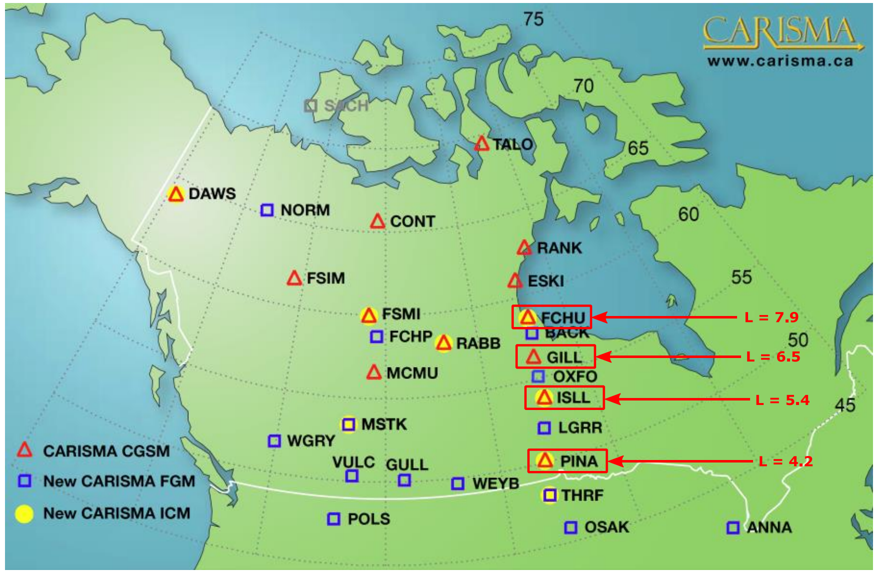
For 1990-2005;

- Solar wind: Hourly OMNI data
  - ➢ Measured at L1 and propagated to the magnetosphere.

- ULF waves: CANOPUS/CARISMA
  - ➢ Power spectral density (PSD) is calculated in hour-long windows
  - ➢ Signatures of ULF waves can be mapped up to get equatorial E and hence D^{E}_{LL} [1] (although this will mean more uncertainty)

These large data sets are well suited to statistical analyses across a large span of the magnetosphere.





[1] Ozeke et al, 2009

The question: solar wind driving of ULF waves

Issues to consider:

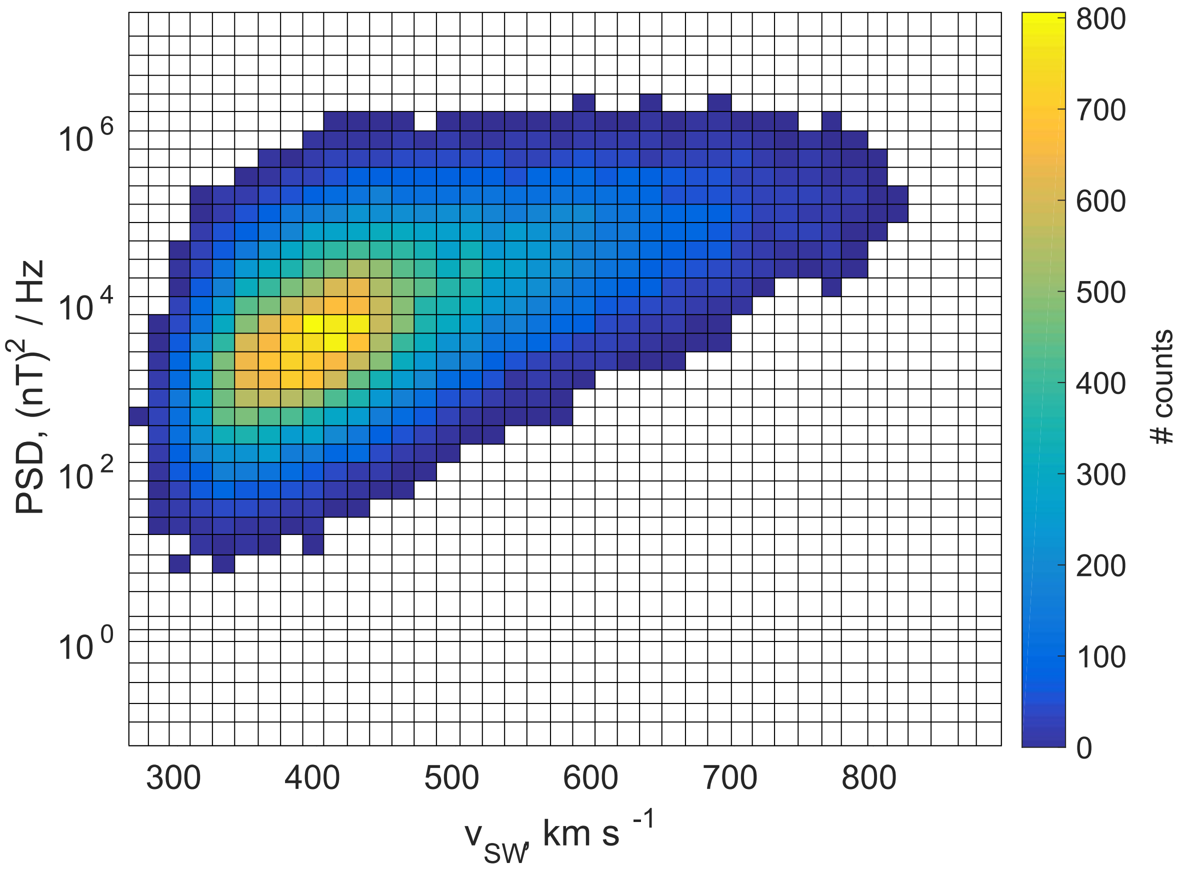
- Data reduction
- Huge parameter space
- Uneven data distributions
- Interdependence of parameters
    - Implicit (e.g. similar solar sources) and explicit (e.g. psw = nv^2)
    - Correlation vs causality

The big ones!

The question: solar wind driving of ULF waves

Issues to consider:

- Data reduction
- Huge parameter space
- Uneven data distributions
- Interdependence of parameters
    - Implicit (e.g. similar solar sources) and explicit (e.g. psw = nv^2)
    - Correlation vs causality



[2] Intensity map showing occurrence of power spectral density and solar wind speed. There is a lot more data at lower speeds.

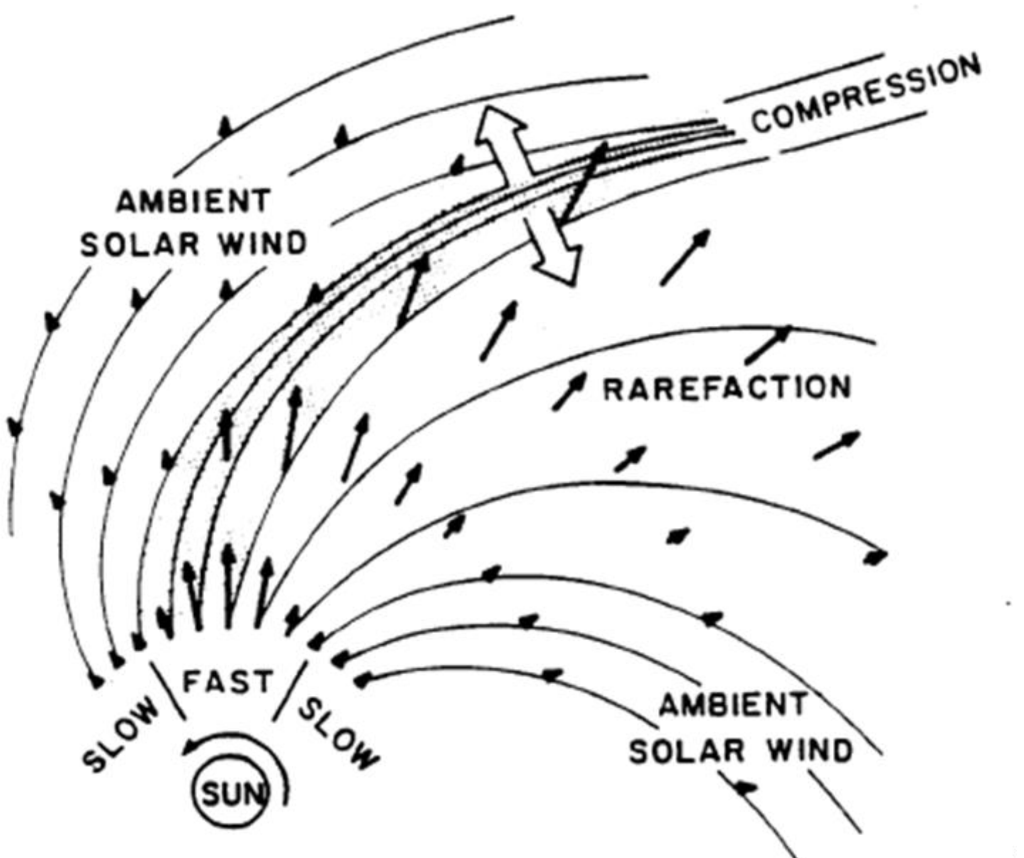
[2] Bentley at al, 2018

Interdependence examples

- Solar wind speed v_{sw} and proton number density *Np* anticorrelate due to the nature of fast/slow solar wind
- Magnetic field orientation, temperature in CMEs
- Sudden changes in magnetic field orientation and Np in compression regions and shock fronts
- Wave processes

Most solar wind properties correlate strongly with speed $v_{sw,}$ because this describes the type of solar wind.

Compression and rarefaction regions of the solar wind.



Pizzo, 1978

Returning to the question: how does the solar wind determine ULF wave power?

We want an analysis technique that still allows us to investigate the physics.

- Input and output must be physical quantities

Interdependence and uneven data make this difficult...

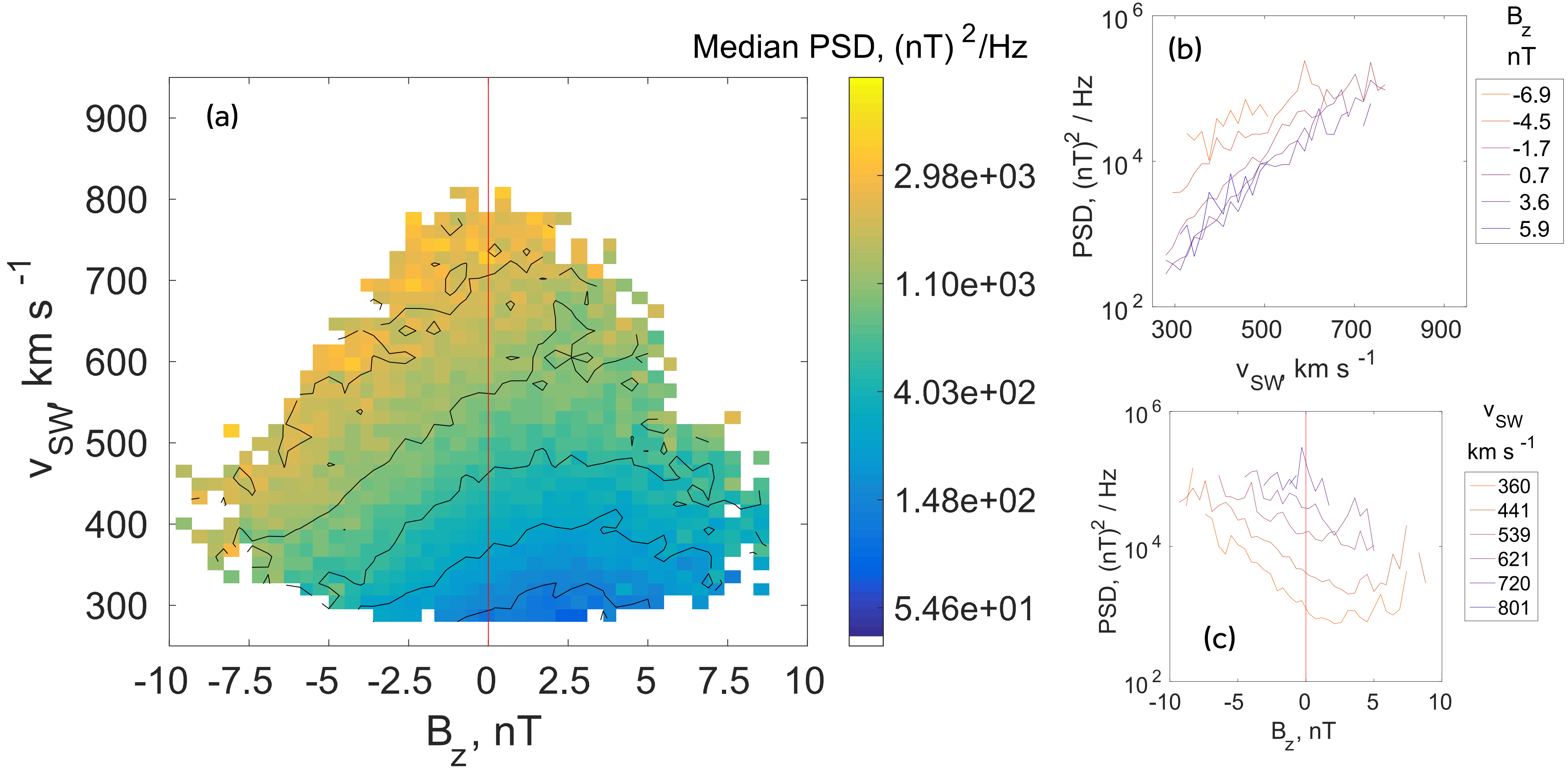
- Uneven data can be solved using conditional probability
- Many analysis techniques require assumptions about interdependence, such as
    1. Input variables are independent
    2. The relationship between any two variables is linear
    3. The relationship between any two variables is consistent, i.e. smooth

Solution

→ Go for the simple, systematic solution!

- What parameters are causal?
- Use more sophisticated techniques when I know which assumptions are valid

Example – how to compare the contribution to ULF power of two solar wind parameters



[2] Median ULF wave power at a single frequency (2.5mHz) at a single magnetometer station (GILL, ~6.6 R_E) changes with v_{sw} and southward interplanetary magnetic field Bz<0. Cut-throughs are shown in (b) and (c).

[2] Bentley at al, 2018

Determining solar wind driving - results

After systematically comparing all available solar wind parameters to identify which have causal correlations to ULF power, we find the following parameters[2] :

- Solar wind speed v_{sw}
- Southward interplanetary magnetic field, $Bz < 0$
- Variance in proton number density $var(Np)$

(and there was at least one non-smooth interparameter relationship!)

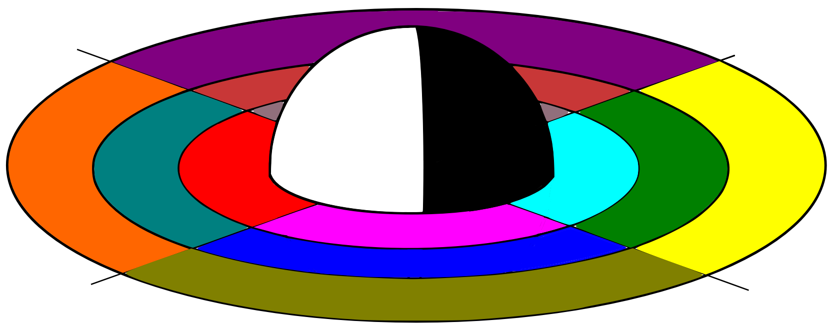
[2] Bentley at al, 2018

Turning this into a model: overview

We want to be able to characterise ULF wave power using these three causal parameters (v_{sw}, *Bz, var(Np))*.

Requirements:

- MLT resolution
- Radial resolution
- Multiple frequencies
- *Bz* </> 0



In each region, parameterise ULF power for frequencies 1-20 mHz using v_{sw}, Bz, var(Np).

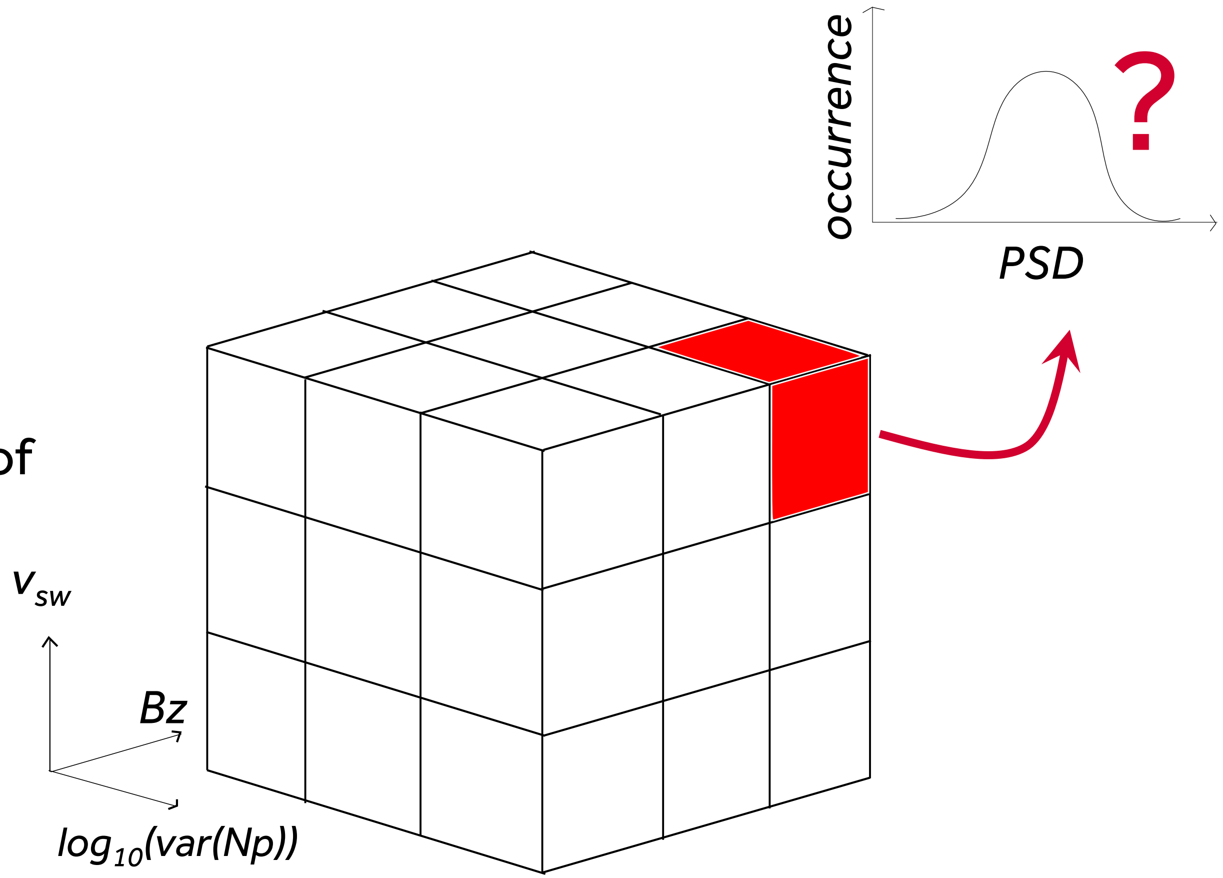
Since these are already discretised in data, do separate models for each physical region and MLT, with *Bz* <0 and *Bz*>0 separately..

Turning this into a model – example of one parameterisation

Take one mini-model: one station, MLT sector, frequency

How can we parameterise?

- Bin by three parameters v_{sw}, *Bz, var(Np)*
- This essentially gives you a 3d grid of bins
- In each bin, find the distribution of values of ULF wave power observed at for this station/frequency/MLT combination



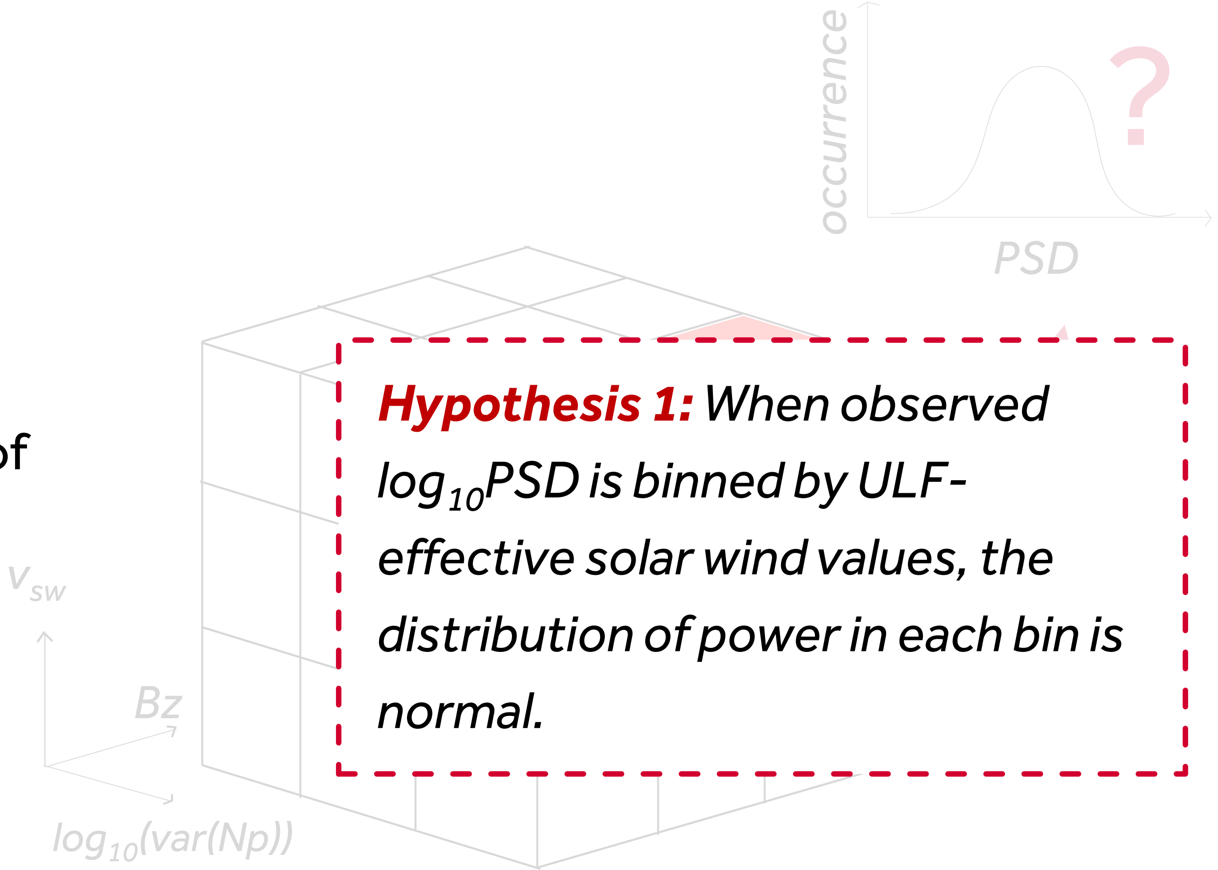
Characterisation of ULF waves using solar wind variables (Bentley, Chapman 2018)

Turning this into a model – example of one parameterisation

Take one mini-model: one station, MLT sector, frequency

How can we parameterise?

- Bin by three parameters v_{sw}, *Bz, var(Np)*
- This essentially gives you a 3d grid of bins
- In each bin, find the distribution of values of ULF wave power observed for this station/frequency/MLT combination

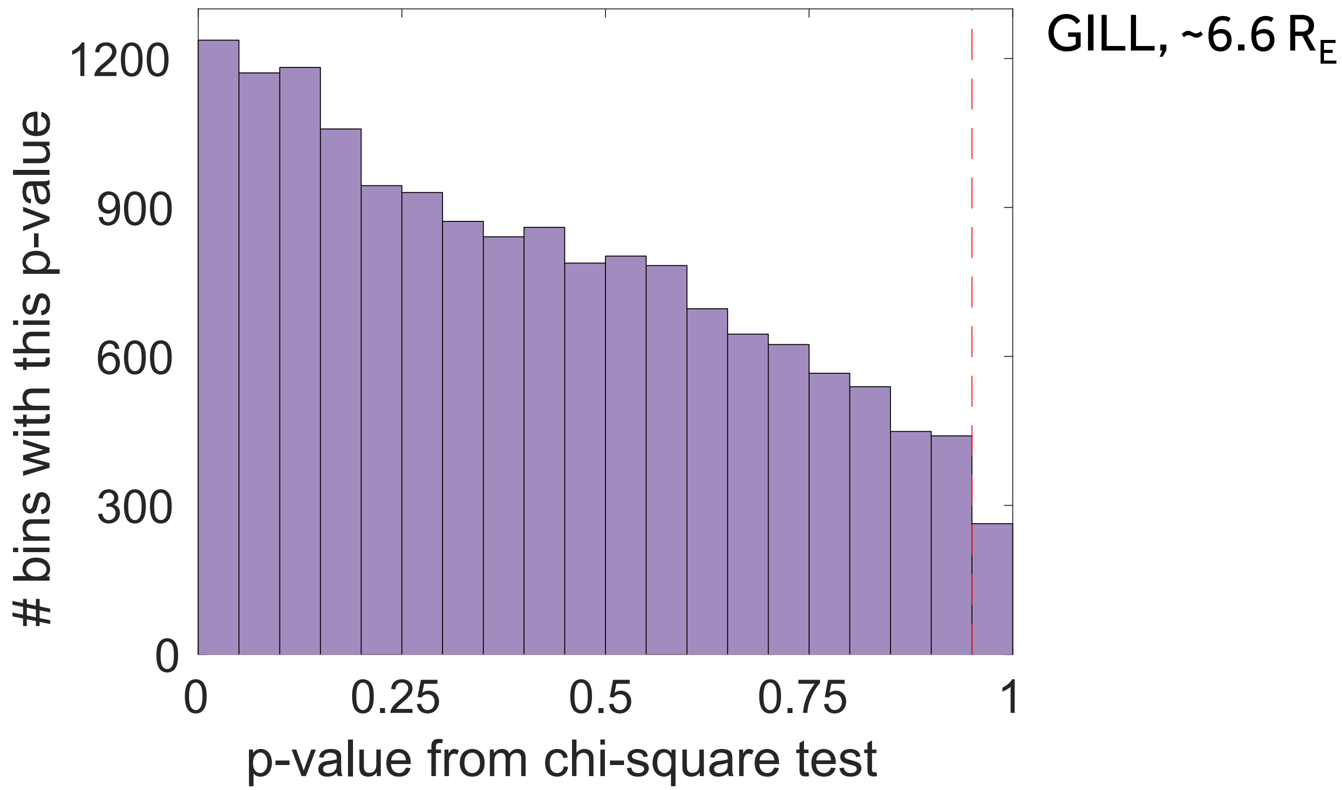


Hypothesis 1: *When observed log_{10}PSD is binned by ULF-effective solar wind values, the distribution of power in each bin is normal.*

Characterisation of ULF waves using solar wind variables (Bentley, Chapman 2018)

Testing lognormal hypothesis

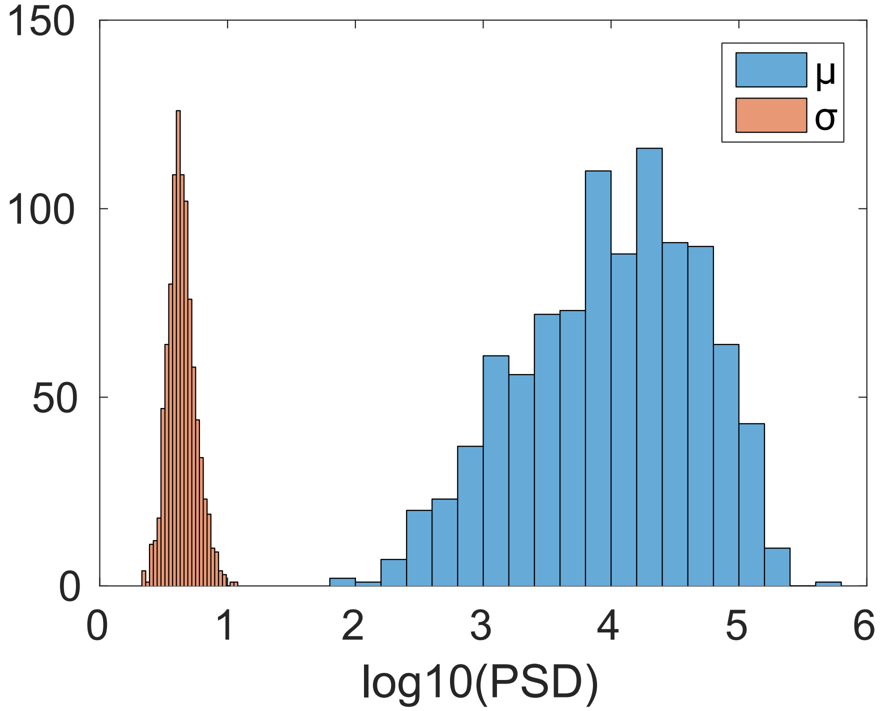
Test this:



Chi-square p-values, testing whether distributions of $\log_{10}$(power) in each bin is normal (in bins with enough data). If $p > 0.95$ in a bin we can reject the lognormal hypothesis for that bin.

Using normal distributions to describe $\log_{10}$power

- Do normal distributions appear to be a good approximation? Yes
- For each distribution, is the mean significantly larger than the variance? Yes – and the variance stays relatively constant



Histogram of the mean and variance of the normal distribution for every bin at GILL, 2.5mHz, all MLT sectors

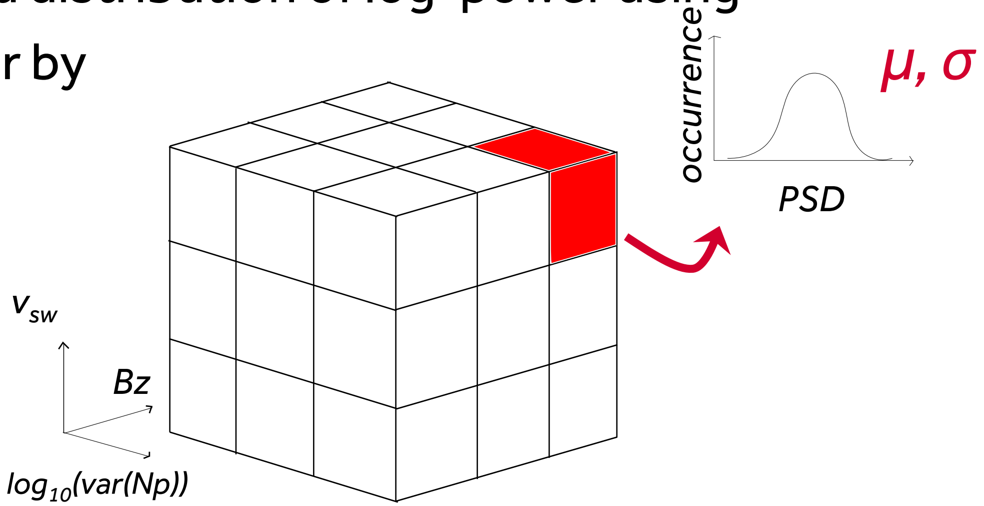
Constructing an empirical model

Hypothesis 2: *log_{10}(PSD) at any station or frequency follows (and can be predicted by) a family of normal distributions whose properties (mu, sigma) are parameterised by (v_{sw}, Bz, var(Np)).*

Example of such a parameterisation for one mini-model:

Use observed values of v_{sw}, *Bz, var(Np)* to identify the expected distribution of log-power using the look-up table. Use this distribution to predict power, either by

1. Sampling it
2. Using the mean value



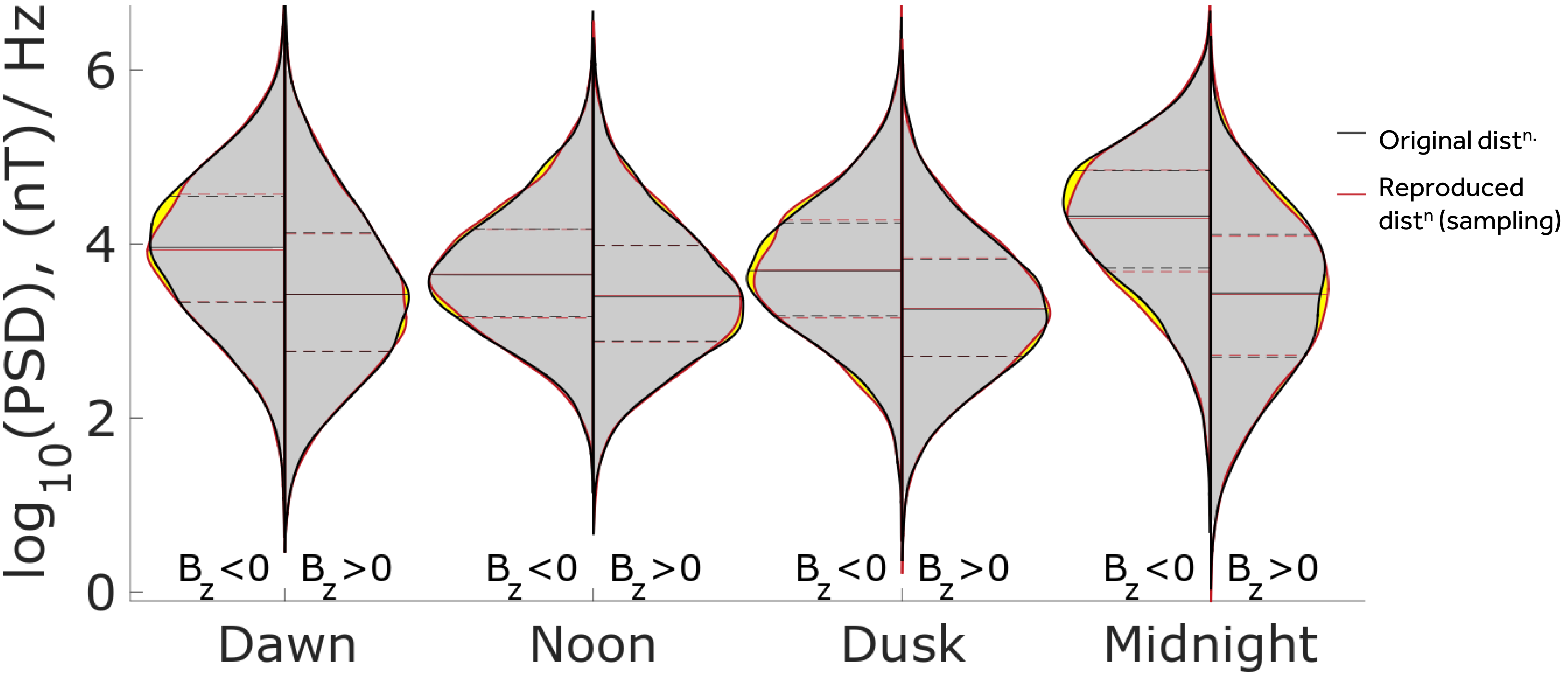
Characterisation of ULF waves using solar wind variables (Bentley, Chapman 2018)

Testing an empirical model

Choose tests appropriate to expected model uses:

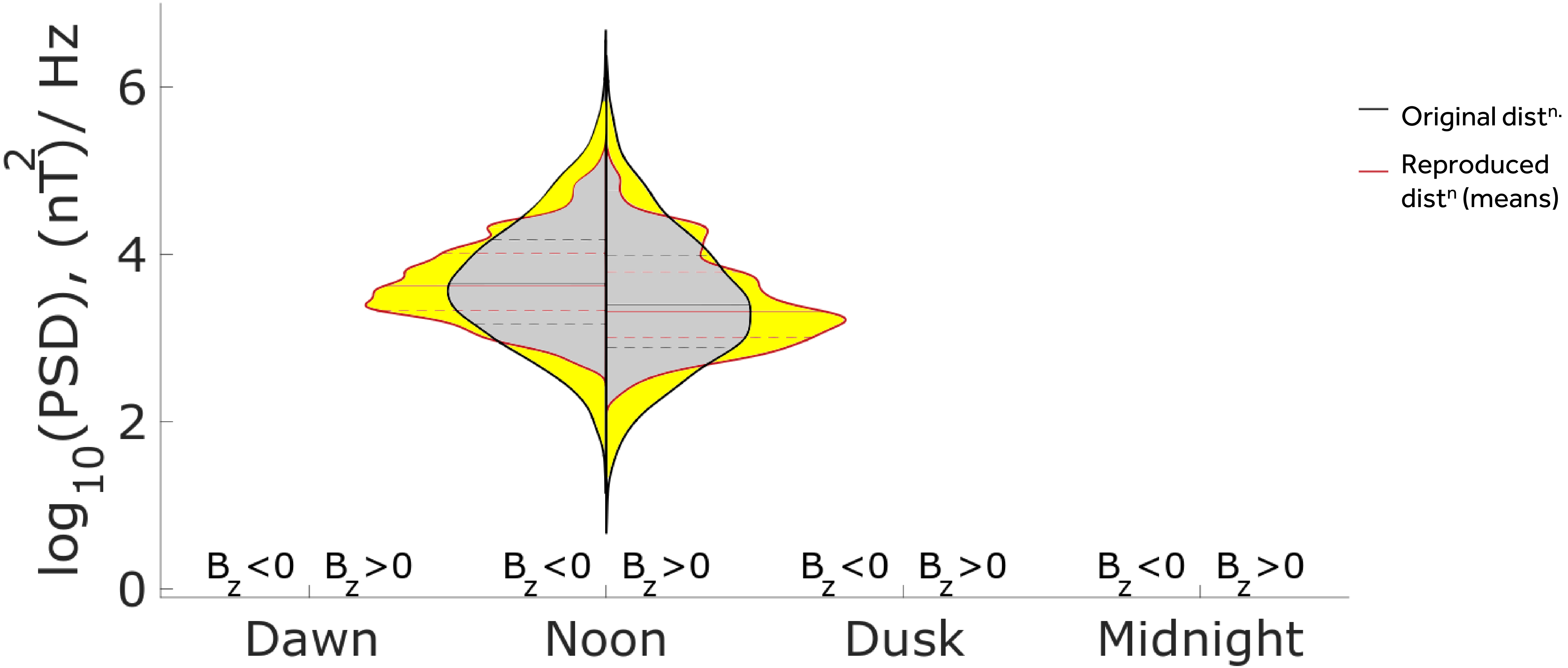
- Distribution of power over an event
- Predicting average power in oncoming hour, compared to existing models

GILL, 2.5 mHz



Violin plots showing the original vs. reproduced power distribution. Discrepancies between the distributions are highlighted yellow.

GILL, 2.5 mHz



Similarly, original distribution compared to reproduced using only means from the look-up tables.

Testing an empirical model: the oncoming hour

Test reproductions for each hour using forecasting skill:

$$Skill = 100\left(1 - \frac{MSE_{model}}{MSE_{ref}}\right)$$

Forecast skill scores can range from -∞ to 100; positive values indicate that the tested model is better than a reference model.

Testing an empirical model: the oncoming hour

Test reproductions for each hour using forecasting skill:

$$Skill = 100\left(1 - \frac{MSE_{model}}{MSE_{ref}}\right)$$

Forecast skill scores can range from -∞ to 100; positive values indicate that the tested model is better than a reference model.

Reference model	Skill (means only)	Skill (sampling)
"Random"	77.51	28.02
24 h persistence	65.73	-9.79
1 h persistence	23.32	-145.70

Output is compared against three reference models, a "random" model sampling the original power distribution and to 24 and 1-hour persistence.

Summary

Predicting the effect of solar wind on magnetospheric ULF waves:

- We found three causal parameters
- These can be used to parameterise ULF power in different physical regions of the magnetosphere via look-up tables
- These parameterisations predict ground-measured ULF power quite well

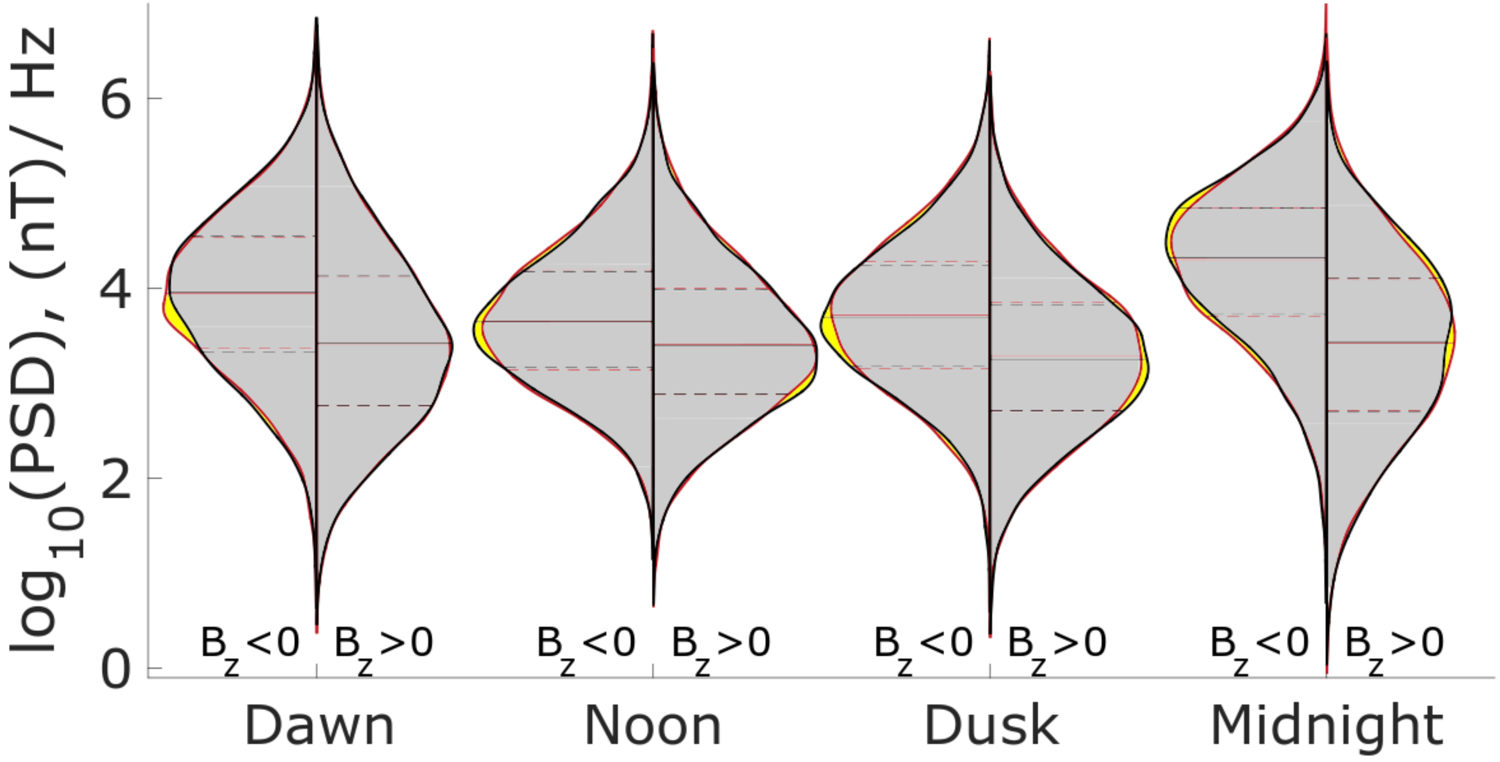
Future work

- Tailor tests
- Analyse these families of lognormals to get physics insights.
- Map ground power to magnetospheric power and calculate their diffusion coefficients

Testing an empirical model CARISMA Jan 2015

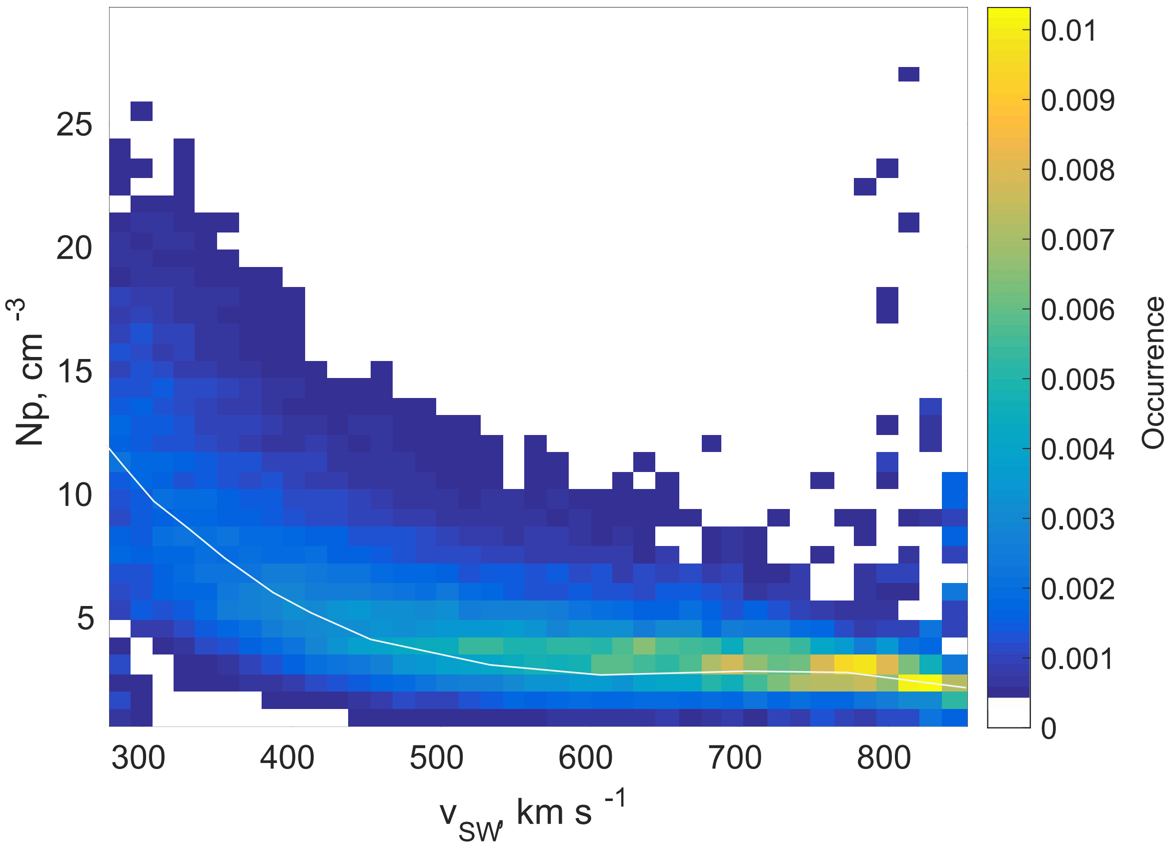
Reference model	Skill (sampling)
"Random"	73.38
24 h persistence	51.14
1 h persistence	11.17

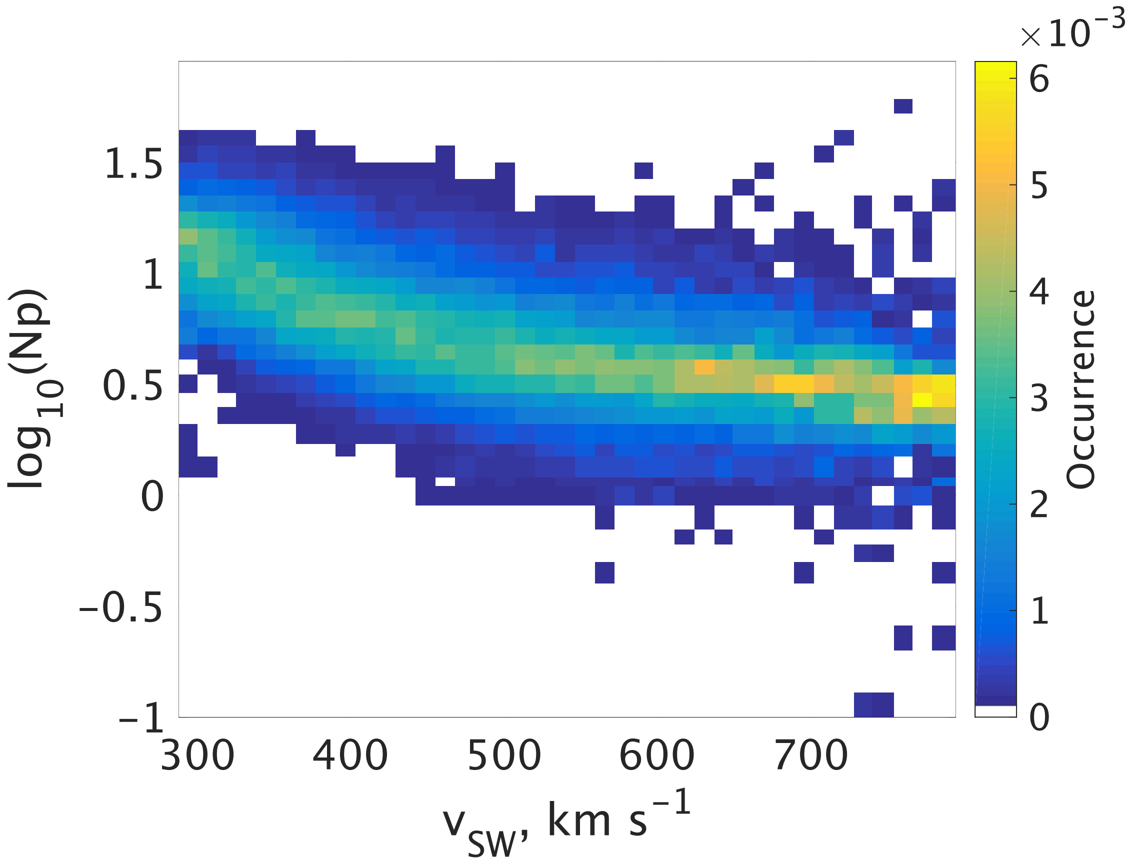
GILL, 2.5 mHz



Characterisation of ULF waves using solar wind variables (Bentley, Chapman 2018)

Interdependence example





Characterisation of ULF waves using solar wind variables (Bentley, Chapman 2018)

Interpreting test results: is it any good?

Test 1: *(reproducing the original distribution)*

Sampling from each distribution gives you a more accurate representation of power over a long period of time. Just taking the mean of the distribution does not.

Test 2: *(predicting the power in the next hour)*

Taking the mean value predicts power in the next hour better than assuming power just persists, and better than sampling.

[1] Horne et al, 2013, *Space weather impacts on satellites and forecasting the Earth's radiation belts with SPACECAST*, Space Weather
[2] Murphy et al 2016, *Accurately characterizing the importance of wave-particle interactions in radiation belt dynamics: The pitfalls of statistical wave representations,* JGR-Space Physics
[3] Ozeke et al 2009, *Mapping guided Alfvén wave magnetic field amplitudes observed on the ground to equatorial electric field amplitudes in space,* JGR
[4] Bentley et al, 2018, *ULF wave activity in the magnetosphere: resolving solar wind interdependencies to identify driving mechanisms,* JGR-Space Physics