Estimating and Validating Diffusion Coefficients with Particle Data

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Introduction

• Many processes in the radiation belts can be described through diffusion
• Diffusion coefficients are usually derived from waves, with large uncertainties due to unknown spatial structure
• Multiple, contradictory simulations seem to reproduce large-scale and small-scale radiation belt features
• We advocate particular uses of particle data to validate and even infer diffusion coefficients

• Method I – constraining $D_{\alpha\alpha}$ with eigen-mode analysis
• Method II – Estimating Inner Zone $D_{LL}$ from radial dynamics
• Method III – Constraining Outer Zone $D_{LL}$ from drift phase structure
Method I – constraining $D_{\alpha\alpha}$ with eigen-mode analysis

- Pitch-angle diffusion is faster than other diffusions (E,L)
- Thus, the pitch-angle distribution should quickly remove any short-lived eigemodes of the pitch-angle diffusion operator
- We can project the observed $f(\alpha)$ onto the eigenmode basis set derived from an hypothesized $D_{\alpha\alpha}$ to determine if $f(\alpha)$ and $D_{\alpha\alpha}$ are mutually consistent
- We can even tune $D_{\alpha\alpha}$ to fit $f(\alpha)$

- The eigenmode $v_i$ satisfies: \[
\frac{1}{xT(y)} \frac{\partial}{\partial x} \left( xT(y) D_{xx} \frac{\partial v_i}{\partial x} \right) = \frac{v_i}{\tau_i} \]
- The PSD evolves as: \[
f(x, t) = \sum_i f_i v_i(x) \exp(-t/\tau_i) \]
- We compute $f_i$ from: \[
f_i = \int_0^1 f(x) v_i(x) xT(y) dx \]
- In the absence of other processes, the observed distribution should mostly be made up of $v_i$ with long $\tau_i$, i.e., $f_i$ should be larger in long $\tau_i$ modes

$D_{aa}$ for multiple wave modes

- Consider chorus alone, or with “plume” waves: EMIC+Hiss
- At L=4.5, 1 MeV
- With the plume waves, a sharp gradient forms around 60°.
- This is a telltale of EMIC waves
In all 7 examples from CRRES MEA, we see that adding “Plume” waves (EMIC+Hiss) reduces the projection of the PAD into $f_1$, which has the longest lifetime.

This suggests that in these cases, there is little EMIC wave activity anywhere on the drift orbit.

This approach can be used to constraint the drift-averaged waves, from a single spin of a spacecraft like CRRES, RBSP, ARASE, etc.
Method I – Summary

• Eigenmode analysis of pitch-angle diffusion provides a tool for constraining drift-average wave parameters from a single angular distribution, even during active times.

• In the example shown, EMIC waves assumed in plumes were likely not present in 7 CRRES passes through L~4.5 [O’Brien, et al., JASTP (2008), doi:10.1016/j.jastp.2008.05.011]

• For quiet times, the pitch angle distribution likely converges to the first eigenmode, making it possible to directly compute $D_{\alpha \alpha}$. [O’Brien, et al. (2014), GRL, doi:10.1002/2013GL058713.]
Method II – Estimating Inner Zone $D_{LL}$ from $f(L,t)$

- Estimated inner zone $D_{LL}$ from MagEIS data, $L<3.5$
  - Quiet times only (by virtue of long term averaging)
  - Allowing for pitch angle scattering
  - Rudimentary CRAND
  - Neglecting energy transport ($D_{EE}$): storm times, outside plasmapause

- Using modified 1st invariant ($\zeta$) and integrating over pitch angle
  - Obtain 1-D diffusion equation in “bundle content” at fixed $\zeta$
  - Solving/integrating radial diffusion equation for $D_{LL}$

- Requires estimate of decay time
  - Difficult at some $L$
  - Modest effect on $D_{LL}$

- Results agree well with Electrostatic $D_{LL}$ from Schulz 1991 and Brautigam and Albert 2000
  - Too high for electromagnetic-only DLL, e.g., Ozeke et al., 2014

**Mathematical Framework**

- Modified first invariant $\zeta = \frac{M}{y^2} = \frac{p^2 L^3}{2m_0 B_0}$
  - All particles on the field line have same $\zeta$, equal to $M$ for equatorially mirroring particles
  - $\zeta$ approximately invariant to radial diffusion at fixed $M,K$
- Using “Bundle Content”
  - $n(\zeta, L) = \int_0^{\infty} f(\zeta, x, L) x T(y) dx$
  - Integrates over equatorial pitch angle, weighted by density of states
  - Invariant to pitch angle diffusion, except flow into loss cone (boundary flux)
  - $nL \propto$ flux tube content
- The Diffusion Equation is then:
  - $\frac{\partial n}{\partial t} = L^2 \frac{\partial}{\partial L} \left[ \frac{D_{LL}}{L^2} \frac{\partial n}{\partial L} \right] = \frac{n}{\tau} + \bar{S}$
  - Assumes pitch angle diffusion coefficient and gradient at edge of loss cone depend only on $L$ and $\zeta$ and not $t$
  - Angle-averaged radial diffusion $\bar{D}_{LL}$ and CRAND source $\bar{S}$
Derivation of $D_{LL}$

- Start with radial and pitch angle diffusion:
  \[
  \frac{\partial \tilde{f}}{\partial t} = \frac{1}{xT(y)} \frac{\partial}{\partial x} \left[ xT(y)D_{xx} \frac{\partial \tilde{f}}{\partial x} \right] + L^2 \frac{\partial}{\partial L} \left[ \frac{D_{LL}}{L^2} \frac{\partial \tilde{f}}{\partial L} \right] + S
  \]

- Integrate $xT(y)dx$:
  
  \[\frac{\partial n}{\partial t} = \int_{x_{LC}} xT(y)D_{xx} \frac{\partial \tilde{f}}{\partial x} |_{\xi, L} \, dx\]

- Assume constant gradient at edge of loss cone:
  
  \[\left[ xT(y)D_{xx} \frac{\partial \tilde{f}}{\partial x} |_{\xi, L} \right]_{x_{LC}} = -\frac{n}{\tau}\]

- Solve for $D_{LL}$:
  
  \[D_{LL} = \left( \frac{\partial n}{\partial L} \right)^{-1} L^2 \left[ \frac{\partial n}{\partial t} L - \frac{\partial n}{\partial L} \right] \int_{L_1}^{L} \frac{1}{L} \left[ \frac{\partial n}{\partial L} \right] + \left( \frac{\partial n}{\partial L} \right)^{-1} L^2 \left[ \frac{\partial n}{\partial t} L - \n + \tilde{S} \right] L^{-\frac{5}{2}} dL\]

  - For each $\zeta$, set $L_1$ to be where $n(L_1)=0$, assume $D_{LL}(L_1) = 0$
**$D_{LL}$ vs $L$**

- No $D_{LL}$ for $\zeta=25$, $L<1.7$ due to low counts/background
- $D_{LL}$ for $\zeta=1,2$ ends at $L=2.4,3$ due to energy dropping below MagEIS range
- Knees in $D_{LL}$ for $\zeta=1,2$ occur at < 70 keV
  - *Instrumental? Assumptions failing?*
- $D_{LL}$ matches *Schulz* [1991] and *Brautigam and Albert* [2000] estimates
  - *Dominated by electrostatic, $L^6$*
  - *Electromagnetic ($L^{10}$) negligible at low $L$*
Conclusion for Method - II

• Inner zone $D_{LL}$ estimates are consistent with prior fits
  – Schulz [1991]
  – Brautigam and Albert [2000]

• Some simulations use only electromagnetic $D_{LL}$, e.g., Ozeke et al., [2014]
  – Dominant in the outer zone
  – Negligible in inner zone

• Simulators should include electrostatic $D_{LL}$ for inner zone studies

• Concepts like “Bundle Content” allow us to reduce dimension of the system:
  specify boundary fluxes rather than PSD over extra dimension
Method III – Estimating Outer Zone $D_{LL}$ from drift phase structure

• Radial transport produces transient drift phase structures
• These drift phase structures scale with $D_{LL}$.

$$D_{LL} \approx \frac{\langle (f - \langle f \rangle_d)^2 \rangle_d}{2T_d} \left( \frac{df}{dL} \right)_{M,K}^{-2}$$

• $\langle (f - \langle f \rangle_d)^2 \rangle_d = \text{variance in PSD over a drift period}$
• $T_d = \text{drift period}$
• A more particular derivation is given in Schulz and Lanzerotti, 1974, eq. 4.17

We can estimate the fluctuation amplitude $\sqrt{\langle (f - \langle f \rangle_d)^2 \rangle_d}$ from $D_{LL}$, and compare to observed wiggles in the detrended time series

$$\langle (f - \langle f \rangle_d)^2 \rangle_d \approx 2T_d D_{LL} \left( \frac{df}{dL} \right)_{M,K}^2$$
**Example Storm: May/June 2013**

- Here we examine the May/June, 2013 storm with ~1 MeV electrons from MagEIS

- The Dst index reaches -100 nT
- The pre-storm flux is high
- There is a dropout followed by a small enhancement on June 1st
- A second enhancement happens mainly at higher L later on the 2\textsuperscript{nd}
- We will examine this second enhancement for drift phase structure
**Example Storm: May/June 2013**

- While the outer zone flux is increasing at L>4.5…
- There is very little drift phase structure (no wiggles)
- The wiggles that are there are generally much smaller than expected from $D_{LL}$ and $df/dL$.

- If radial diffusion is playing a role, it’s doing it without drift phase structure.
- This rules out diffusion primarily by:
  - A sequence of impulses
  - Narrow-band drift resonance
- What’s left?
- Diffusion by broadband power with random phase (quasilinear radial diffusion)
Summary

- The particles can constrain or even provide the diffusion coefficients
- Pitch-angle distributions provide strong constraints on $D_{\alpha\alpha}$
- Inner zone electron $D_{LL}$ estimate from observed radial profiles are consistent with prior fits
  - *Simulations that extend to the inner zone should include electrostatic $D_{LL}$*
- Outer zone drift phase flux variations are not consistently present during flux enhancements
  - *Either there are many tiny radial perturbations (quasilinear $D_{LL}$)*
  - *Or $D_{LL}$ is way too large*
BACKUPS
Time Series of Bundle Content

- **Fast variations**
  - *Only affects highest Ls*
  - *More dramatic at higher energy (ζ)*
  - *Caused by storms*

- **Slow variations**
  - *Pitch angle scattering into atmosphere (~exponential decay)*
  - *Radial transport (undulations)*

- **Compute** $\frac{dn}{dL}$ and $\frac{dn}{dt}$ on weekly timescales
First Increase (Step 1 - no wiggles)

- The left panels show detrended MagEIS data in several energies near 1 MeV.
- The right panels provide L profiles (no detrending) over the day.
- The green L profile is the one plotted in the adjacent time series.
- These are very smooth profiles for L>4.
- An increase without significant drift phase structure.
First Increase (Step two with wiggles) and small drop out

- Here we have a dropout
- There are wiggles!
- The wiggles are sometimes associated with changes in B, other times not

- This could be radial diffusion pushing particles outward toward the magnetopause, contributing to the dropout