Estimating and Validating Diffusion Coefficients with Particle Data

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Introduction

- Many processes in the radiation belts can be described through diffusion
- Diffusion coefficients are usually derived from waves, with large uncertainties due to unknown spatial structure
- Multiple, contradictory simulations seem to reproduce large-scale and small-scale radiation belt features
- We advocate particular uses of particle data to validate and even infer diffusion coefficients

- Method I – constraining $D_{\alpha\alpha}$ with eigen-mode analysis
- Method II – Estimating Inner Zone D_{LL} from radial dynamics
- Method III – Constraining Outer Zone D_{LL} from drift phase structure

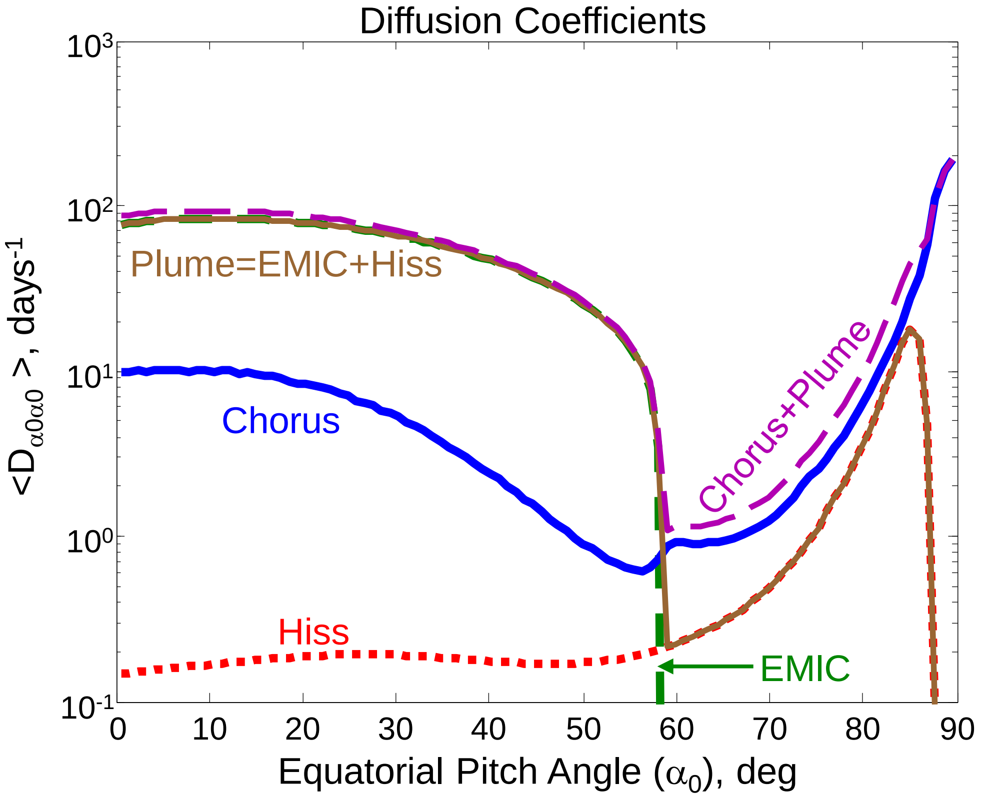
Method I – constraining $D_{\alpha\alpha}$ with eigen-mode analysis

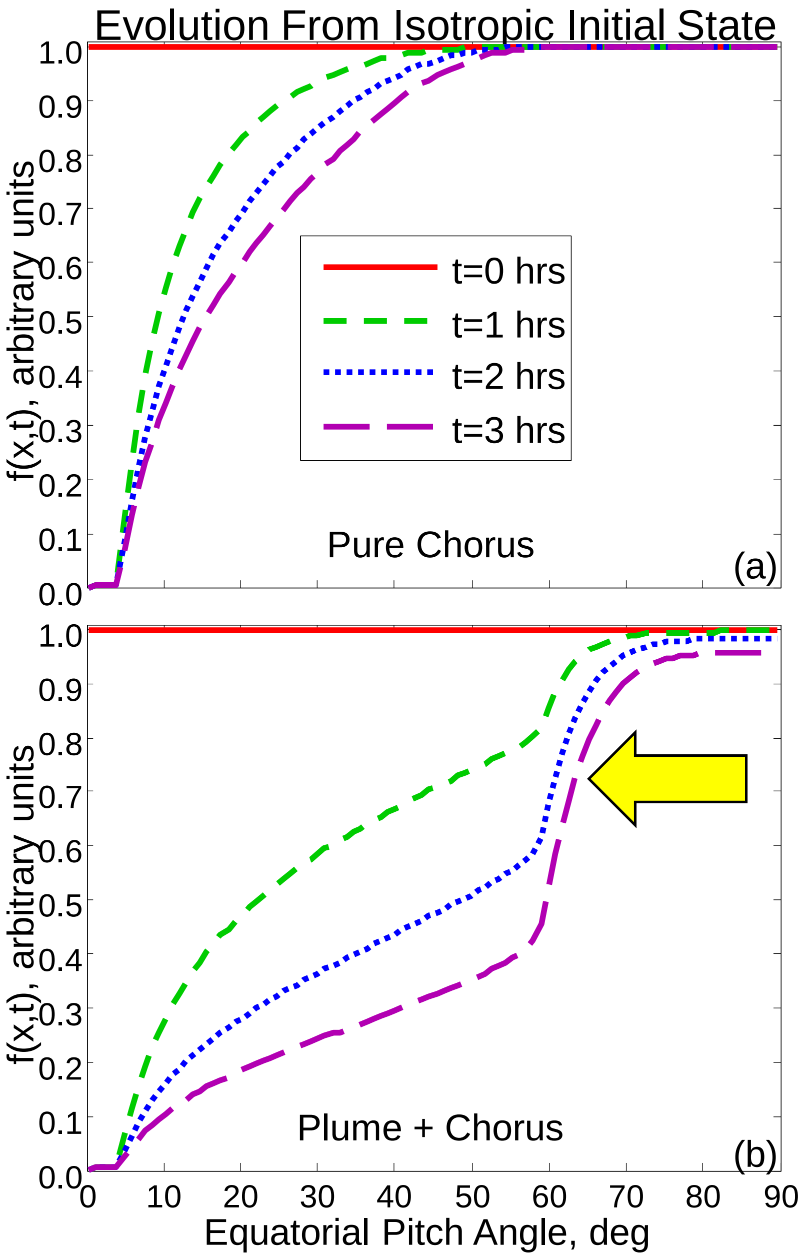
- Pitch-angle diffusion is faster than other diffusions (E,L)
- Thus, the pitch-angle distribution should quickly remove any short-lived eignemodes of the pitch-angle diffusion operator
- We can project the observed $f(\alpha)$ onto the eigenmode basis set derived from an hypothesized $D_{\alpha\alpha}$ to determine if $f(\alpha)$ and $D_{\alpha\alpha}$ are mutually consistent
- We can even tune $D_{\alpha\alpha}$ to fit $f(\alpha)$

- The eigenmode v_i satisfies : $\frac{1}{xT(y)}\frac{\partial}{\partial x}\left(xT(y)D_{xx}\frac{\partial v_i}{\partial x}\right) = \frac{v_i}{\tau_i}$
- The PSD evolves as: $f(x,t) = \sum_i f_i v_i(x) \exp(-t/\tau_i)$
- We compute f_i from: $f_i = \int_0^1 f(x) v_i(x) xT(y) dx$
- In the absence of other processes, the observed distribution should mostly be made up of v_i with long τ_i, i.e., f_i should be larger in long τ_i modes

- See O’Brien, et al., JASTP (2008), doi:10.1016/j.jastp.2008.05.011

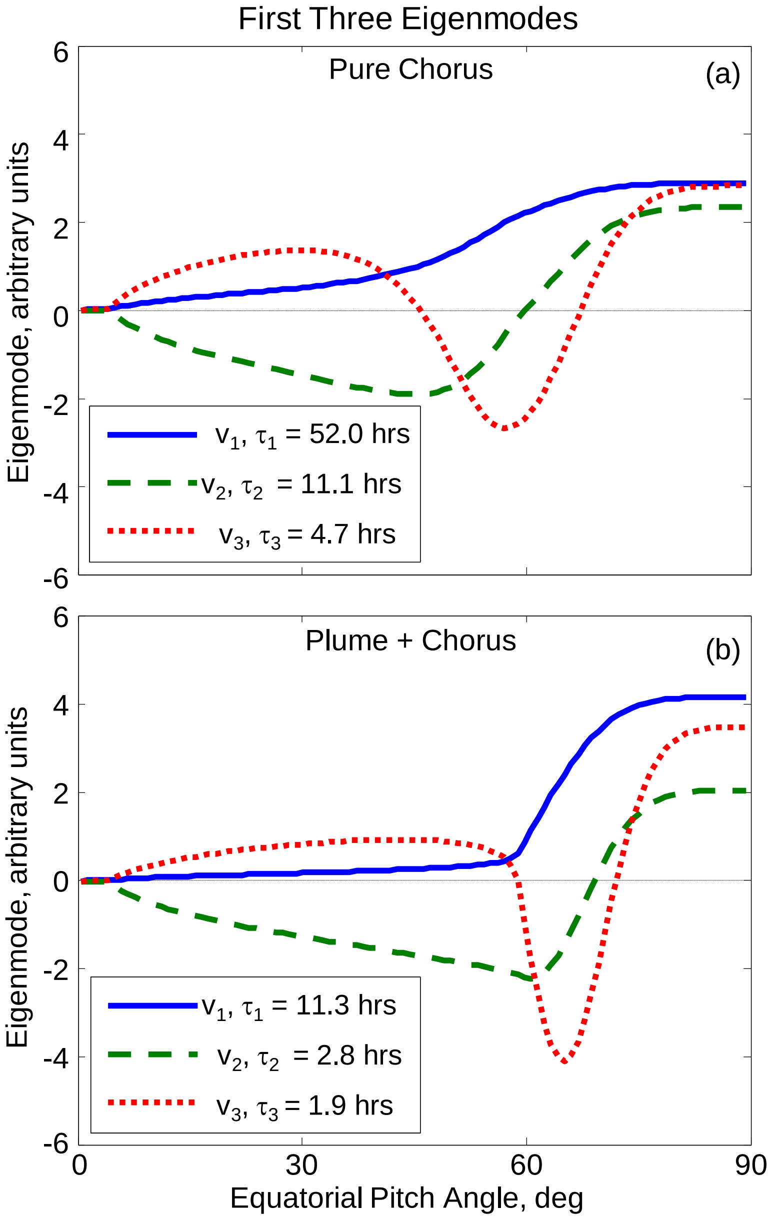
D_{aa} for multiple wave modes

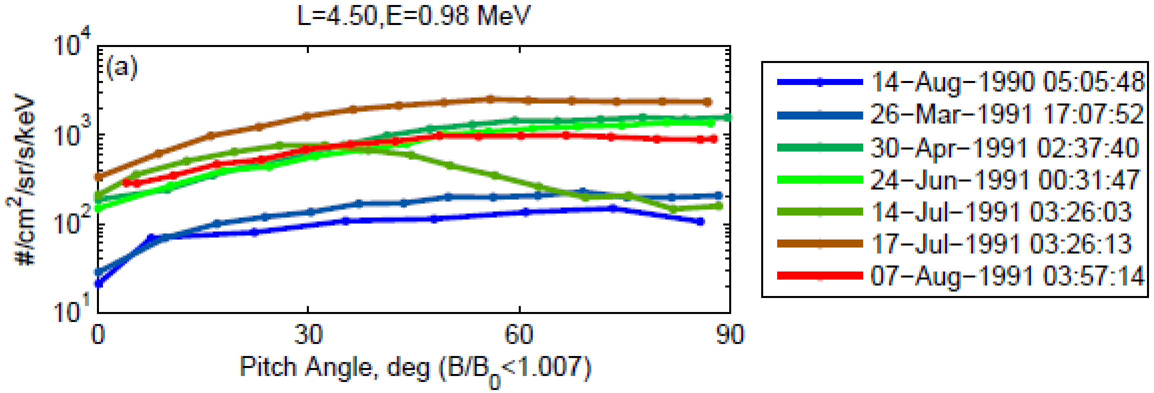


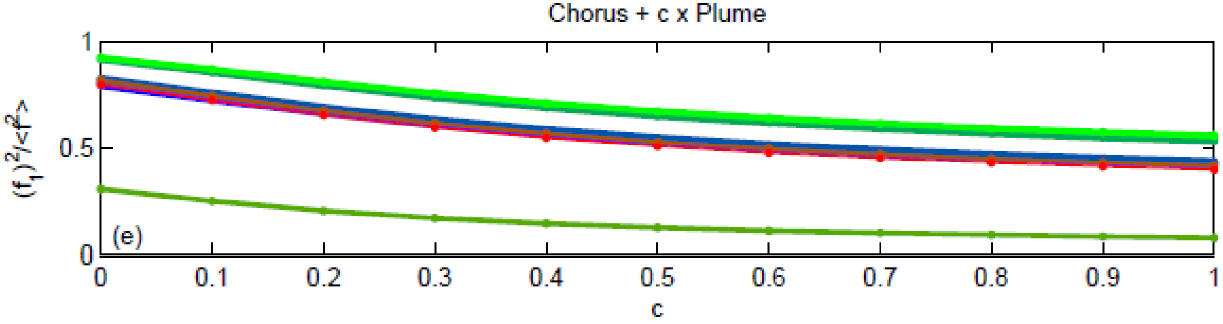


- Consider chorus alone, or with “plume” waves: EMIC+Hiss
- At L=4.5, 1 MeV
- With the plume waves, a sharp gradient forms around 60^o.
- This is a telltale of EMIC waves

Eigenmode analysis







- In all 7 examples from CRRES MEA, we see that adding “Plume” waves (EMIC+Hiss) reduces the projection of the PAD into f_1, which has the longest lifetime.
- This suggests that in these cases, there is little EMIC wave activity *anywhere on the drift orbit*.
- This approach can be used to constraint the drift-averaged waves, from a single spin of a spacecraft like CRRES, RBSP, ARASE, etc.

Method I – Summary

- Eigenmode analysis of pitch-angle diffusion provides a tool for constraining drift-average wave parameters from a single angular distribution, even during active times.

- In the example shown, EMIC waves assumed in plumes were likely not present in 7 CRRES passes through L~4.5 [O'Brien, et al., JASTP (2008), doi:10.1016/j.jastp.2008.05.011]

- For quiet times, the pitch angle distribution likely converges to the first eigenmode, making it possible to directly compute $D\alpha\alpha$. [O'Brien, et al. (2014), GRL, doi:10.1002/2013GL058713.]

Method II – Estimating Inner Zone D_{LL} from f(L,t)

- Estimated inner zone D_{LL} from MagEIS data, L<3.5
  - *Quiet times only (by virtue of long term averaging)*
  - *Allowing for pitch angle scattering*
  - *Rudimentary CRAND*
  - *Neglecting energy transport (D_{EE}): storm times, outside plasmapause*
- Using modified 1st invariant (ζ) and integrating over pitch angle
  - *Obtain 1-D diffusion equation in "bundle content" at fixed ζ*
  - *Solving/integrating radial diffusion equation for D_{LL}*
- Requires estimate of decay time
  - *Difficult at some L*
  - *Modest effect on D_{LL}*
- Results agree well with Electrostatic D_{LL} from *Schulz* 1991 and *Brautigam and Albert* 2000
  - *Too high for electromagnetic-only DLL, e.g., Ozeke et al., 2014*

- See: O'Brien et al., (2016), GRL, doi:10.1002/2016GL069749

Mathematical Framework

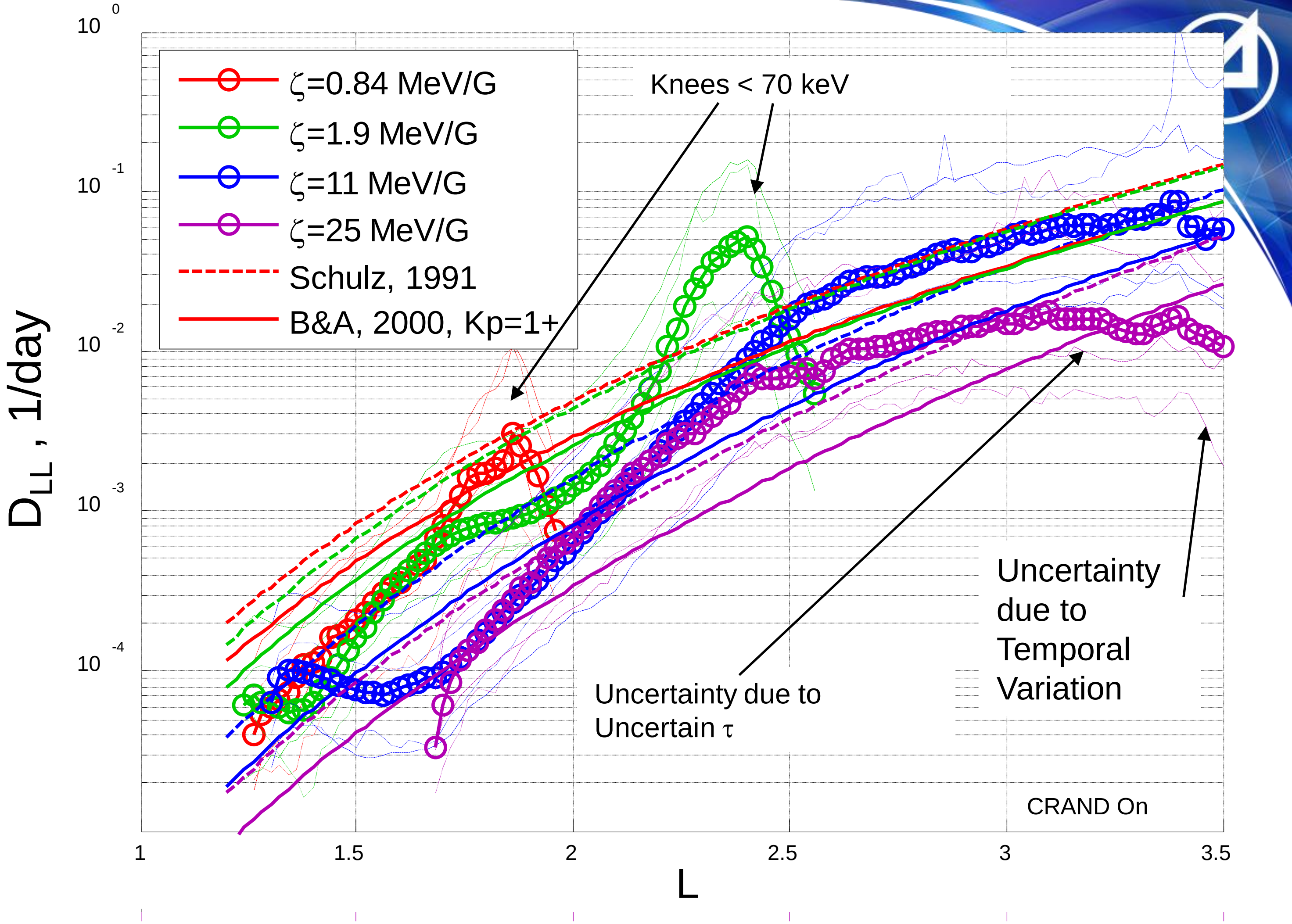
- Modified first invariant $\zeta = \frac{M}{y^2} = \frac{p^2 L^3}{2 m_0 B_0}$
  - *All particles on the field line have same ζ, equal to M for equatorially mirroring particles*
  - *ζ approximately invariant to radial diffusion at fixed M,K*
- Using "Bundle Content"
  - $n(\zeta, L) = \int_0^{x_{LC}} f(\zeta, x, L) x T(y) dx$
  - *Integrates over equatorial pitch angle, weighted by density of states*
  - *Invariant to pitch angle diffusion, except flow into loss cone (boundary flux)*
  - *$nL \propto$ flux tube content*
- The Diffusion Equation is then:
  - $\frac{\partial n}{\partial t} = L^{\frac{5}{2}} \frac{\partial}{\partial L}\Big|_\zeta \left[\frac{\bar{D}_{LL}}{L^{\frac{5}{2}}} \frac{\partial n}{\partial L}\Big|_\zeta \right] - \frac{n}{\tau} + \bar{S}$
  - *Assumes pitch angle diffusion coefficient and gradient at edge of loss cone depend only on L and ζ and not t*
  - *Angle-averaged radial diffusion $\bar{D}_{LL}$ and CRAND source $\bar{S}$*

Derivation of D_{LL}

- Start with radial and pitch angle diffusion: $\frac{\partial \bar{f}}{\partial t} = \frac{1}{xT(y)}\frac{\partial}{\partial x}\left[xT(y)D_{xx}\frac{\partial \bar{f}}{\partial x}\right] + L^{\frac{5}{2}}\frac{\partial}{\partial L}\left[\frac{D_{LL}}{L^{\frac{5}{2}}}\frac{\partial \bar{f}}{\partial L}\right] + S$
- Integrate xT(y)dx:
  - $n(\zeta, L) = \int_0^{x_{LC}} \bar{f}(\zeta, x, L)xT(y)dx$
  - $\frac{\partial n}{\partial t} = \left[xT(y)D_{xx}\frac{\partial \bar{f}}{\partial x}\Big|_{\zeta,L}\right]_{x_{LC}} + L^{\frac{5}{2}}\frac{\partial}{\partial L}\Big|_{\zeta}\left[\frac{\bar{D}_{LL}}{L^{\frac{5}{2}}}\frac{\partial n}{\partial L}\Big|_{\zeta}\right] + \bar{S}$
- Assume constant gradient at edge of loss cone:
  - $\left[xT(y)D_{xx}\frac{\partial \bar{f}}{\partial x}\Big|_{\zeta,L}\right]_{x_{LC}} = -\frac{n}{\tau}$
  - $\frac{\partial n}{\partial t} = L^{\frac{5}{2}}\frac{\partial}{\partial L}\Big|_{\zeta}\left[\frac{\bar{D}_{LL}}{L^{\frac{5}{2}}}\frac{\partial n}{\partial L}\Big|_{\zeta}\right] - \frac{n}{\tau} + \bar{S}$
- Solve for D_{LL}: $\bar{D}_{LL} = \left(\frac{\partial n}{\partial L}\Big|_{\zeta}\right)^{-1} L^{\frac{5}{2}} \int_{L_1}^{L}\left[\frac{\partial n}{\partial t} + \frac{n}{\tau} - \bar{S}\right] L^{-\frac{5}{2}} dL$
  - *For each ζ, set L_1 to be where $n(L_1)$=0, assume $D_{LL}(L_1)$ = 0*

D_{LL} vs L

- No D_{LL} for ζ=25, L<1.7 due to low counts/background
- D_{LL} for ζ=1,2 ends at L=2.4, 3 due to energy dropping below MagEIS range
- Knees in D_{LL} for ζ=1,2 occur at < 70 keV
  - *Instrumental? Assumptions failing?*
- D_{LL} matches *Schulz* [1991] and *Brautigam and Albert* [2000] estimates
  - *Dominated by electrostatic, L^6*
  - *Electromagnetic (L^{10}) negligible at low L*



2330 keV	1094	602	361	231	155
1390 keV	609	314	179	110	71
426 keV	155	71	37	22	14
214 keV	72	31	16	10	6

Conclusion for Method - II

- Inner zone D_{LL} estimates are consistent with prior fits
  - *Schulz [1991]*
  - *Brautigam and Albert [2000]*
- Some simulations use only electromagnetic D_{LL}, e.g., *Ozeke et al.*, [2014]
  - *Dominant in the outer zone*
  - *Negligible in inner zone*
- Simulators should include electrostatic D_{LL} for inner zone studies
- Concepts like “Bundle Content” allow us to reduce dimension of the system: specify boundary fluxes rather than PSD over extra dimension

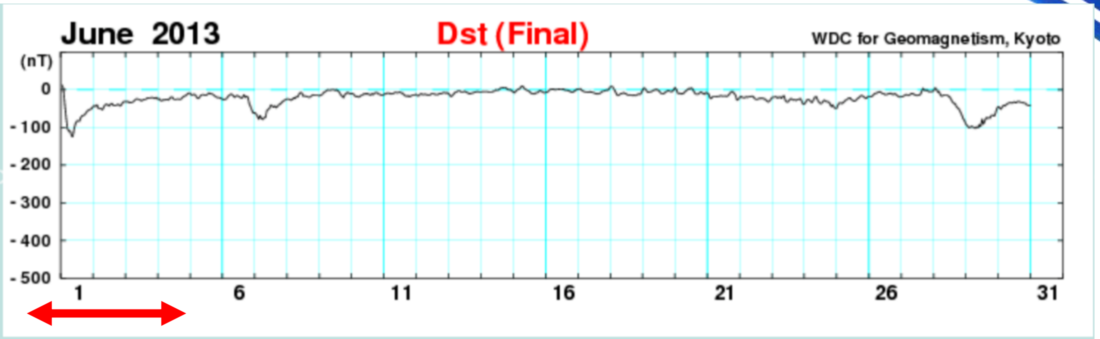
Method III – Estimating Outer Zone D_{LL} from drift phase structure

- Radial transport produces transient drift phase structures
- These drift phase structures scale with D_{LL}.
  - $D_{LL} \approx \frac{\langle (f - \langle f \rangle_d)^2 \rangle_d}{2T_d} \left(\frac{df}{dL}\right)_{M,K}^{-2}$
  - $\langle (f - \langle f \rangle_d)^2 \rangle_d$ = *variance in PSD over a drift period*
  - T_d = drift period
  - A more particular derivation is given in *Schulz and Lanzerotti*, 1974, eq. 4.17

  - We can estimate the fluctuation amplitude $\sqrt{\langle (f - \langle f \rangle_d)^2 \rangle_d}$ from D_{LL}, and compare to observed wiggles in the detrended time series
  - $\langle (f - \langle f \rangle_d)^2 \rangle_d \approx 2T_d D_{LL} \left(\frac{df}{dL}\right)_{M,K}^{2}$

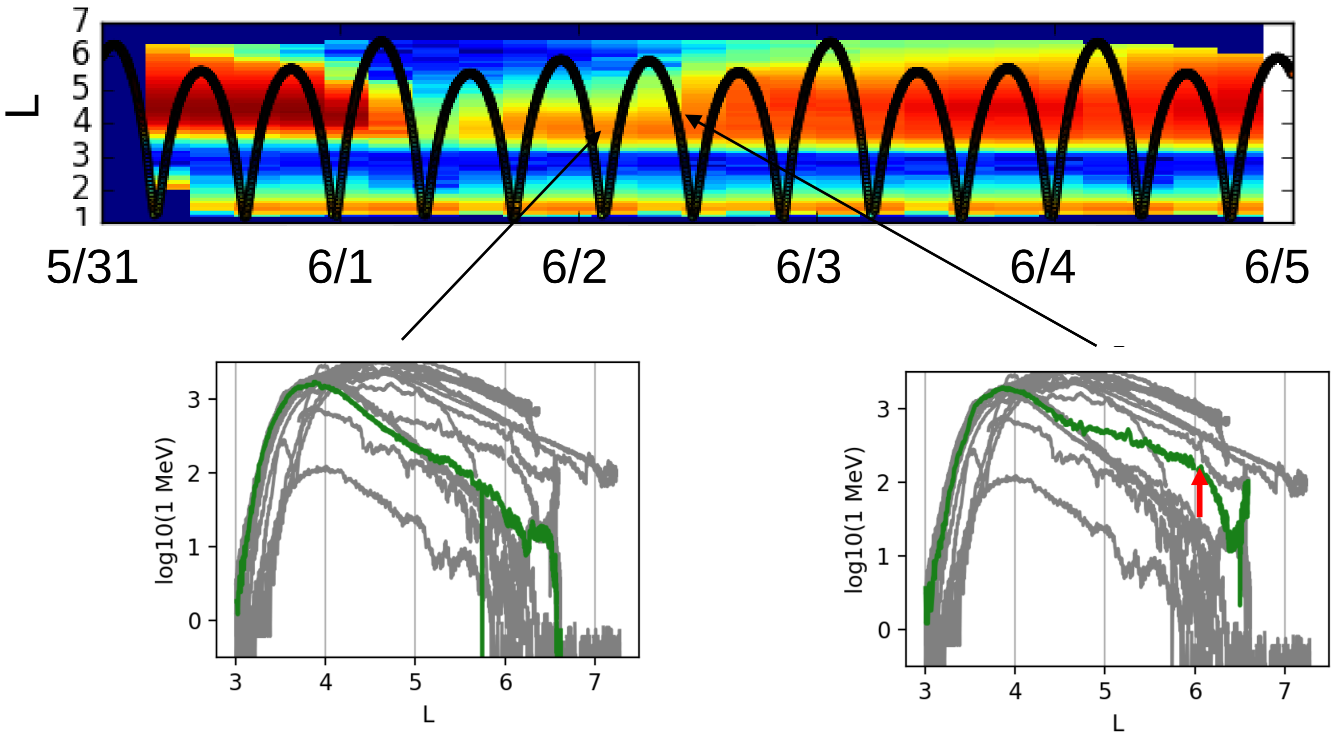
Example Storm: May/June 2013

- Here we examine the May/June, 2013 storm with ~1 MeV electrons from MagEIS



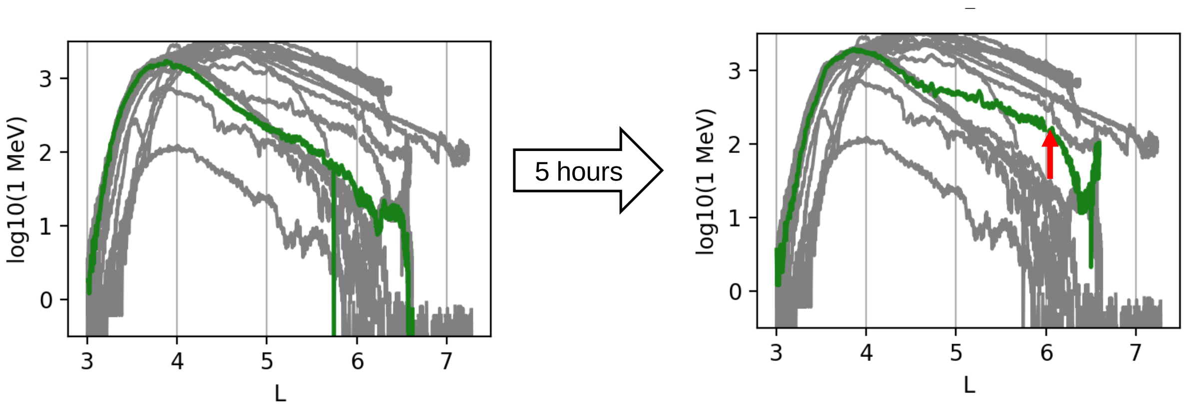
- The Dst index reaches -100 nT
- The pre-storm flux is high
- There is a dropout followed by a small enhancement on June 1st
- A second enhancement happens mainly at higher L later on the 2nd
- We will examine this second enhancement for drift phase structure

1 MeV Electron Flux

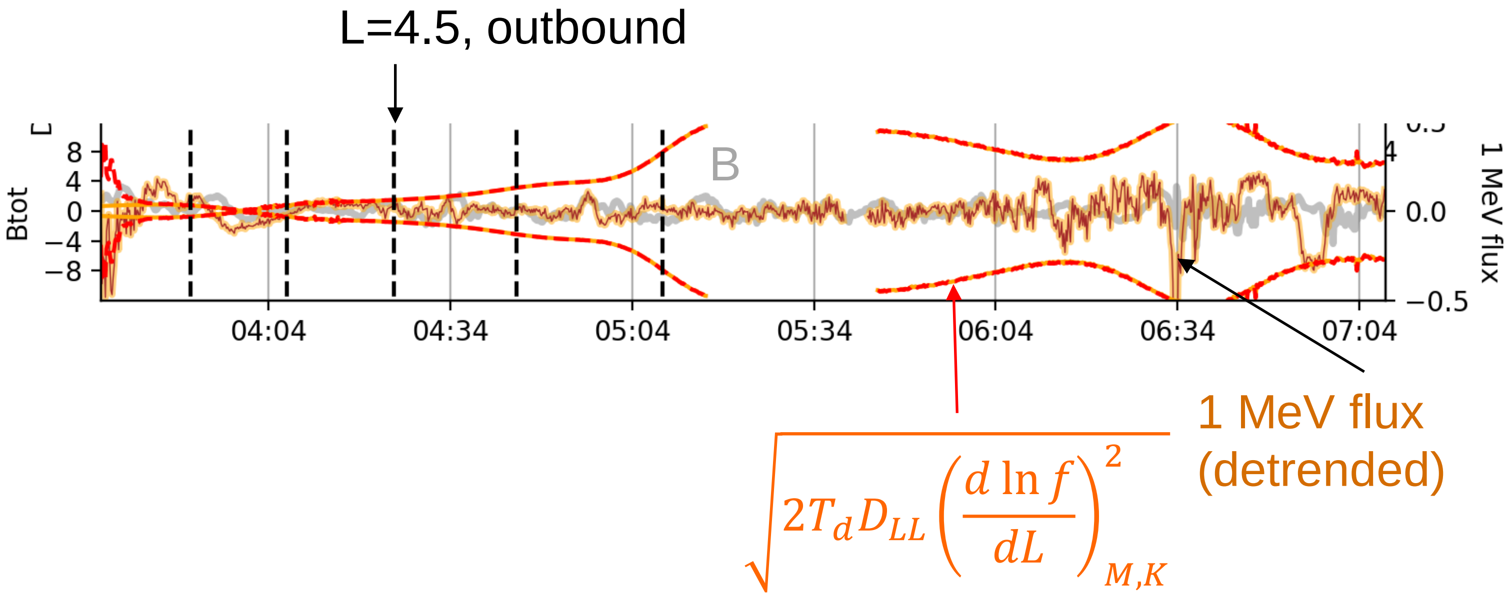


Example Storm: May/June 2013

- While the outer zone flux is increasing at L>4.5…
- There is very little drift phase structure (no wiggles)
- The wiggles that are there are generally much smaller than expected from D_{LL} and *df/dL*.



- If radial diffusion is playing a role, it's doing it without drift phase structure.
- This rules out diffusion primarily by:
  - *A sequence of impulses*
  - *Narrow-band drift resonance*
- What's left?
- Diffusion by broadband power with random phase (quasilinear radial diffusion)



Summary

- The particles can constrain or even provide the diffusion coefficients

- Pitch-angle distributions provide strong constraints on $D_{\alpha\alpha}$

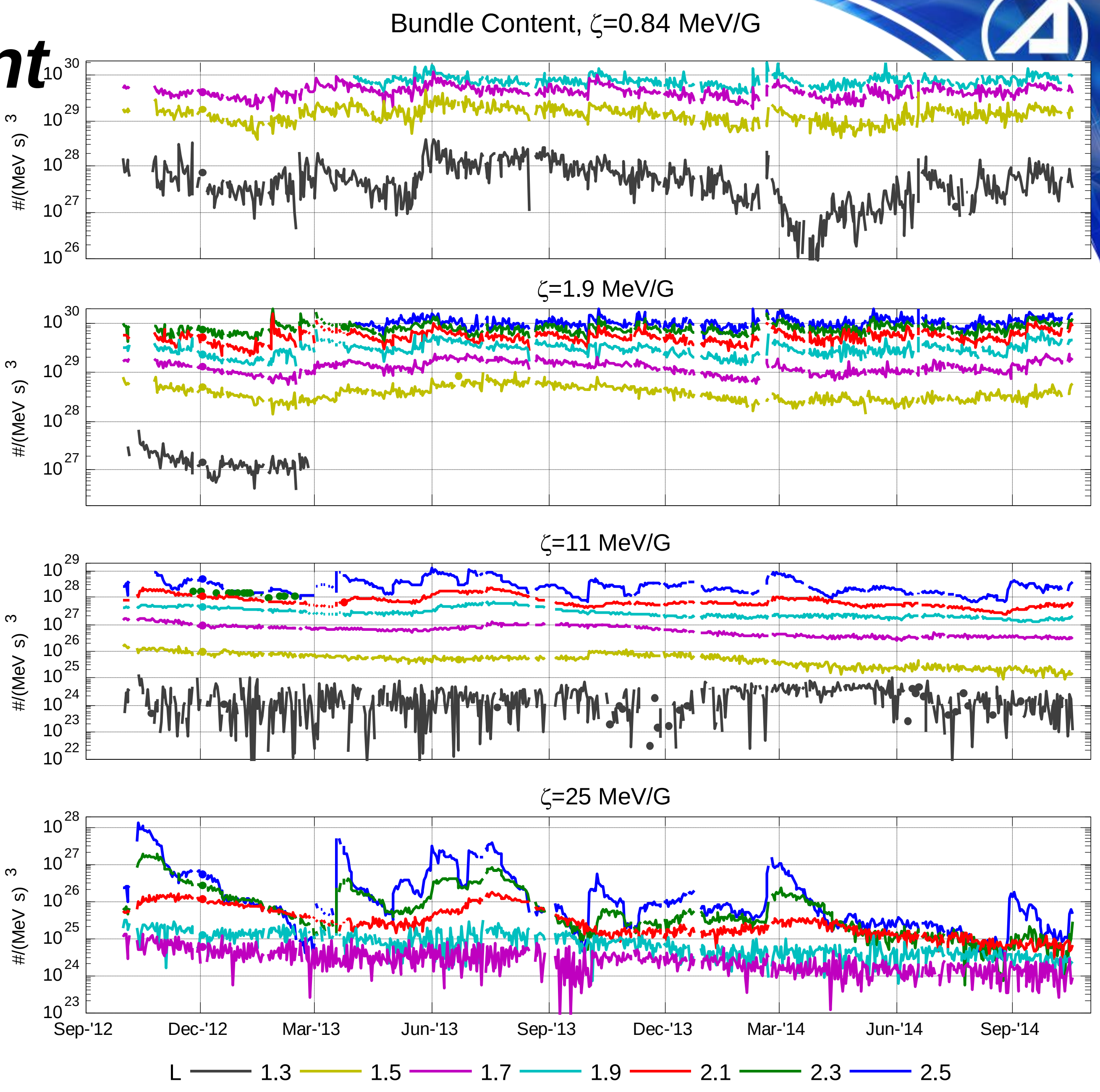
- Inner zone electron D_{LL} estimate from observed radial profiles are consistent with prior fits
  - *Simulations that extend to the inner zone should include electrostatic D_{LL}*

- Outer zone drift phase flux variations are not consistently present during flux enhancements
  - *Either there are many tiny radial perturbations (quasilinear D_{LL})*
  - *Or D_{LL} is way too large*

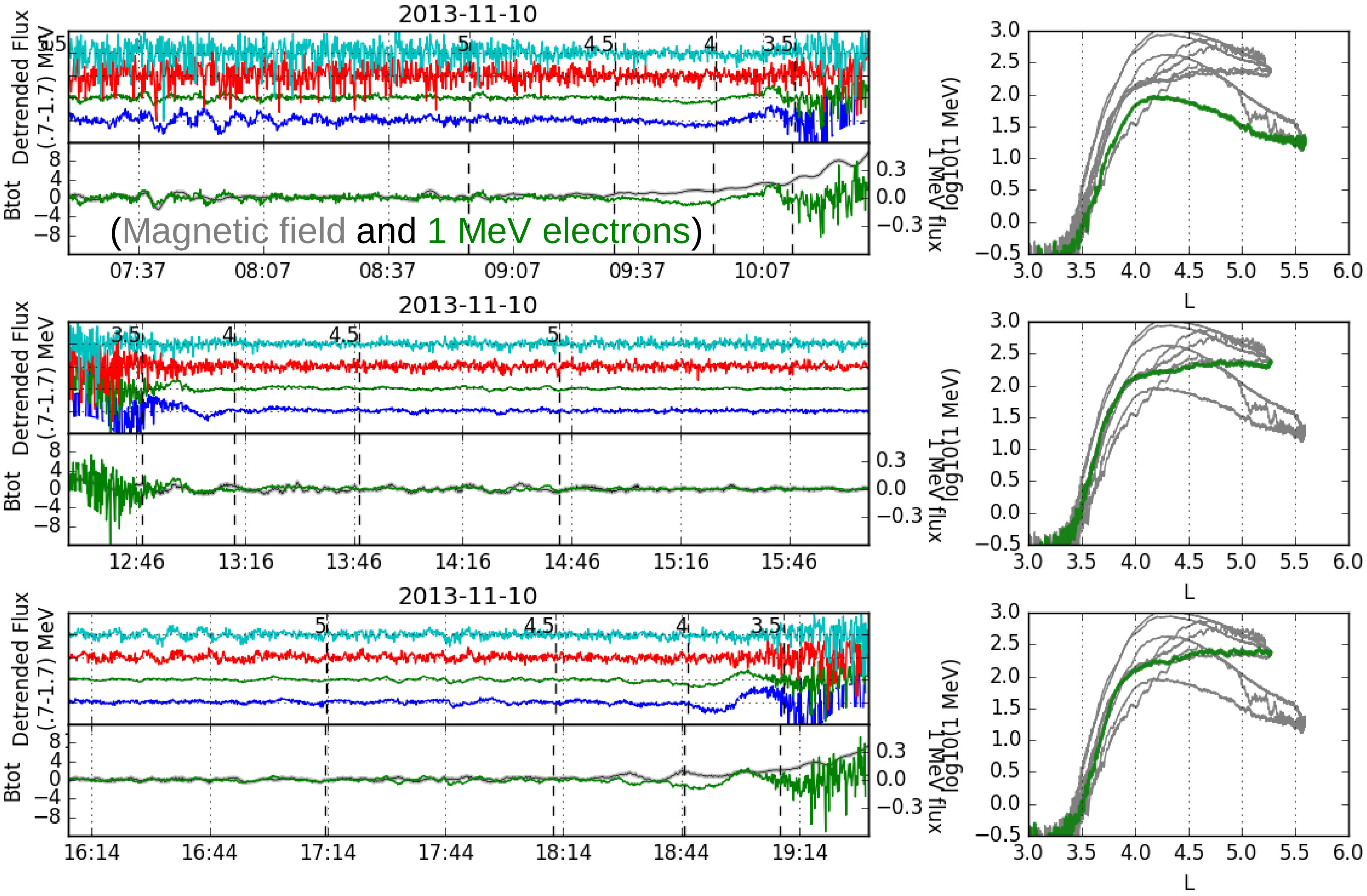
BACKUPS

Time Series of Bundle Content

- Fast variations
  - *Only affects highest Ls*
  - *More dramatic at higher energy (ζ)*
  - *Caused by storms*
- Slow variations
  - *Pitch angle scattering into atmosphere (~exponential decay)*
  - *Radial transport (undulations)*
- Compute *dn/dL* and *dn/dt* on weekly timescales

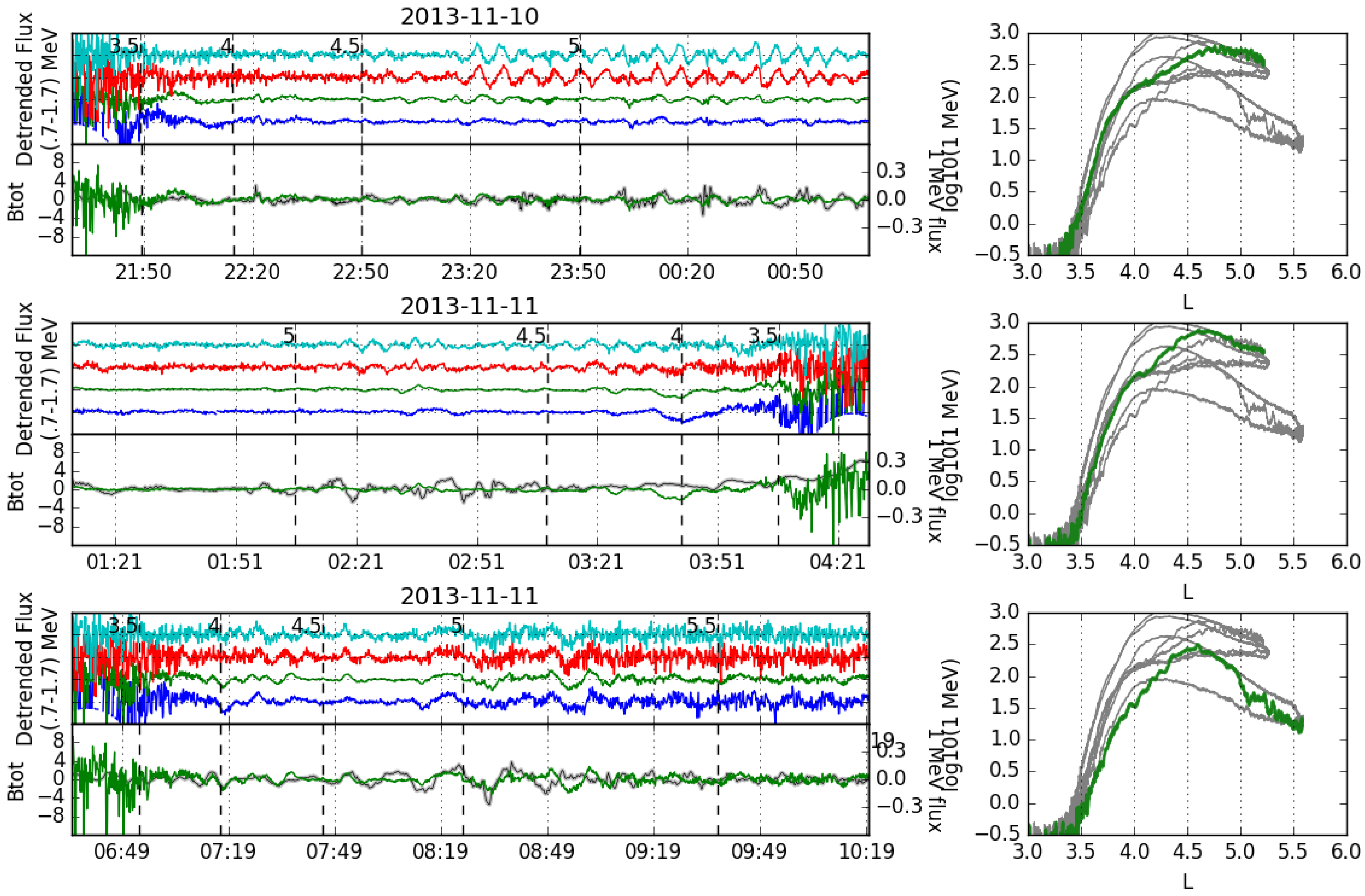


First Increase (Step 1- no wiggles)



- The left panels show detrended MagEIS data in several energies near 1 MeV
- The right panels provide L profiles (no detrending) over the day
- The green L profile is the one plotted in the adjacent time series
- These are very smooth profiles for L>4
- An increase without significant drift phase structure

First Increase(Step two with wiggles) and small drop out



- Here we have a dropout
- There are wiggles!
- The wiggles are sometimes associated with changes in B, other times not

- This could be radial diffusion pushing particles outward toward the magnetopause, contributing to the dropout