Particle-in-cell simulations of diffusion due to whistler mode waves: comparing quasi-monochromatic to broadband waves

Direct diffusion coefficients for transmitter waves: Classical diffusion or else?

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Quasilinear theory of wave-particle interactions

$$\frac{\partial f_s}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f_s}{\partial \boldsymbol{x}} + \frac{q_s}{m_s}(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \cdot \frac{\partial f_s}{\partial \boldsymbol{v}} = 0$$

$$\frac{\partial f}{\partial t} = \frac{1}{g(\alpha)} \frac{\partial}{\partial \alpha}\bigg|_{E,L} \left(g(\alpha) D_{\alpha\alpha} \frac{\partial f}{\partial \alpha}\bigg|_{E,L} \right) + \frac{1}{A(E)} \frac{\partial}{\partial E}\bigg|_{\alpha,L} \left(A(E) D_{EE} \frac{\partial f}{\partial E}\bigg|_{\alpha,L} \right) + L^2 \frac{\partial}{\partial L}\bigg|_{\mu,J} \left(\frac{D_{LL}}{L^2} \frac{\partial f}{\partial L}\bigg|_{\mu,J} \right) - \frac{f}{\tau_L}$$

Main Assumptions:

- Waves have random phase/are incoherent, and broadband
- Waves have small amplitudes
- Hence no particle trapping within the waves
- a.k.a "Weak turbulence" theory

Classical Diffusion in

- Energy
- Pitch angle
- (And L – radial diffusion)

$$D_{EE}(E,\alpha) = \frac{\langle (\Delta E)^2 \rangle}{\Delta t} \qquad \langle (\Delta E)^2 \rangle \propto \Delta t$$

$$D_{\alpha\alpha}(E,\alpha) = \frac{\langle (\Delta \alpha)^2 \rangle}{\Delta t} \qquad \langle (\Delta \alpha)^2 \rangle \propto \Delta t$$

$$D_{E\alpha}(E,\alpha) = \frac{\langle \Delta\alpha \Delta E \rangle}{\Delta t}$$

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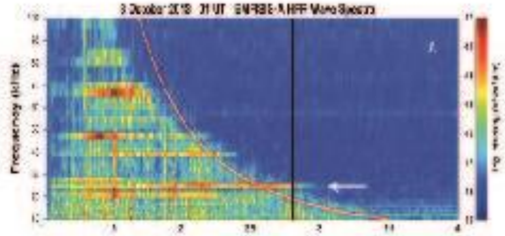
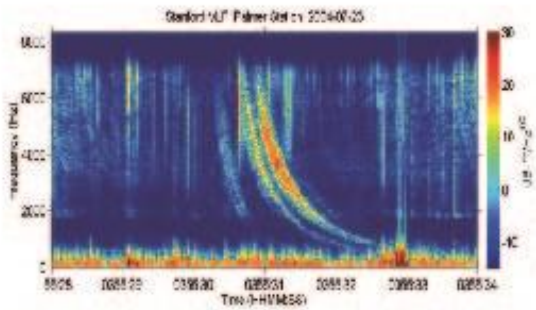
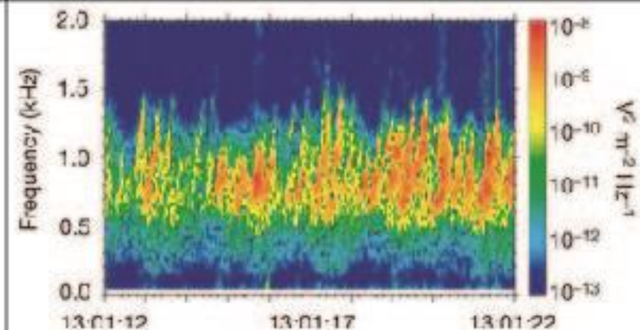
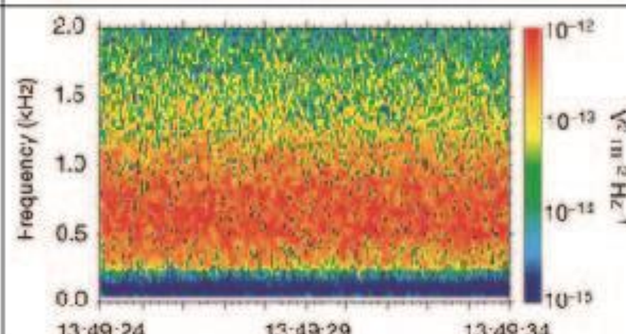
Line of Quasilinearliness

Whistler-mode wave structure in the magnetosphere

Transmitter waves: - Large amplitude - Nearly monochromatic due to artificial source at very well-defined frequency Figure: Foster et al., [2016]	
Lightning-generated whistlers: - Large amplitude - Narrow-band varying frequency structure due to dispersion of broadband source Figure: Stanford VLF Group	
Whistler-mode chorus: - Structured waves in frequancy space due to nonlinear generation mechanism - range of amplitudes Figure: Bortnik et al., [2008]	
Whistler-mode hiss: - Incoherent waves - Range of observed amplitudes Figure: Bortnik et al., [2008]	

Downward pointing arrow indicates increasing likelihood of observed wave type fulfilling all the assumptions of quaslinear theory

Less quasilinear:
Coherent
Monochromatic
Large(r) amplitude?

QL Diffusion valid/ invalid?

More quasilinear:
Incoherent
Broadband
Small(er) Amplitude?

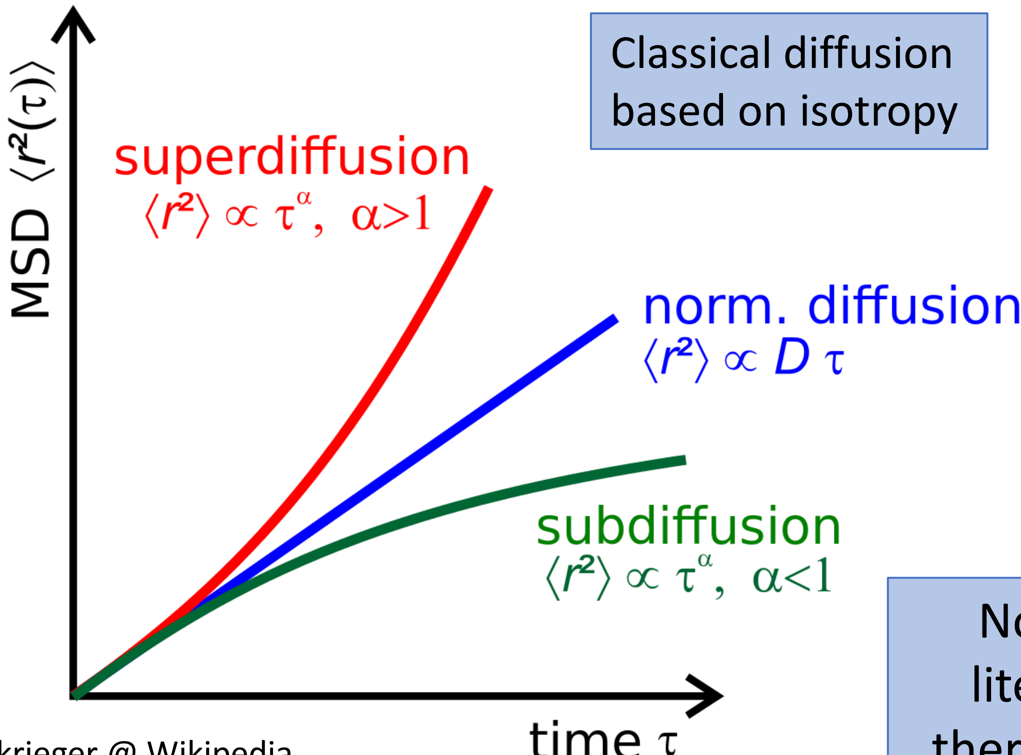
Figure credit: Clare Watt

The science question

$$\frac{\partial f}{\partial t} = \frac{1}{g(\alpha)} \frac{\partial}{\partial \alpha}\bigg|_{E,L} \left(g(\alpha) D_{\alpha\alpha} \frac{\partial f}{\partial \alpha}\bigg|_{E,L} \right) + \frac{1}{A(E)} \frac{\partial}{\partial E}\bigg|_{\alpha,L} \left(A(E) D_{EE} \frac{\partial f}{\partial E}\bigg|_{\alpha,L} \right) + L^2 \frac{\partial}{\partial L}\bigg|_{\mu,J} \left(\frac{D_{LL}}{L^2} \frac{\partial f}{\partial L}\bigg|_{\mu,J} \right) - \frac{f}{\tau_L}$$

$$\text{Correct Theory} = \begin{cases} \text{QL (diffusion)} & \text{plasma/wave properties} = ? \\ \text{NL (diffusion or else)} & \text{plasma/wave properties} = ? \end{cases}$$

B, n, **V**, T,
f, psi, ampl.



Jkrieger @ Wikipedia

Classical diffusion based on isotropy

- Super-/sub-diffusion?

... Non-classical/anomalous diffusion coefficients?

- If classical then diffusion with modified results
- Something else entirely?

Not yet explored the literature on this, but there is one! (any hints?)

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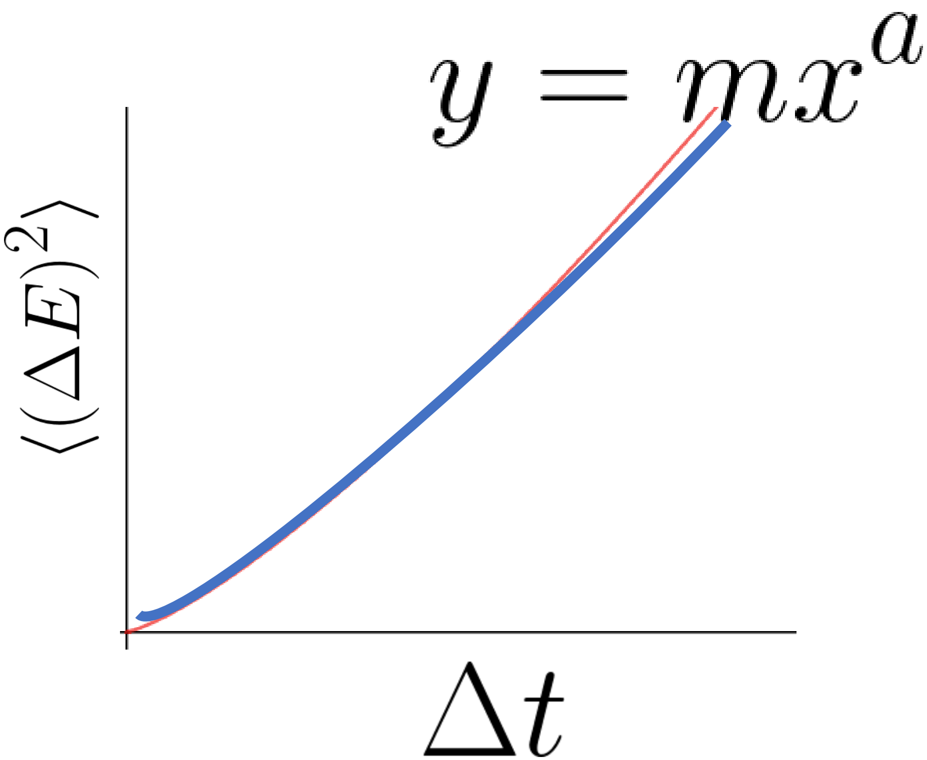
Diagnostic: in a nutshell

Track individual particles

- Directly track diffusion in E, α space
- "Use y=mx^a" to empirically measure the power of Δt and directly construct diffusion coefficients D_{EE} etc



$$\langle(\Delta X)^2\rangle = m(\Delta t)^a$$

$$\text{if } a = \begin{cases} = 1 \implies D_{XX} = m & : \text{ Classical diffusion} \\ \neq 1 \implies D_{XX} = ?,\ m = ? & : \text{ Anomalous diffusion} \end{cases}$$

If a ≠ 1 => can we use a diffusion equation at all?
D_{XX} ~ d <(Δ X)^2> / dt ?

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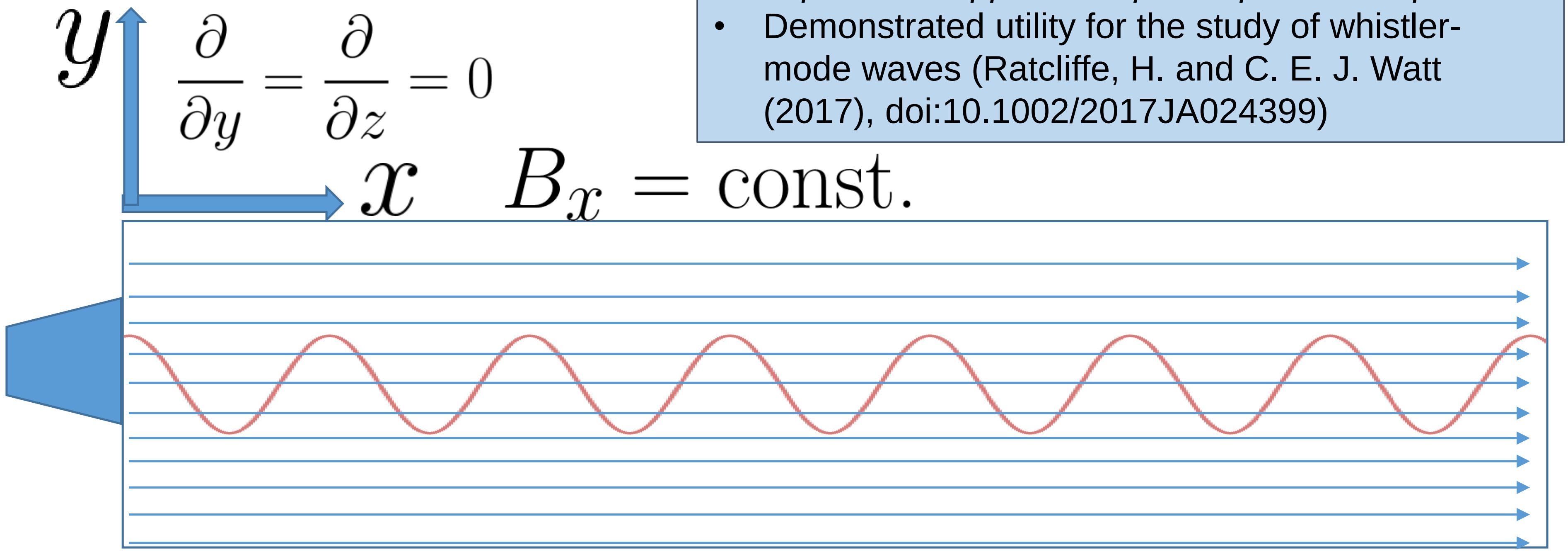
Method: Prelim. 1D PIC experiments to test quasilinear assumption

EPOCH

- Open-source, explicit, parallelised (~>90%), relativistic
- Easily customised with a variety of different boundary conditions. c=c, mi/me = REAL VALUE.
- *http://www.ccpp.ac.uk/epoch/epoch_user.pdf*
- Demonstrated utility for the study of whistler-mode waves (Ratcliffe, H. and C. E. J. Watt (2017), doi:10.1002/2017JA024399)

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0$$

$$B_x = \text{const.}$$



Stationary & almost entirely cold prototypical L~2.5 population

$$E_y \sim 1,\, 10mV/m \quad B_z \sim 0.03,\, 0.3nT$$

24kHz @ L~2.5 (e.g. Clilverd et al 2008)

Basic simulation parameters

- Equatorial B_x~ 2000nT @ L~2.5 => w/w_{ce}~0.4
- IC: Driven experiment, not an instability.
- BC: Open .. Ok since large domain
- Wave travels 1/100 box:

=> L~600km ~ 8.5 Wavelengths p/point ~<5% leave box

- T_{world}=20t_{ce} =3x10^{-4}s : $T_{wall\text{-}time}$~2hrs on 10 cores

!

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The plasma populations in the box

200 or 1000 tracers
Can/will be increased

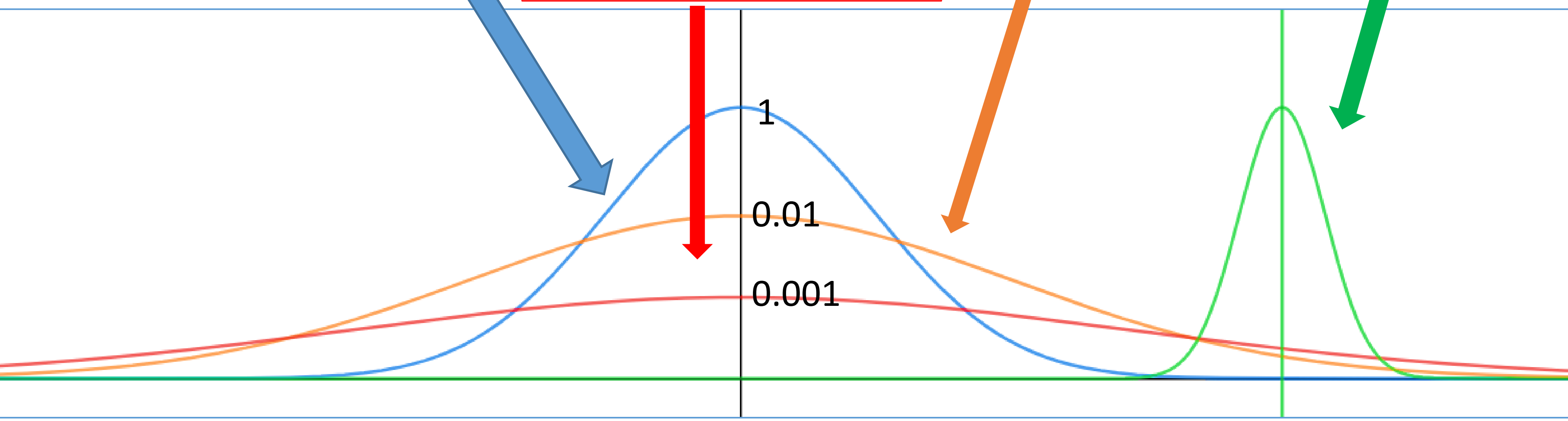
Tracers on hot plasma population at 100keV (well samples phase space)

Tracers population narrowly focussed on bulk flow: vph from cold plasma DR (Stix)

e: 1eV cold @ 98.9%
i: 1eV cold @ 100%

e: 100keV hot @ 0.1%

e: 10keV warm @ 1%



$v_{\rm phase}$

- n ~ 10^3cm^{-3}: fixed by w_{pe}/w_{ce}= 8 (from CRRES data)
- Resonant ("res") tracers: T ~ 20eV (narrowly focussed) on v_{ph}/c ~ 0.06

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Tracer statistics (better than they sound)

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MANIPULATING DATA into "delta parameter versus delta time" form

if number_of_files = 11, say, then "n"=11, and we have an 11x11 matrix

dv(0,n-1)=v(n-1)-v(n-1); dv(1,n-1)=v(n-1)-v(n-2); dv(2,n-1)=v(n-1)-v(n-3); dv(3,n-1)=v(n-1)-v(n-4); ... ; dv(n-1,n-1)=v(n-1)-v(0)
.                      ; .                      ; .                      ; ...                    ;
.                      ; .                      ; .                      ; ...                    ;
.                      ; .                      ; dv(2,3)=v(3)-v(1)      ; dv(3,3)=v(3)-v(0)      ;
dv(0,2)=v(2)-v(2)      ; dv(1,2)=v(2)-v(1)      ; dv(2,2)=v(2)-v(0)      ; dv(3,2)=0              ; ... ; 0
dv(0,1)=v(1)-v(1)      ; dv(1,1)=v(1)-v(0)      ; dv(2,1)=0              ; dv(3,1)=0              ; ... ; 0
dv(0,0)=v(0)-v(0)      ; dv(1,0)=0              ; dv(2,0)=0              ; dv(3,0)=0

 deltat=0              ; deltat=1               ; .............................................
         columns increasing with i  ------->


COLUMN 0 is deltat=0 (i=0), COLUMN 1 is deltat=1 (i=1) etc
;ROW    0 is j=0, ... up to row (n-2) is j=n-2
```

Tracers:

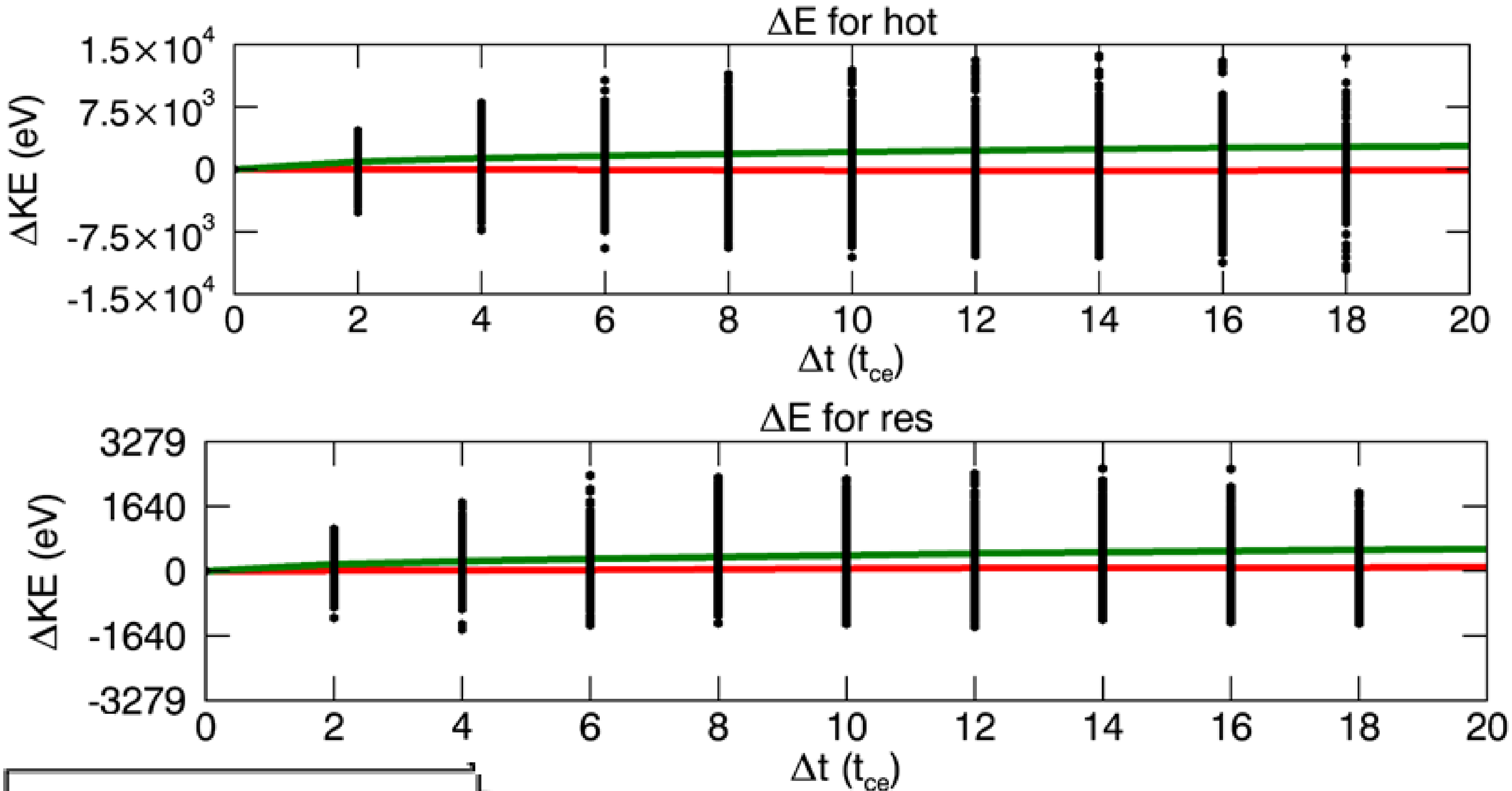
Cut up data

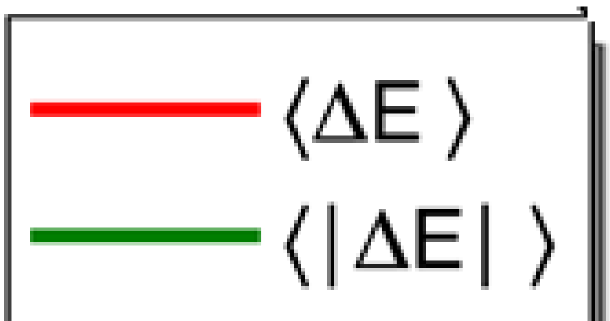
-> ~T(T+1)/2 * n data points

Statistics at each step

- 66 * n data points
- (T-t)*n at each delta t
- ~(T(T+1)/2 * n/n_bin^2) in each E,alpha bin
- In each bin these samples are distributed ~ as above

200 tracers, 1mV/m laser: Naïve scatters and averages





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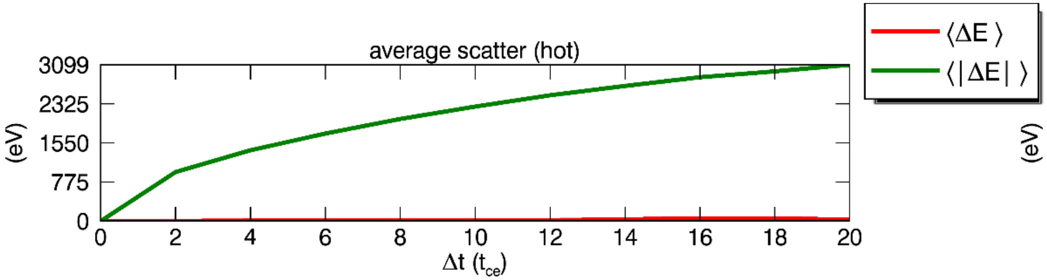
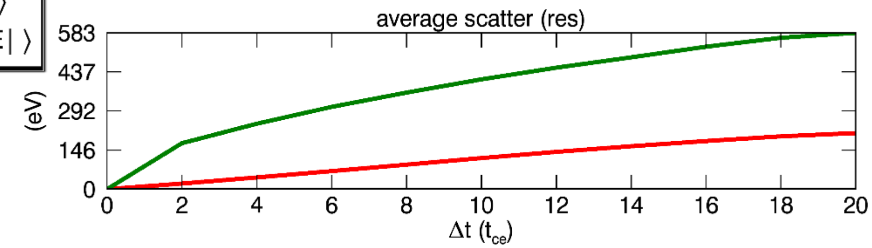
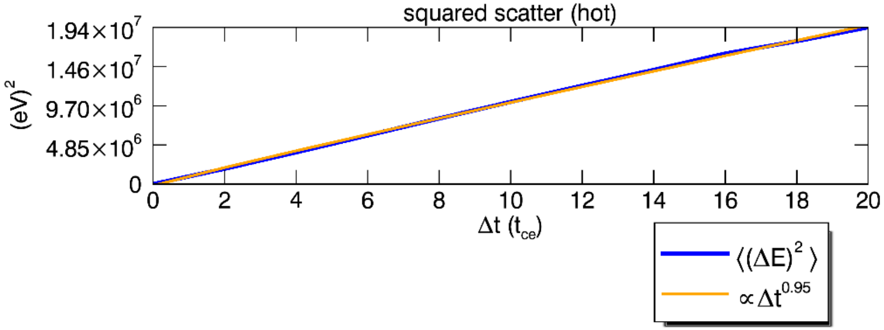
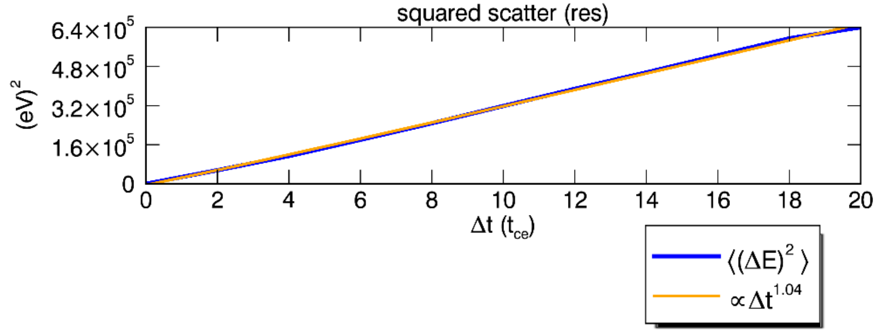
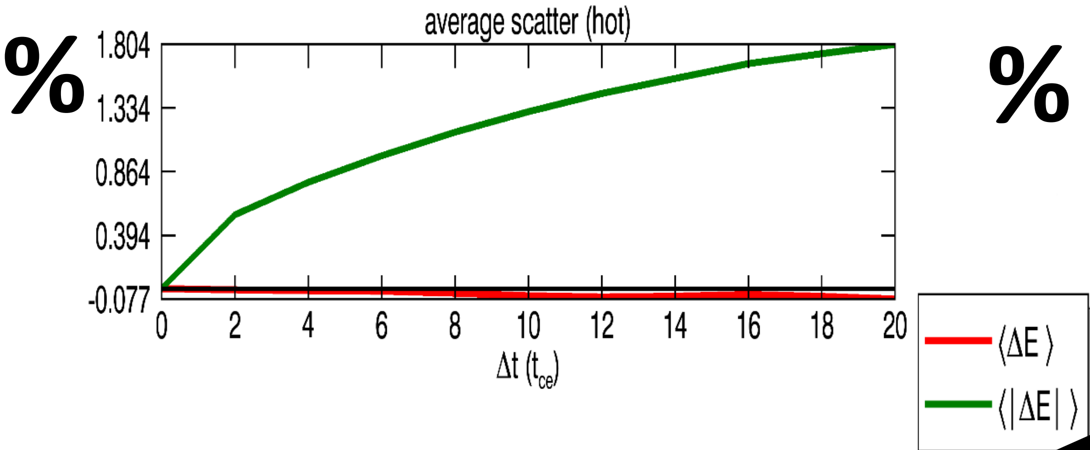
1000 tracers, 1mV/m laser: Averages, and some reassurance

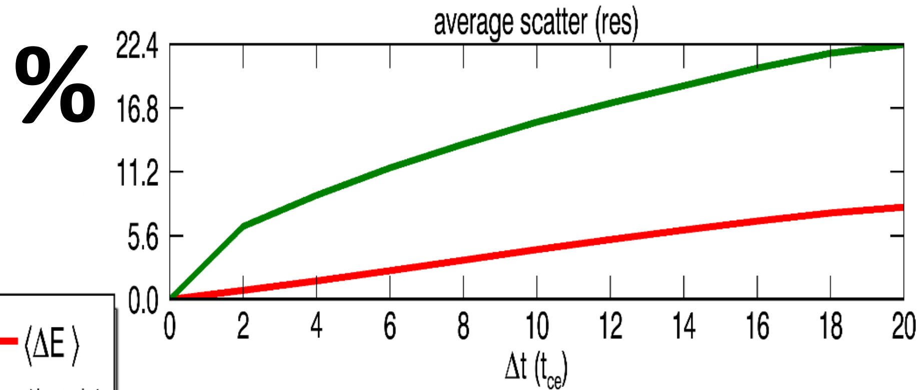






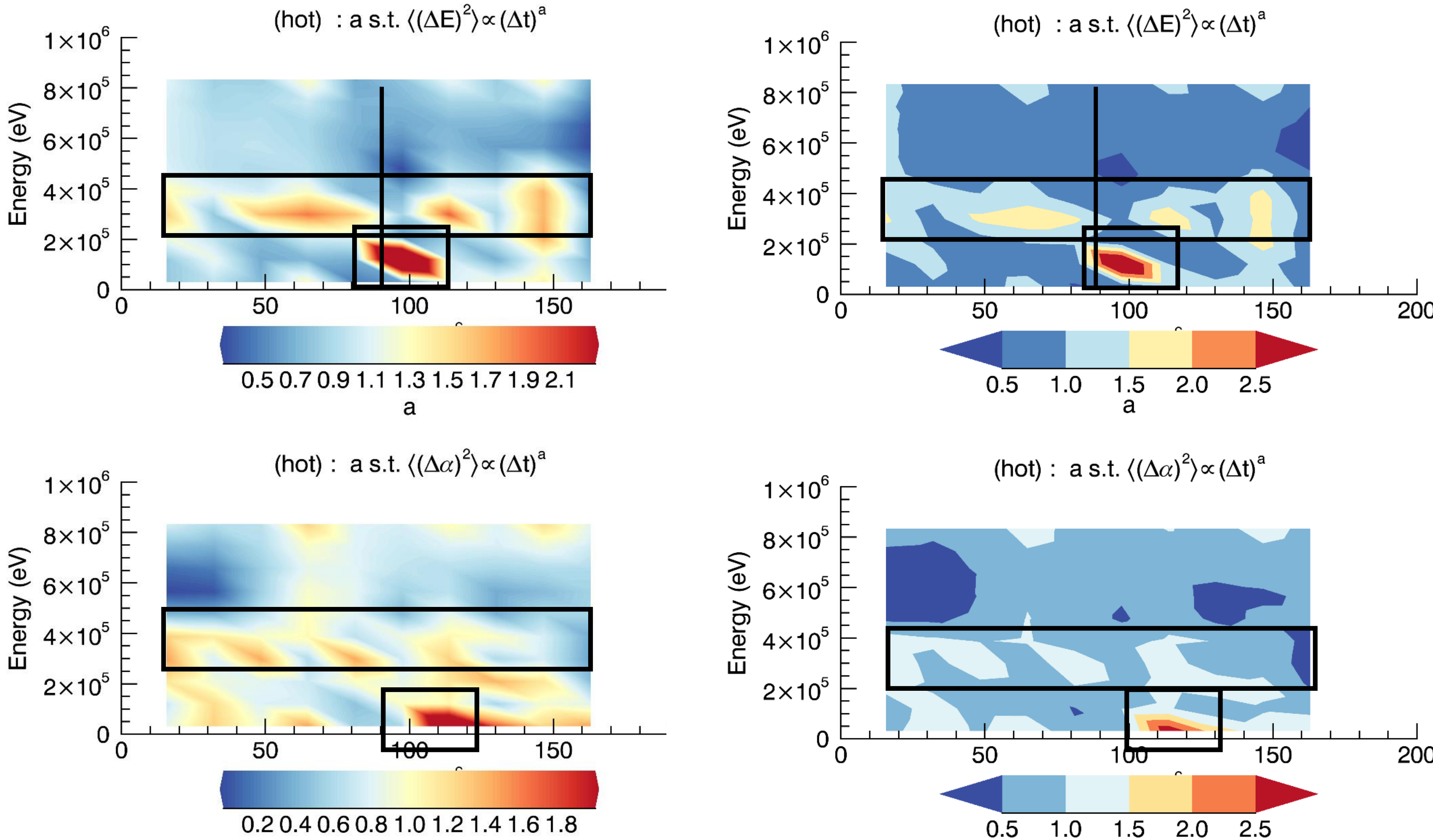






E

1000 tracers, 10mV/m laser: How classical is the diffusion?



Next steps

- 2D : oblique waves
- Parameter space studies … B, wave angle., ampl., other whistler mode waves (hiss first at L~4-5)
- Longer time runs
- All this requires > 10 cores!
- Convergence testing of code etc

Conclusions

<u>Very preliminary conclusions:</u>

- We are making a tool that can "directly" support/cast doubt upon the assumption of classical diffusion.
- ***We see that whilst classical diffusion seems to hold on aggregate (for our waves), it is on shaky ground once you start to bin data (for short timescales at least).***
- Sometimes the algorithm doesn't work, and so we cannot say behaviour is diffusive at all
- Results seem qualitatively similar for different amplitudes of wave thus far (e.g. 10 – 100 mV/m)

Due diligence (Other recent work in a similar vein on whistler diffusion coeffs) e.g.:

- Tao et al 2011 (test particle approach)
- Albert 2010 (numerical/analytical)
- R. Denton (hybrid simulations) poster @ this conference

Questions
(actually suggestions and hints please…!)

- This is a talk about a tool/new project, not really about a whole raft of new results:

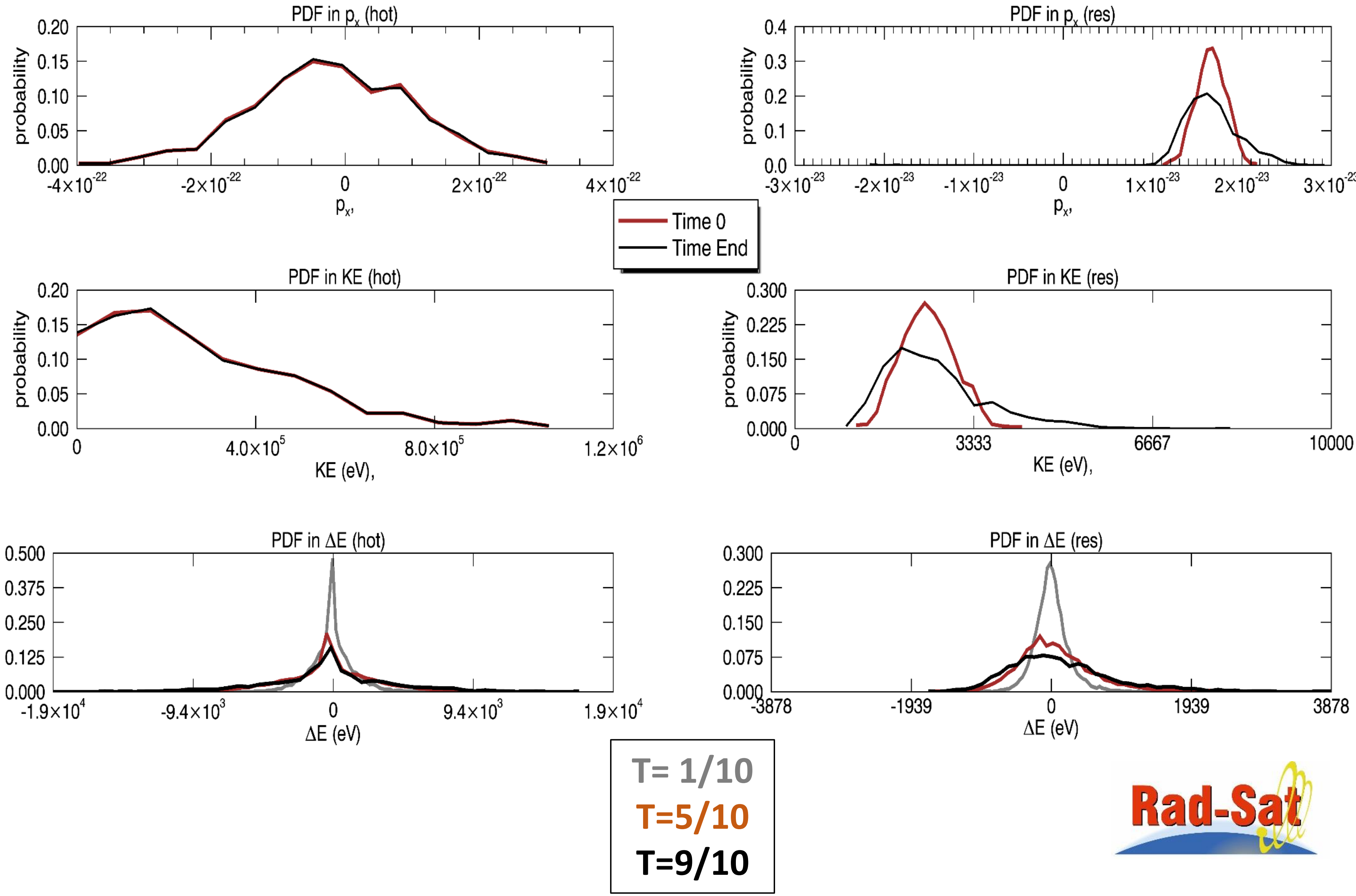
How could we best use the tool to find out interesting things?

(a) Best signatures/metrics?

(b) Most important waves / phenomena /plasma conditions to the community?

Thanks!

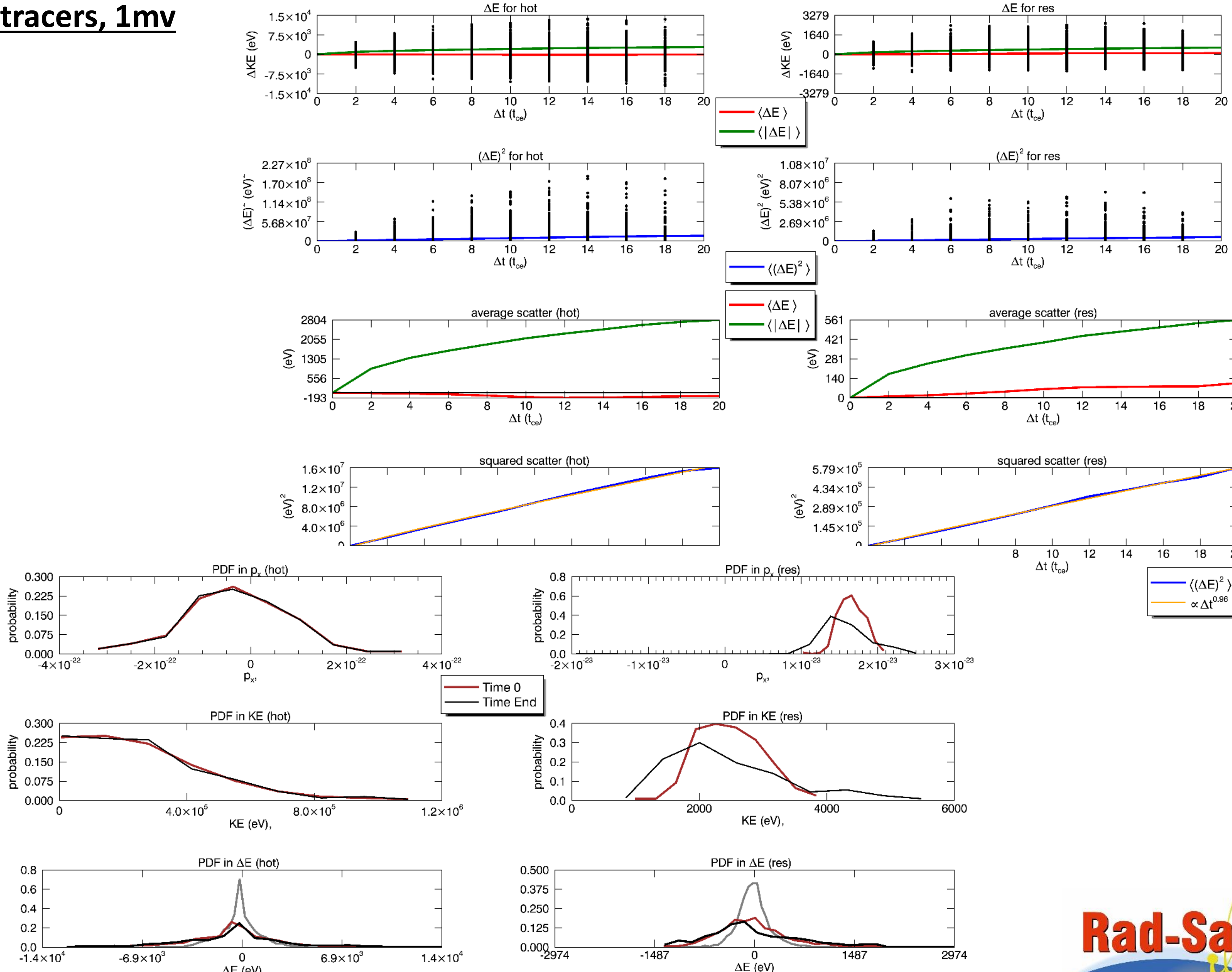
1000 tracers, 1mV/m laser: Distributions



Why PIC?

- **Why kinetic:** wave-particle interactions in principle require fully relativistic kinetic theory (i.e. with up to six dimensions in phase space), timescales range from microseconds to tens of seconds. Non-linear studies of diffusion in whistler-mode waves have so far focussed on the test-particle approach [e.g. *Tao and Bortnik, 2010*]. Natural extension is to use PIC.
- **Why not Vlasov:** Problems with velocity-space filamentation over long domains: mitigating lamentation requires high resolution in phase space which is computationally expensive in 1-D and prohibitively so in 2-D.
- **Why not hybrid:** (e.g. Katoh and Omura, 2013) treat lower energy populations of electrons and ions as fluid, and the fast electron population as particles (electron timescales cannot be neglected). However, there are known problems in the modelling of short (grid scale comparable) wavelength whistlers in electron fluid hybrid schemes, leading to unphysical energy build up. Furthermore, evolution of the higher energy tails of the lower energy distributions can in fact be important for large wave amplitudes, and so a fully kinetic treatment is required.

200 tracers, 1mv



1000 tracers, 10mv