AGU Chapman Conference on
Particle Dynamics in the Earth's Radiation Belts
5-9 March 2018
Cascais, Portugal

Simulation study of the nonlinear processes of whistler-mode chorus generation in the Earth's inner magnetosphere

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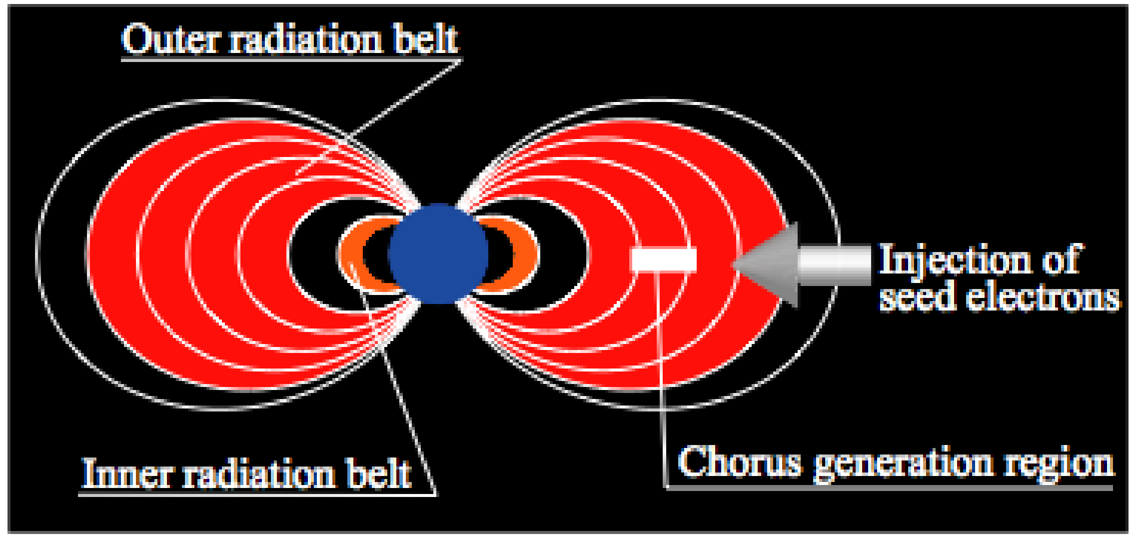
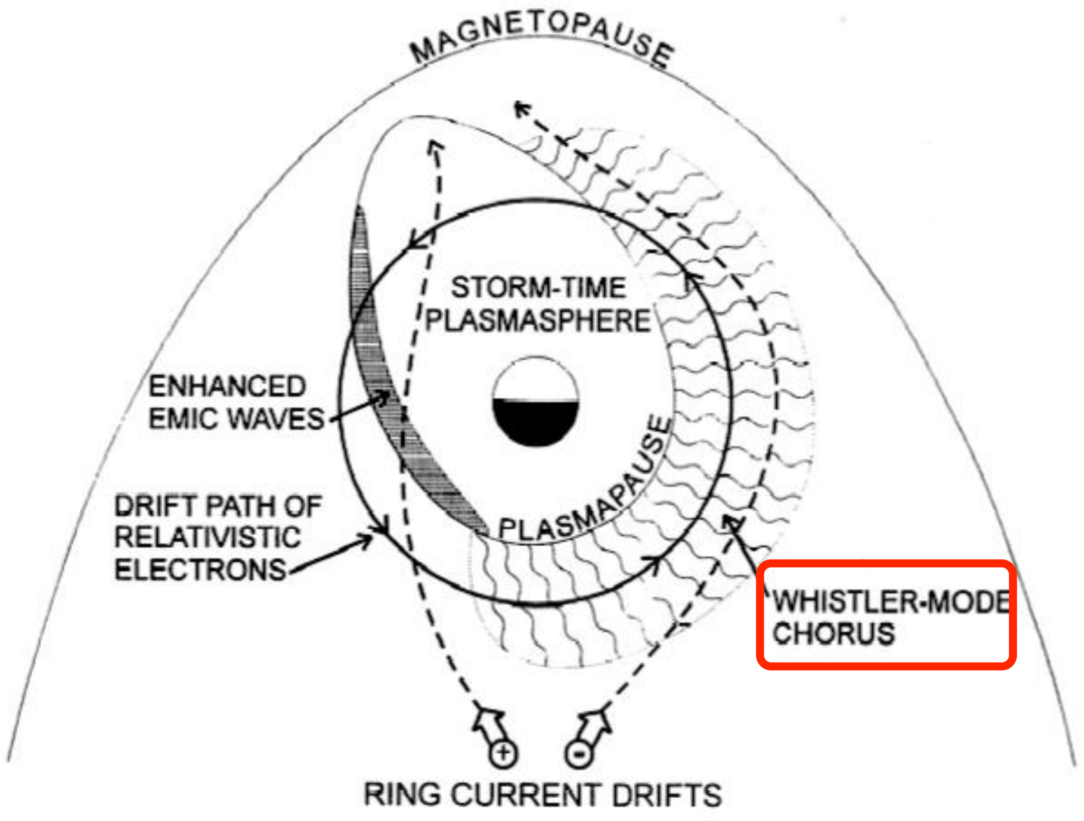
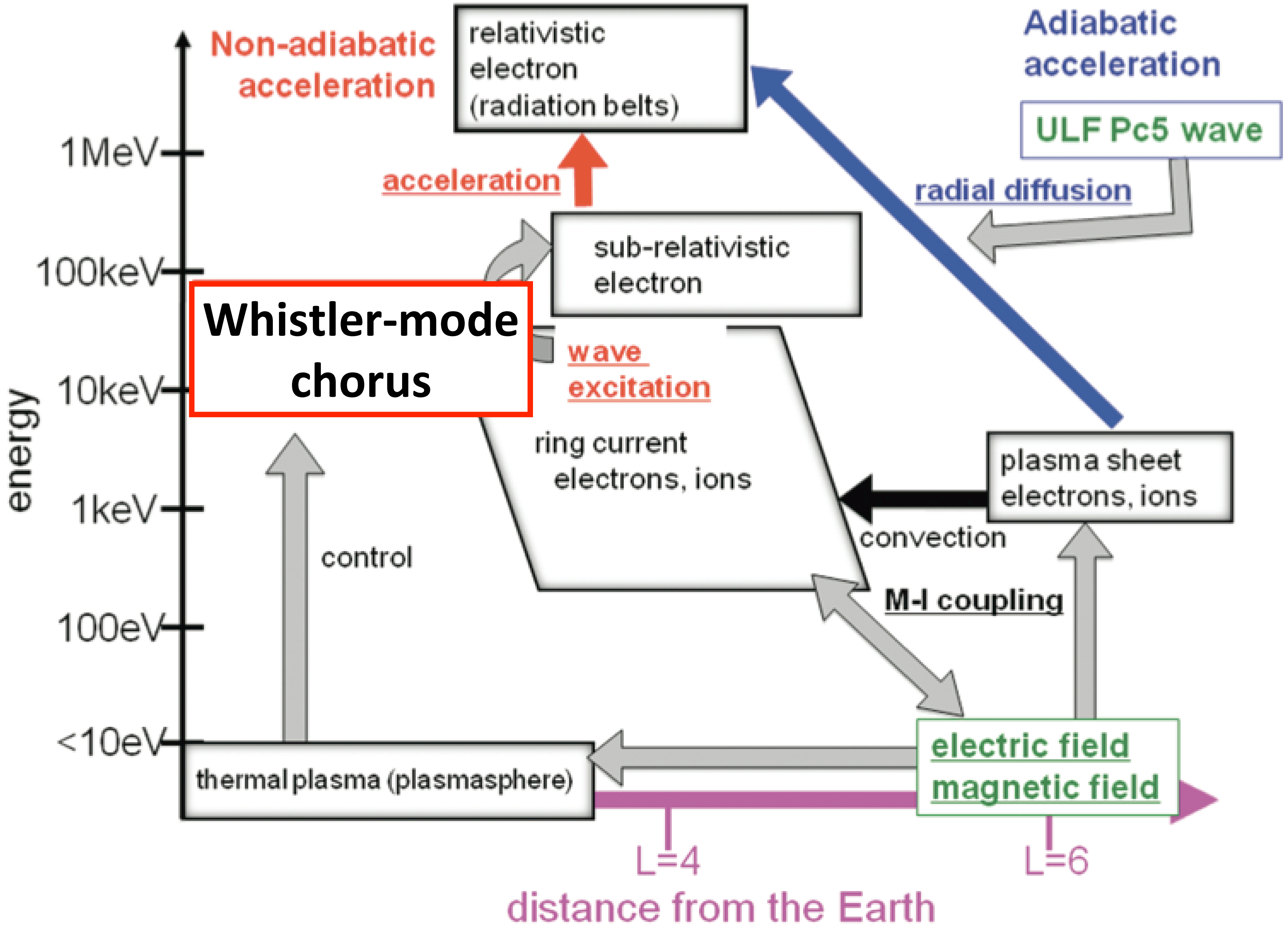
Whistler-mode chorus:
nonlinear wave-particle interactions in Geospace



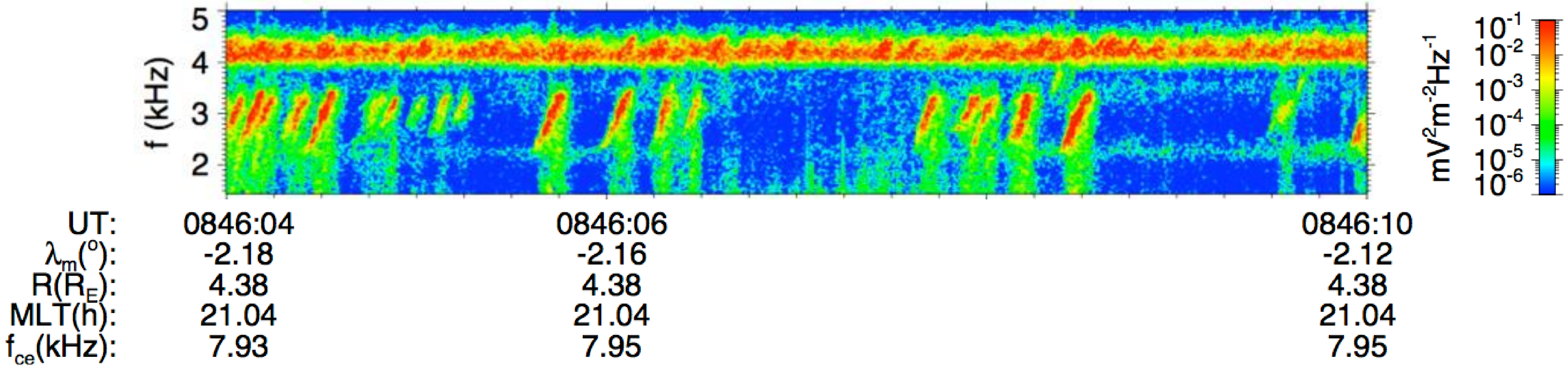


[*Summers et al.*, JGR 1998]

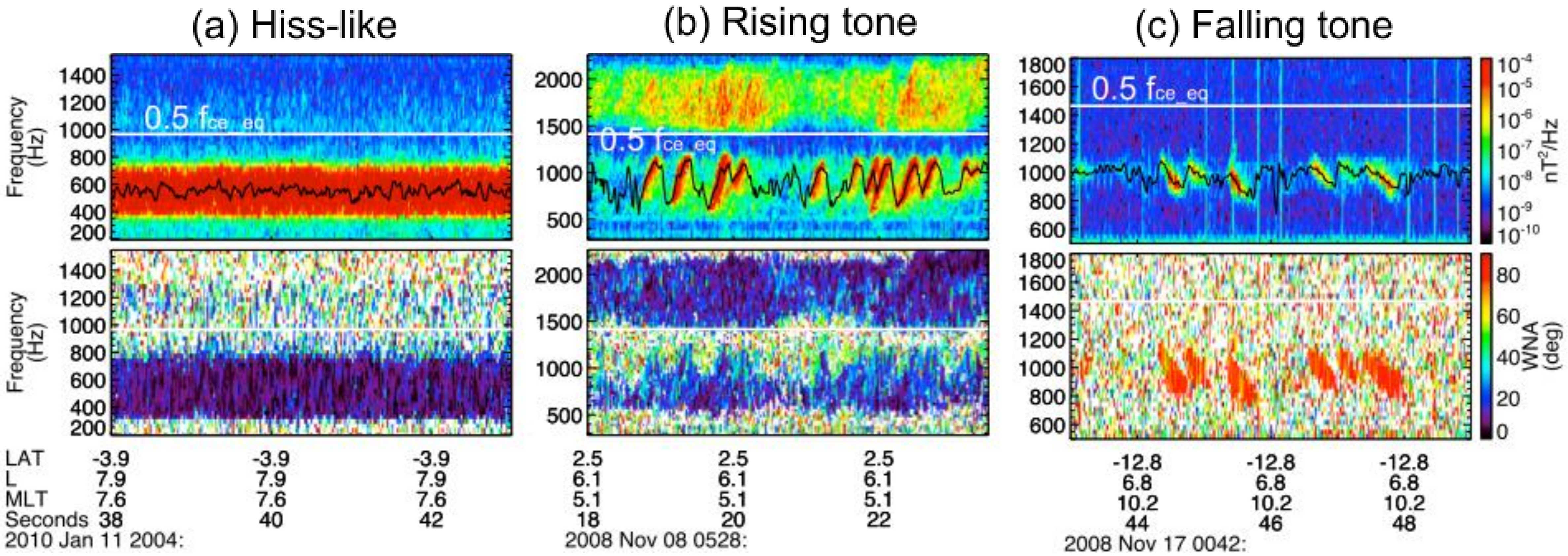


[*Miyoshi et al.*, AGU Monogr. 2012]

A variety of chorus in the Earth's magnetosphere



[*Santolik et al.*, 2008]



[*Li et al.*, 2012]

Generation process of whistler-mode chorus

1) Linear growth phase

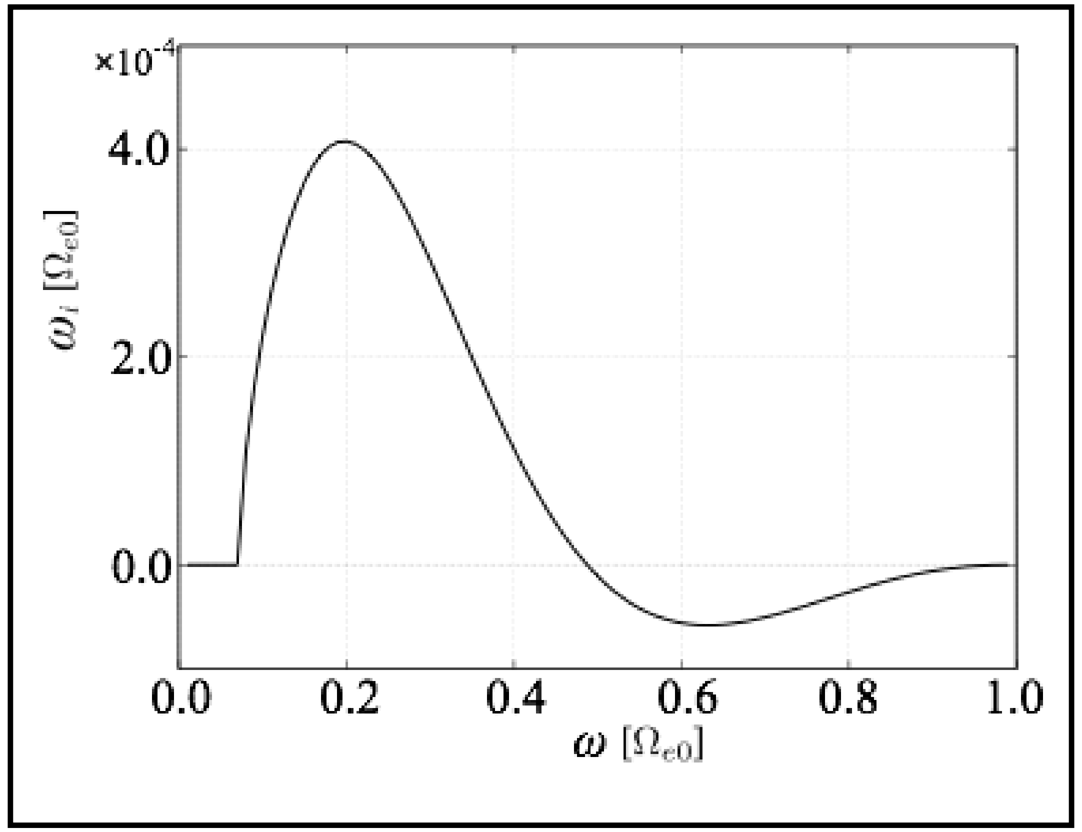
$$\boxed{f(v_{\|}, v_{\perp}, \psi), \quad \partial f/\partial \psi = 0}$$

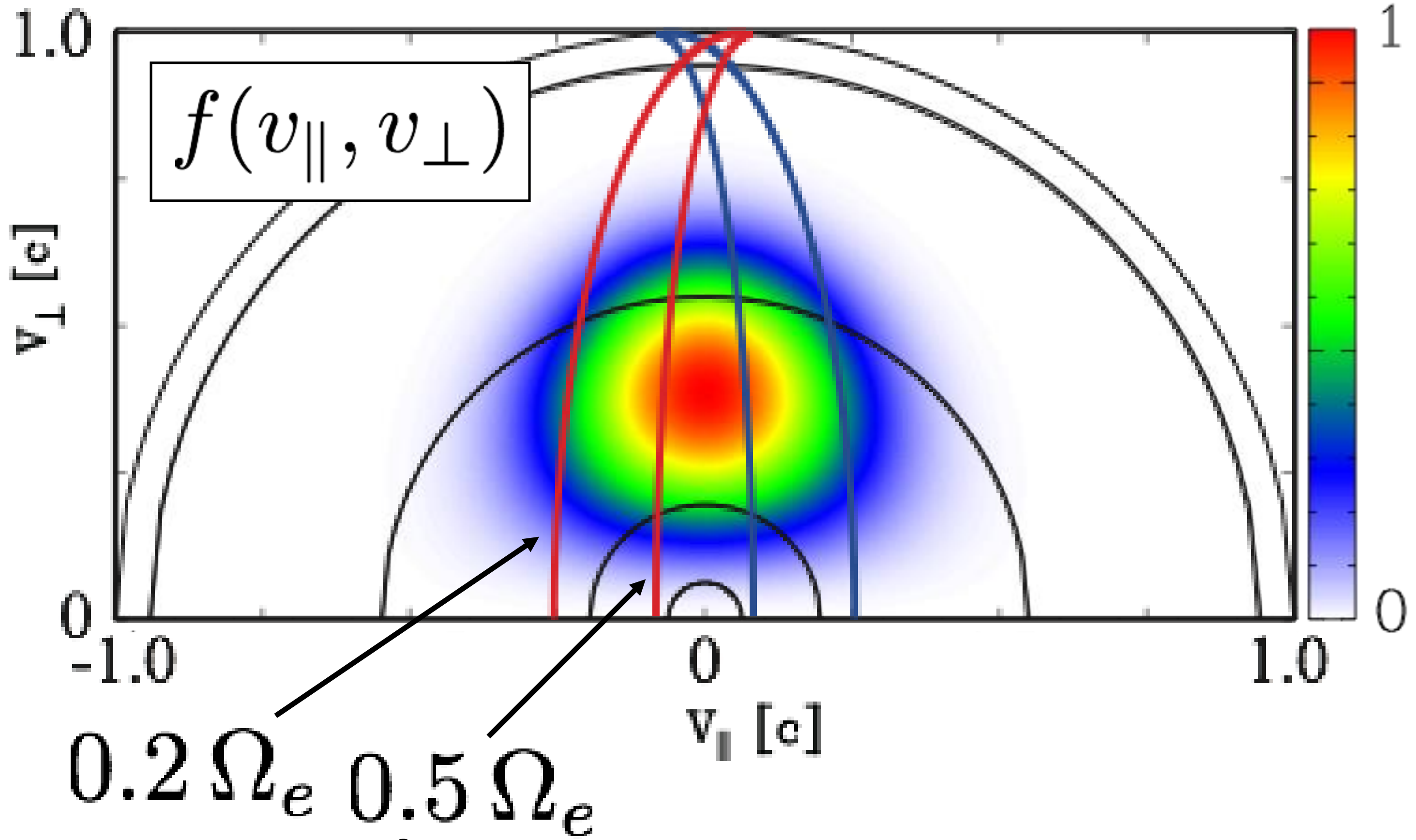
Excitation of a band of whistler-mode waves through the instability driven by a temperature anisotropy of energetic electrons

$$\omega_i \propto \int_0^{\infty} 2\pi v_{\perp} f(V_R, v_{\perp}) dv_{\perp}$$

$$V_R = \frac{\omega - \Omega_e/\gamma}{k}$$

[cf., *Kennel and Petschek, 1966*]





Forward/Backward propagating waves

--> A finite amplitude coherent wave element emerges from the band of waves

Generation process of whistler-mode chorus

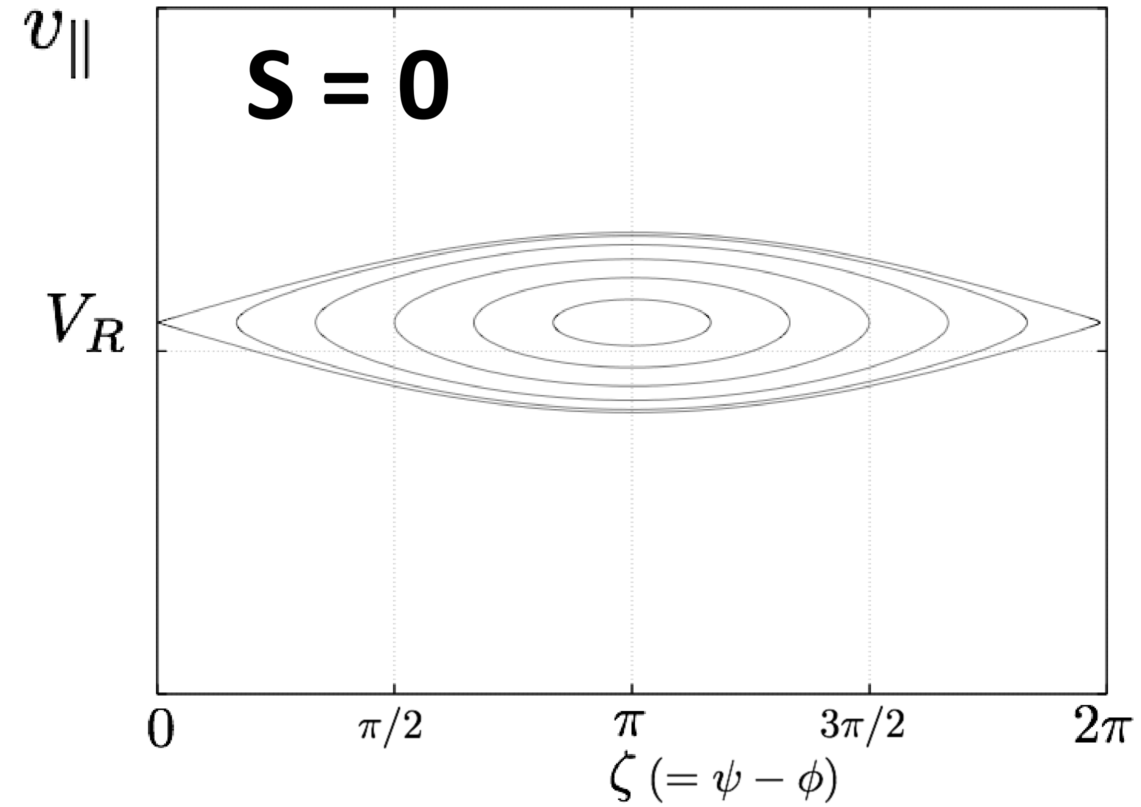
2) Non-linear phase

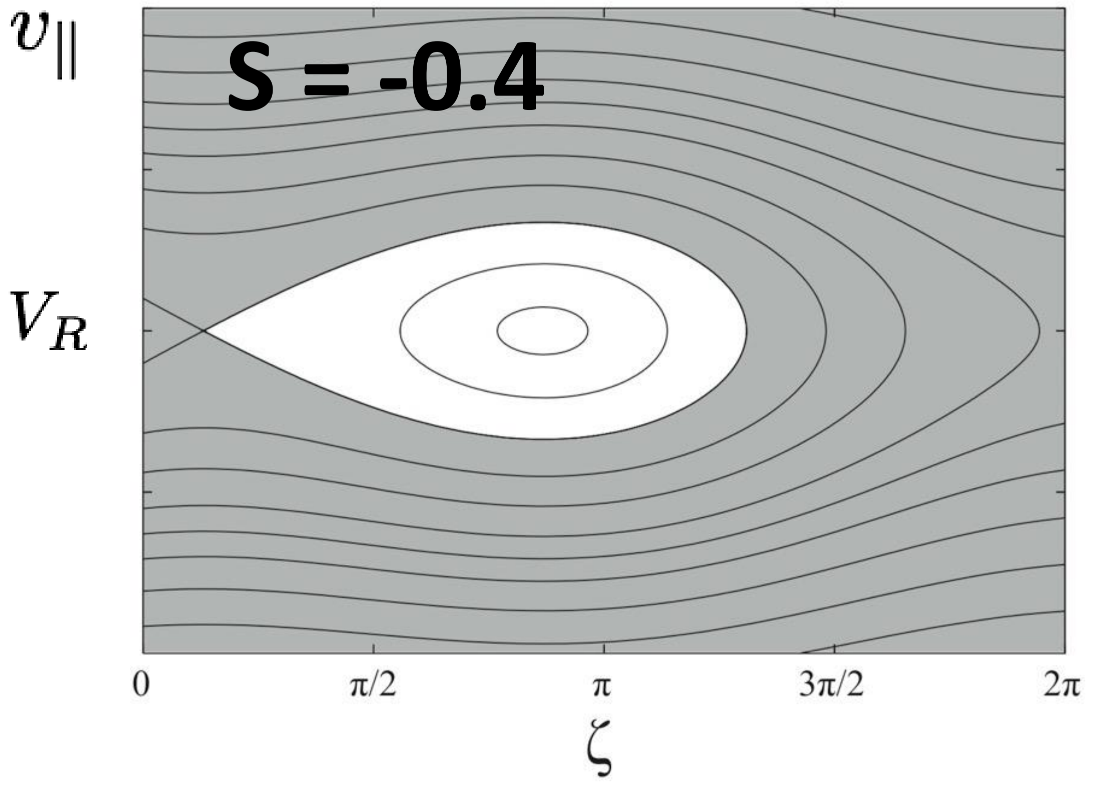
$$\boxed{f(v_{\|}, v_{\perp}, \psi), \quad \partial f / \partial \psi \neq 0}$$

Resonant electrons are trapped/untrapped by a coherent wave element, resulting in the formation of “resonant current”

$$\frac{d^2\zeta}{dt^2} = \frac{\omega_t^2 \chi^2}{\gamma}(\sin\zeta + S) \qquad \omega_t = \sqrt{k v_{\perp} \Omega_W} \qquad \chi^2 = 1 - \frac{\omega^2}{c^2 k^2}$$

[*cf. Omura et al., 2008, 2009, 2012*]





--> Amplification by the resonant current maximizes for a wave element whose frequency increases in time

Purpose of the present study

- In order to investigate the condition required for triggering non-linear effects of the chorus generation, we carry out a series of self-consistent electron hybrid simulations

- Dependences on (1) the temperature anisotropy and density of energetic electrons and (2) gradient of the background magnetic field have been studied

Electron Hybrid code: Basic equations

[*cf. Katoh and Omura, 2004, 2006*]

Cold electrons are treated as a fluid
Energetic electrons are treated as particles

$$\frac{\partial \boldsymbol{V}}{\partial t} = -(\boldsymbol{V} \cdot \nabla)\boldsymbol{V} + \frac{q}{m}(\boldsymbol{E} + \boldsymbol{V} \times \boldsymbol{B})$$

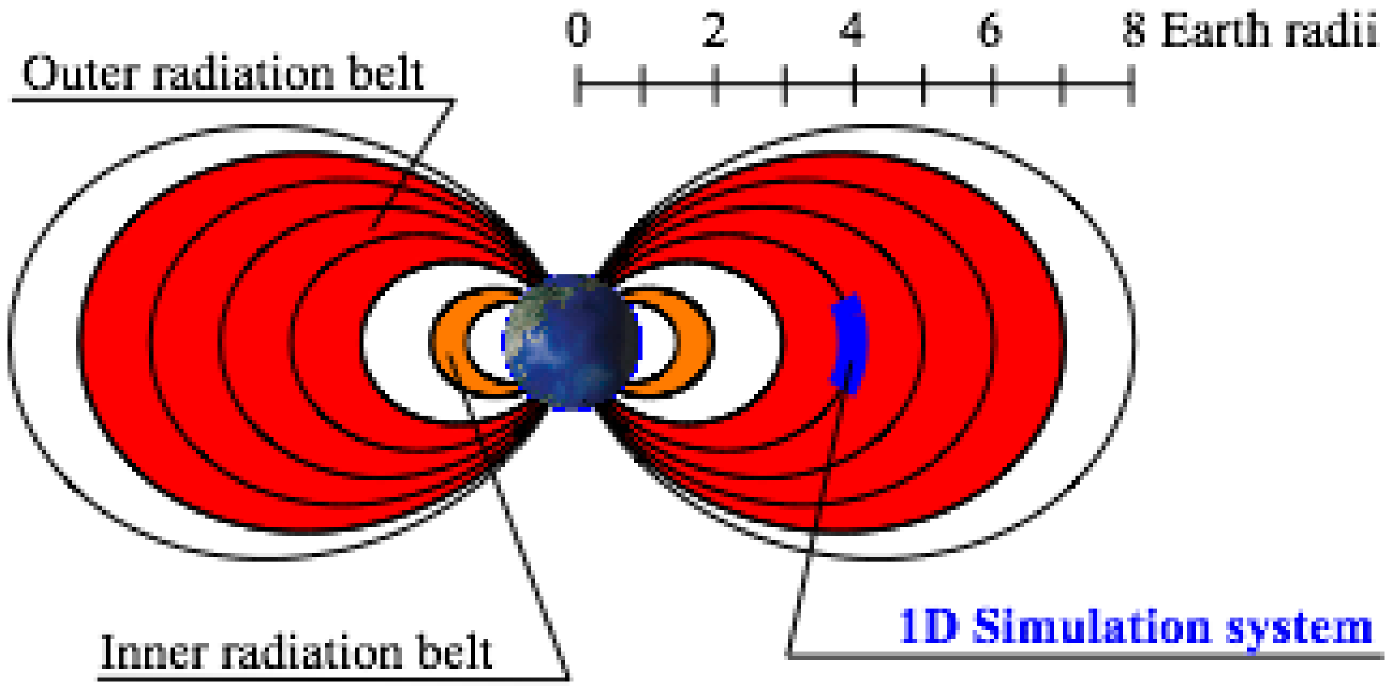
$$\frac{d(m\boldsymbol{v})}{dt} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

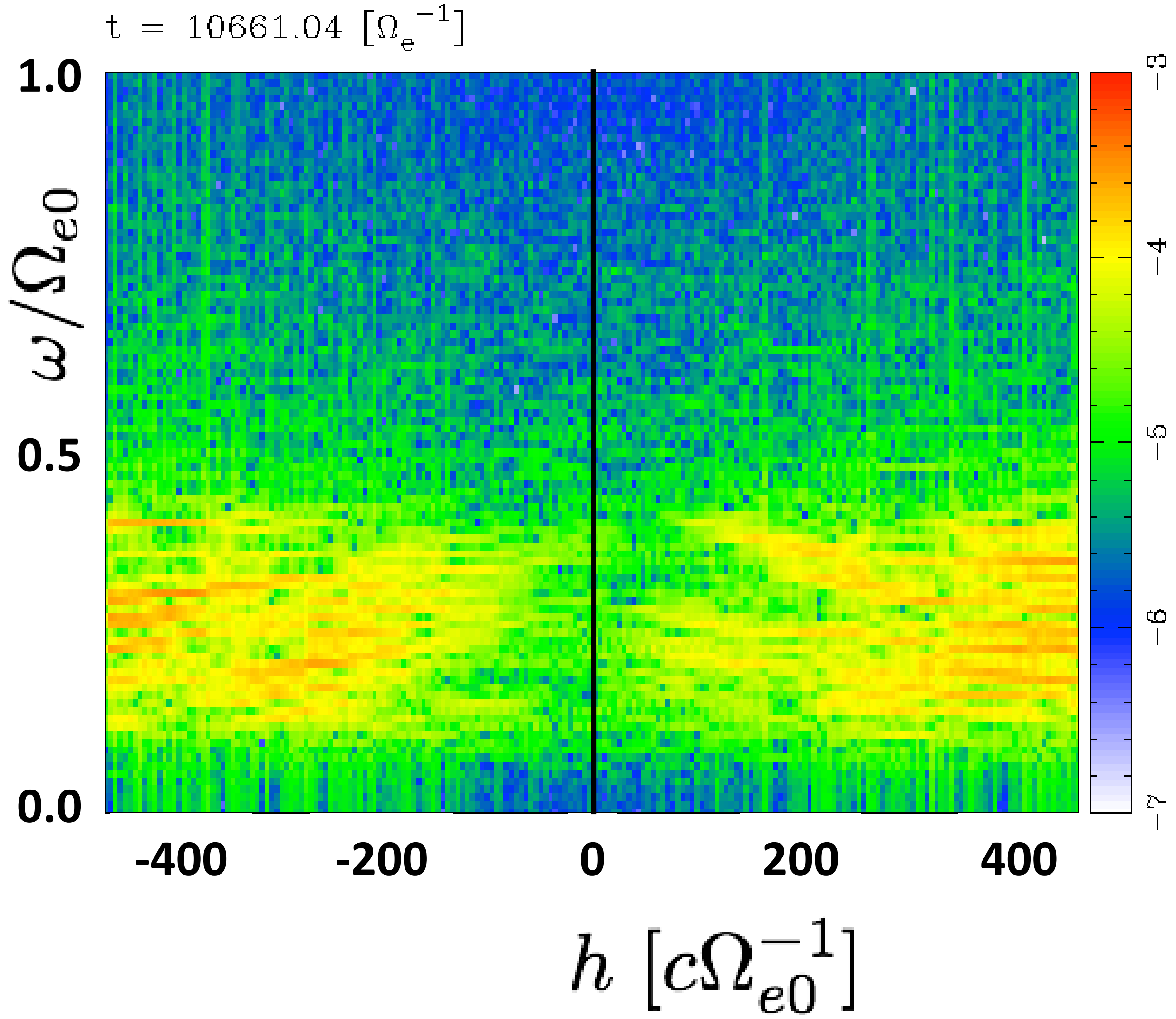
$$\boldsymbol{J} = qN\boldsymbol{V} + \sum_i q n_i \boldsymbol{v}_i$$

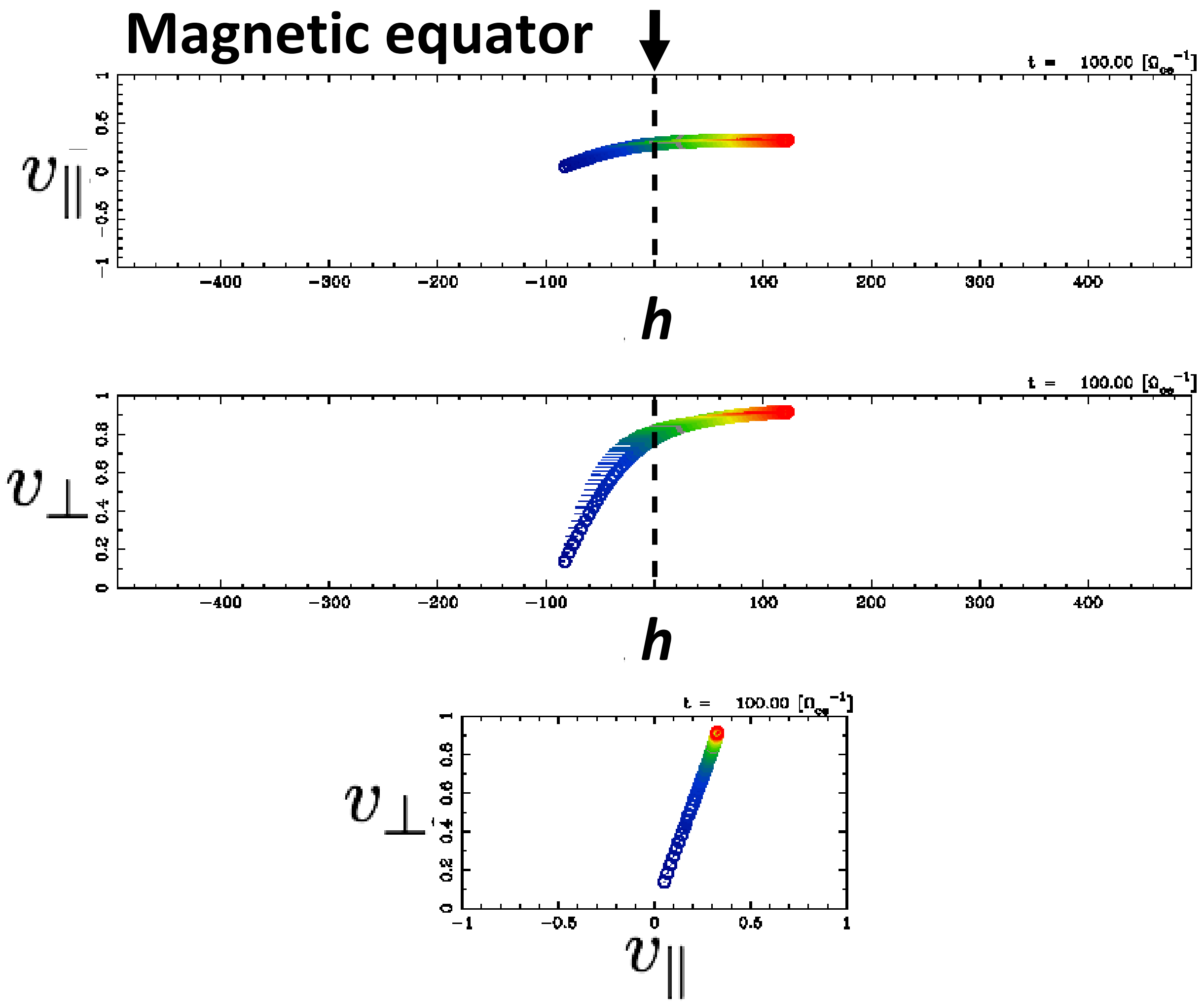
$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\mu_0 \varepsilon_0}\nabla \times \mathbf{B} - \frac{1}{\varepsilon_0}\mathbf{J}$$

Simulation model & initial settings







- Electron Hybrid code
- 1D, field aligned system
- mirror motion of electrons is taken into account
- neglecting electrostatic waves

Initial conditions for the study of properties of chorus generation

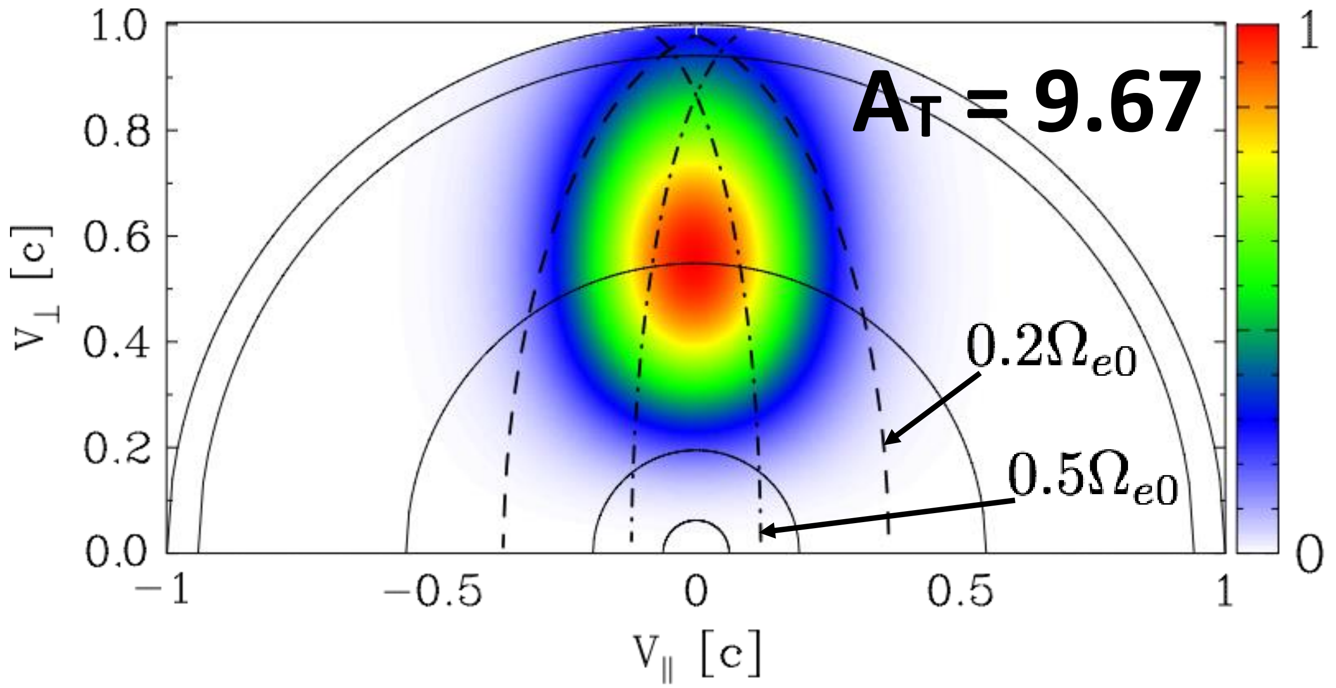
- **Case 1**: Different temperature anisotropy of energetic electrons at the magnetic equator

- **Case 2**: Different magnetic field inhomogeneity with the same property of energetic electrons at the magnetic equator

Initial velocity (momentum) distribution of energetic electrons used in electron hybrid simulations

$$f(p_{\parallel}, p_{\perp}) = C \exp\left(-\frac{p_{\parallel}^2}{2U_{t\parallel}^2}\right) g(p_{\perp})$$

$$g(p_{\perp}) = \frac{1}{1-\beta}\left\{\exp\left(-\frac{p_{\perp}^2}{2U_{t\perp}^2}\right) - \exp\left(-\frac{p_{\perp}^2}{2\beta U_{t\perp}^2}\right)\right\}$$



Initial velocity distribution of energetic electrons at the magnetic equator [e.g., *Katoh and Omura*, JGR 2011]

$$A_T = (1+\beta)\frac{U_{t\perp}^2}{U_{t\parallel}^2} - 1$$

[*cf*. *Tang et al.*, AnG 2014]

Simulation results of different temperature anisotropies of energetic electrons

- Run 1: A_T = 9.0
- Run 2: A_T = 7.0
- Run 3: A_T = 6.0
- Run 4: A_T = 5.0
- Run 5: A_T = 4.0

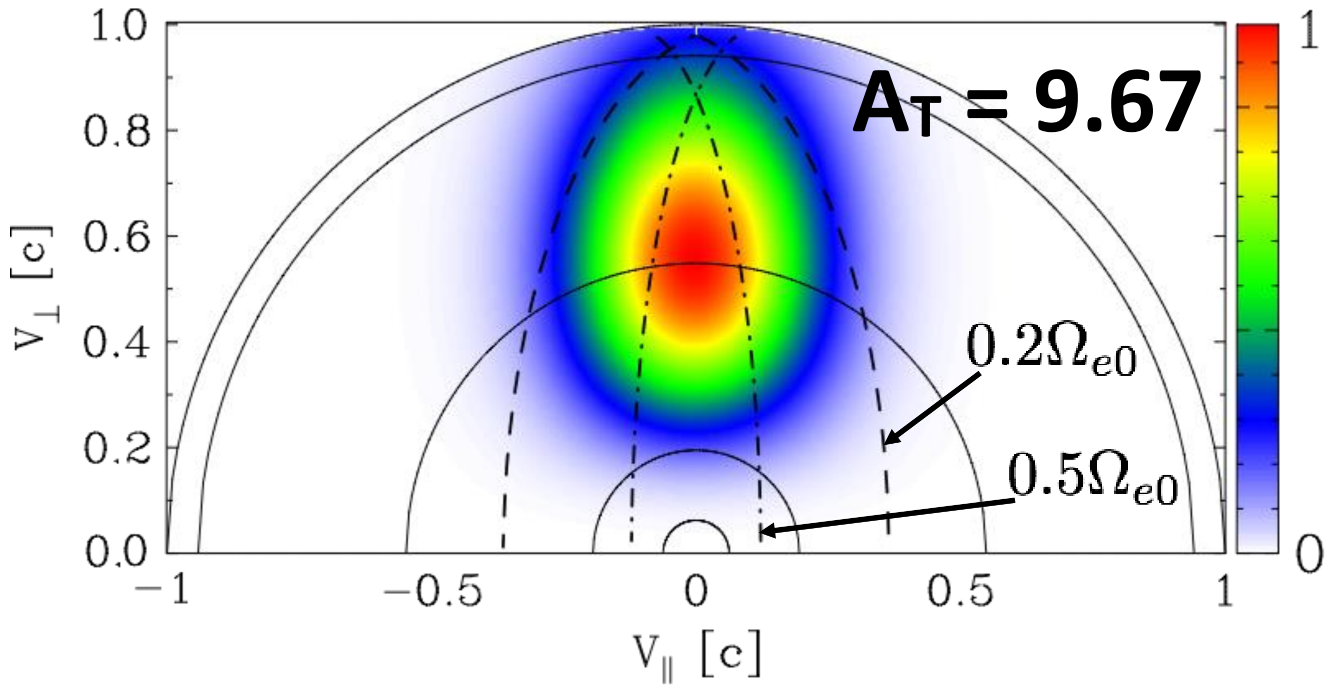
N_h = 2.99 x 10^{-4} N_0

5.98 x 10^{-4} N_0

6.72 x 10^{-4} N_0

7.47 x 10^{-4} N_0

8.22 x 10^{-4} N_0



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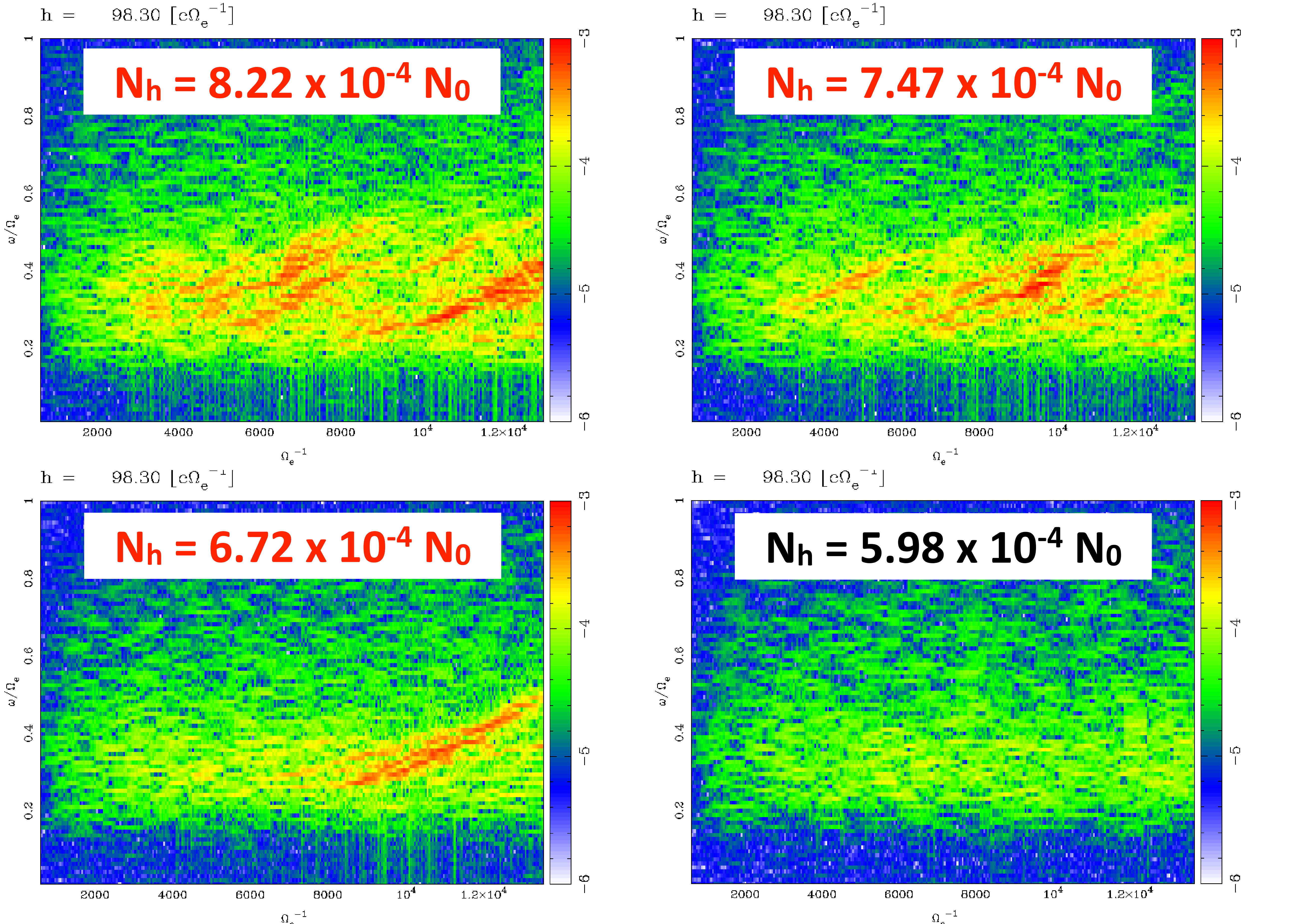
$$A_T = (1+\beta)\frac{U_{t\perp}^2}{U_{t\parallel}^2} - 1$$

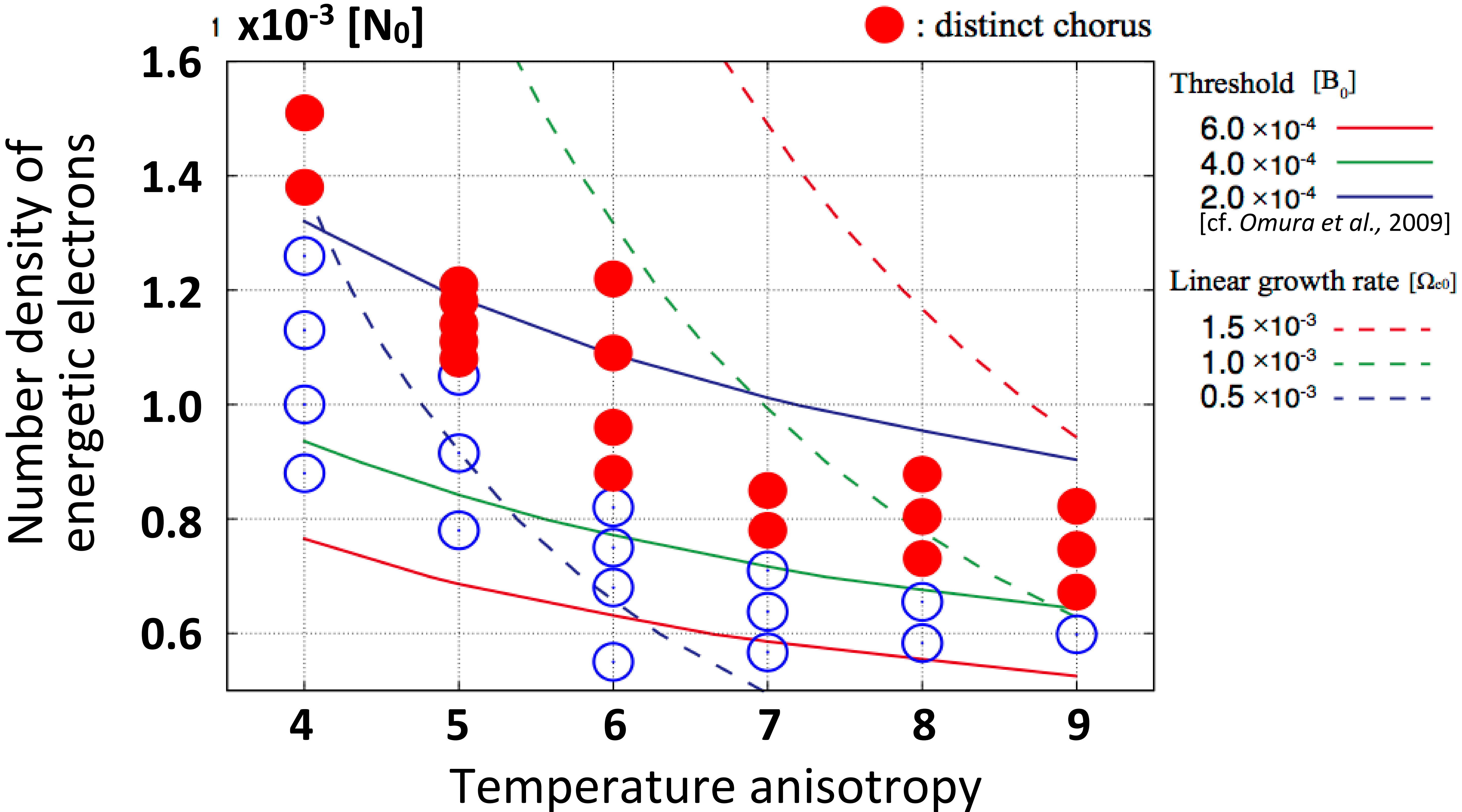
[*cf. Tang et al.*, AnG 2014]

Run 1 (A_T = 9.0): spectra observed at northern hemisphere



Summary of Case 1: Chorus emissions are reproduced when the wave amplitude exceeds the threshold amplitude

[*Katoh et al.*, JGR 2018]



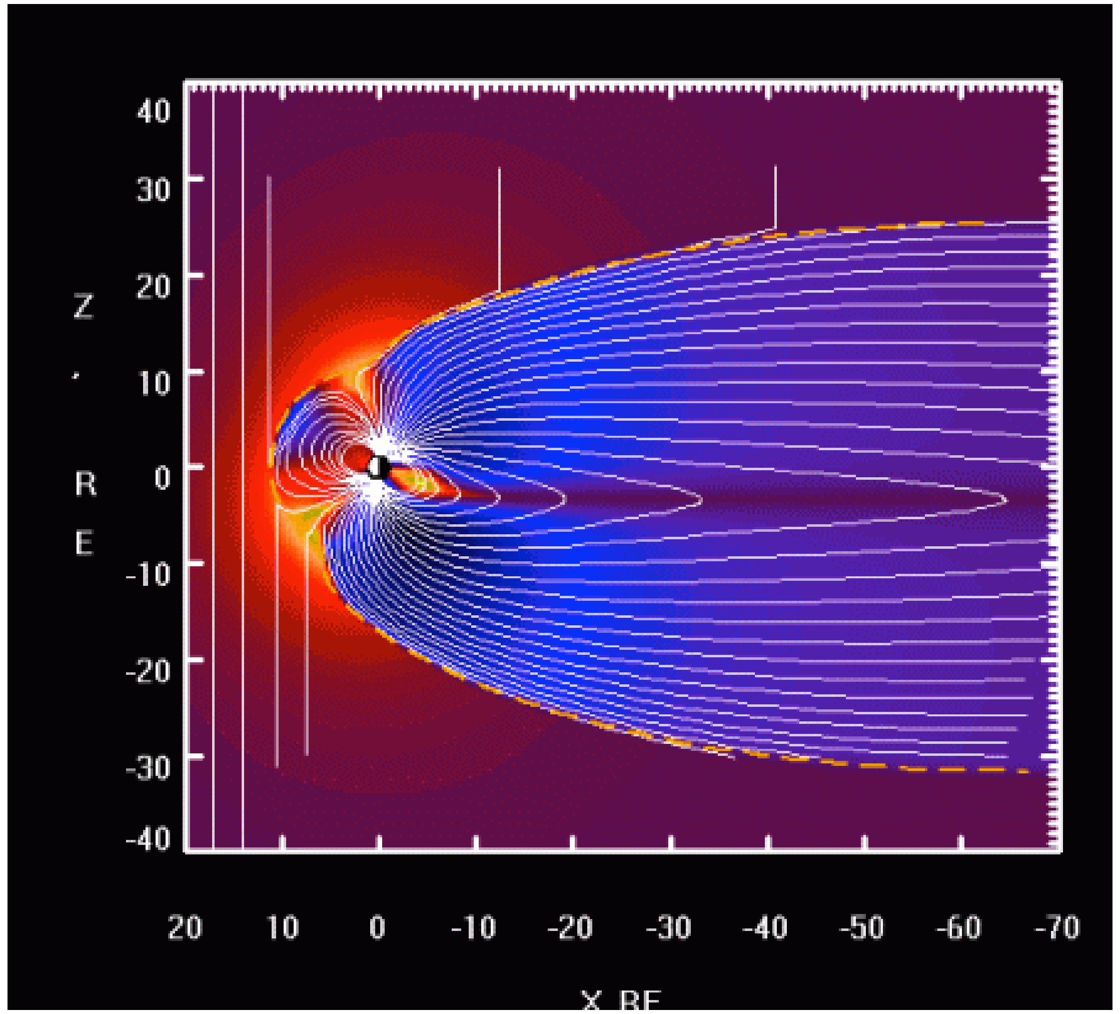
Initial conditions for the study of properties of chorus generation

- **Case 1**: Different temperature anisotropy of energetic electrons at the magnetic equator

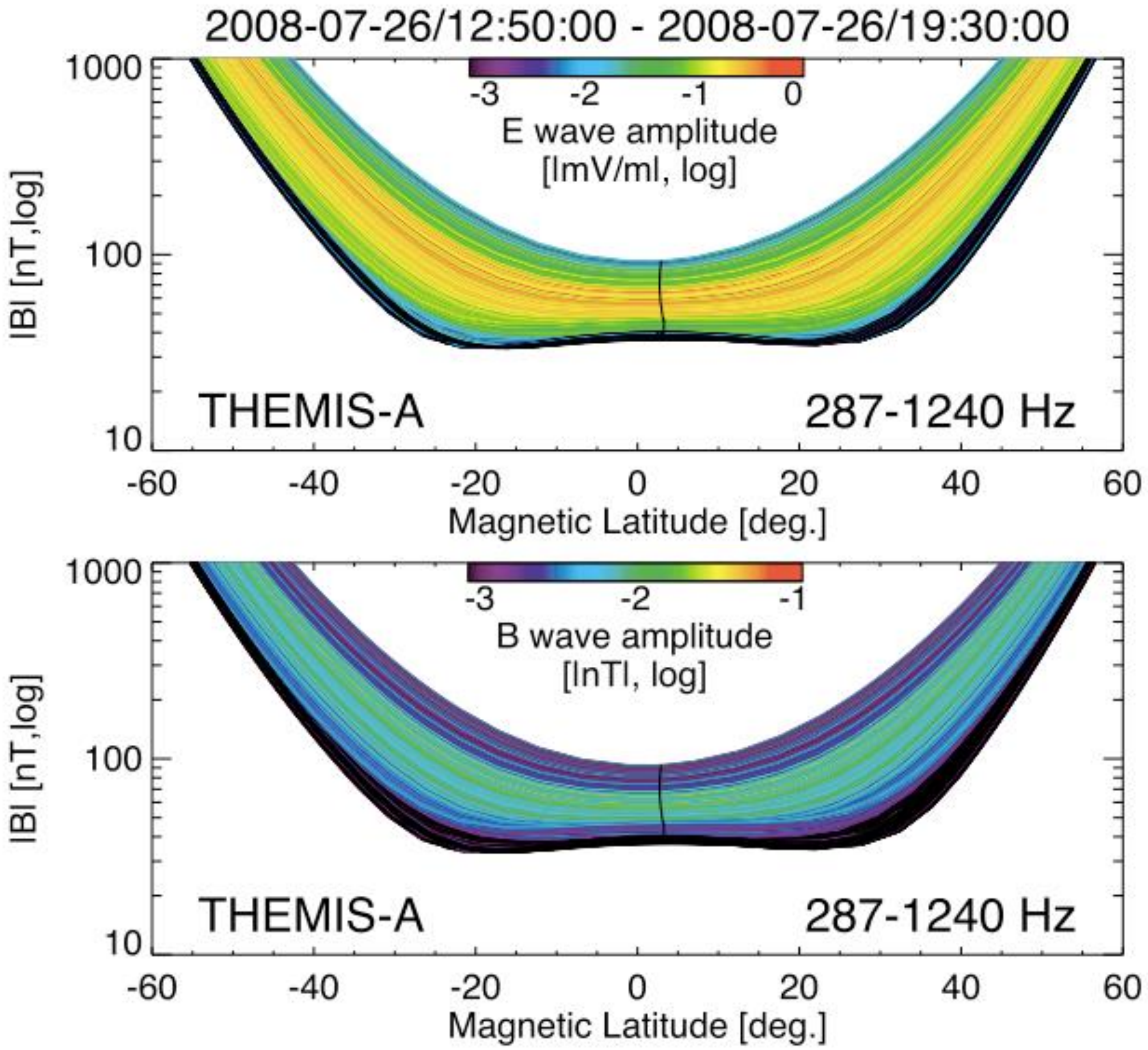
- **Case 2**: Different gradient of the background magnetic field with the same property of energetic electrons at the magnetic equator

Spatial gradient of the background magnetic field

- Nightside: magnetic field lines are stretched during a disturbed time



[*Tsyganenko*, 1989]

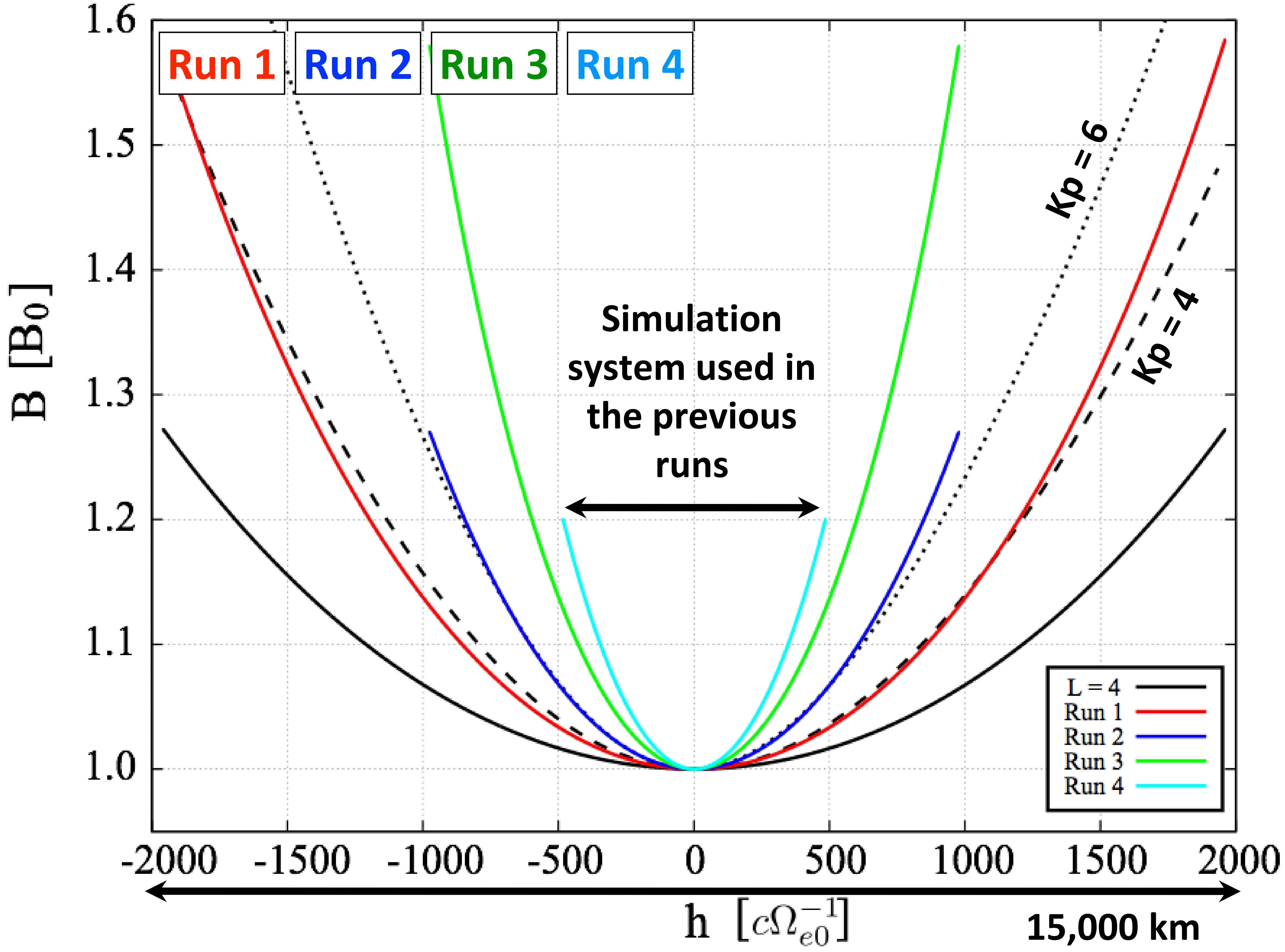


[*Keika et al.*, 2012]

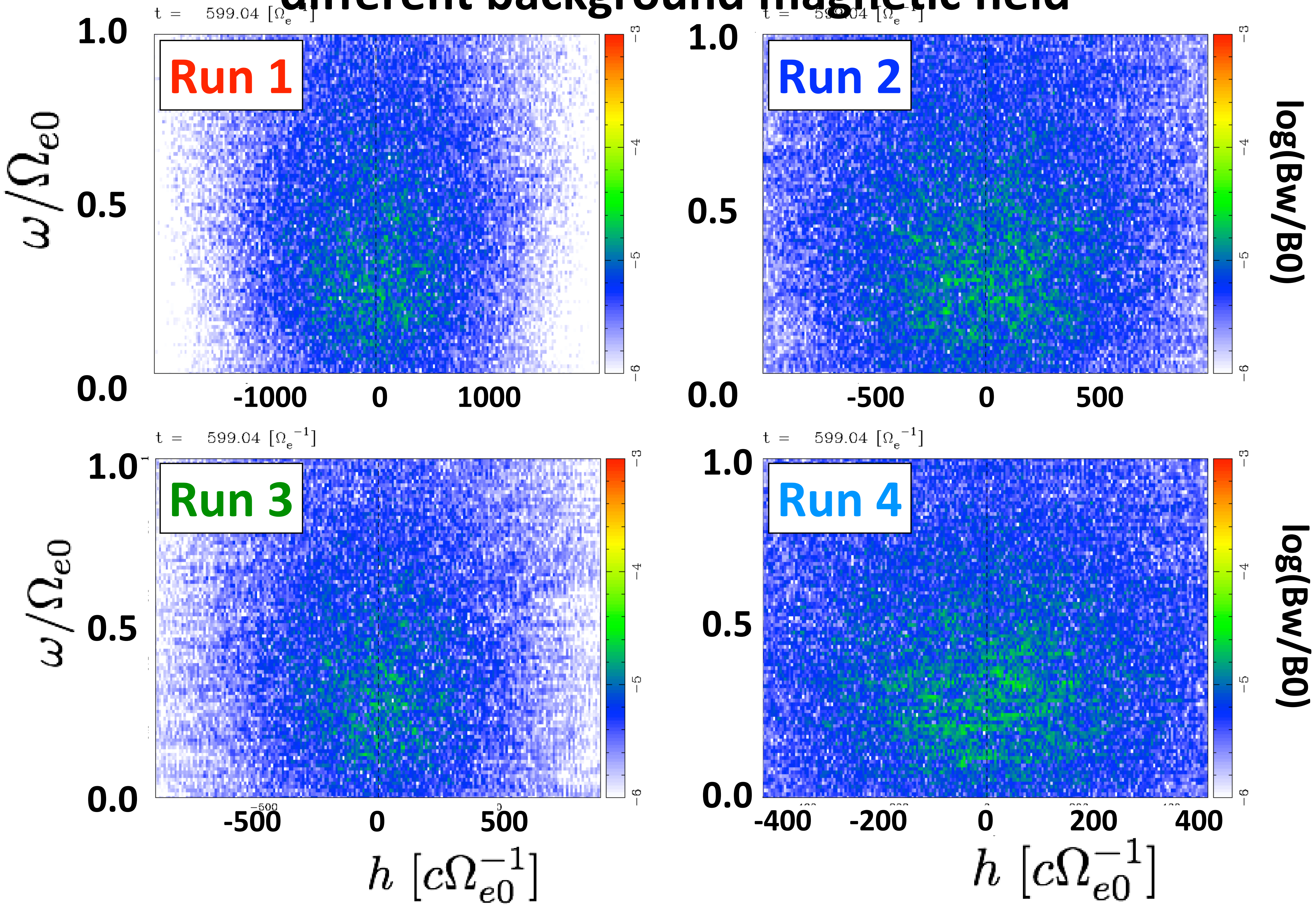
- Dayside: magnetospheric configuration is distorted by solar wind compression

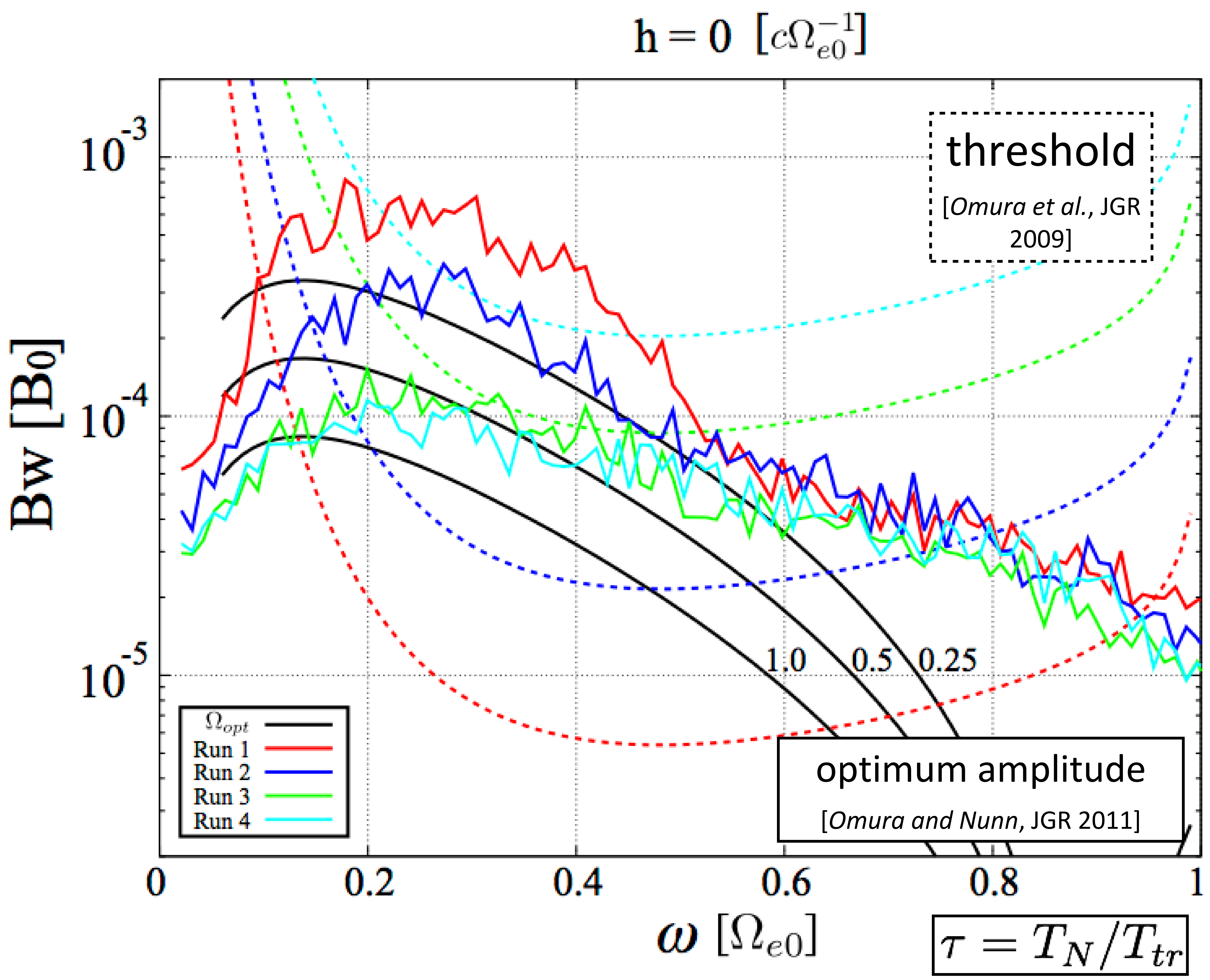
Simulations of the same initial condition of energetic electrons and different background magnetic field

[*Katoh and Omura*, JGR 2013]

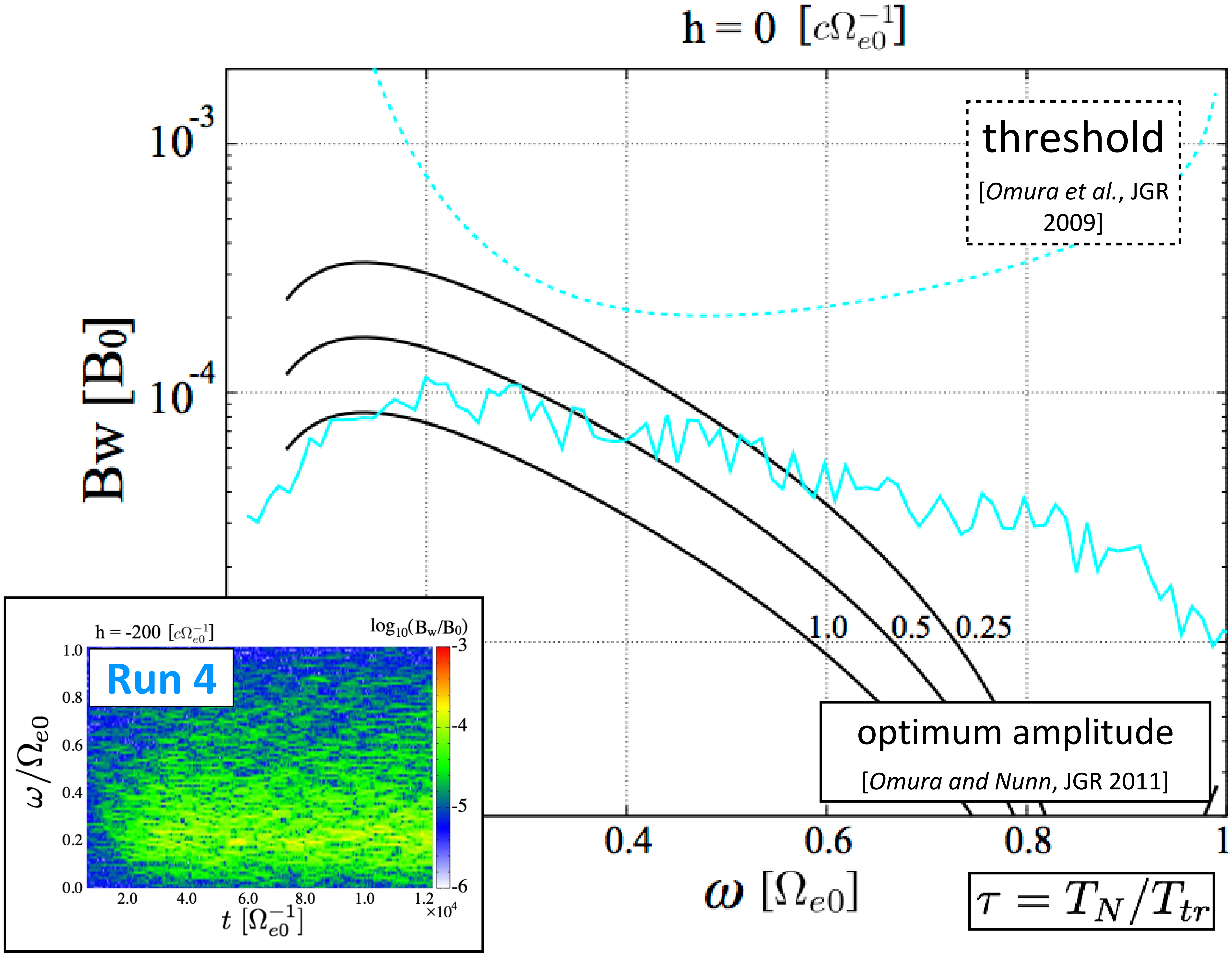


Simulations of the same initial condition of energetic electrons and different background magnetic field

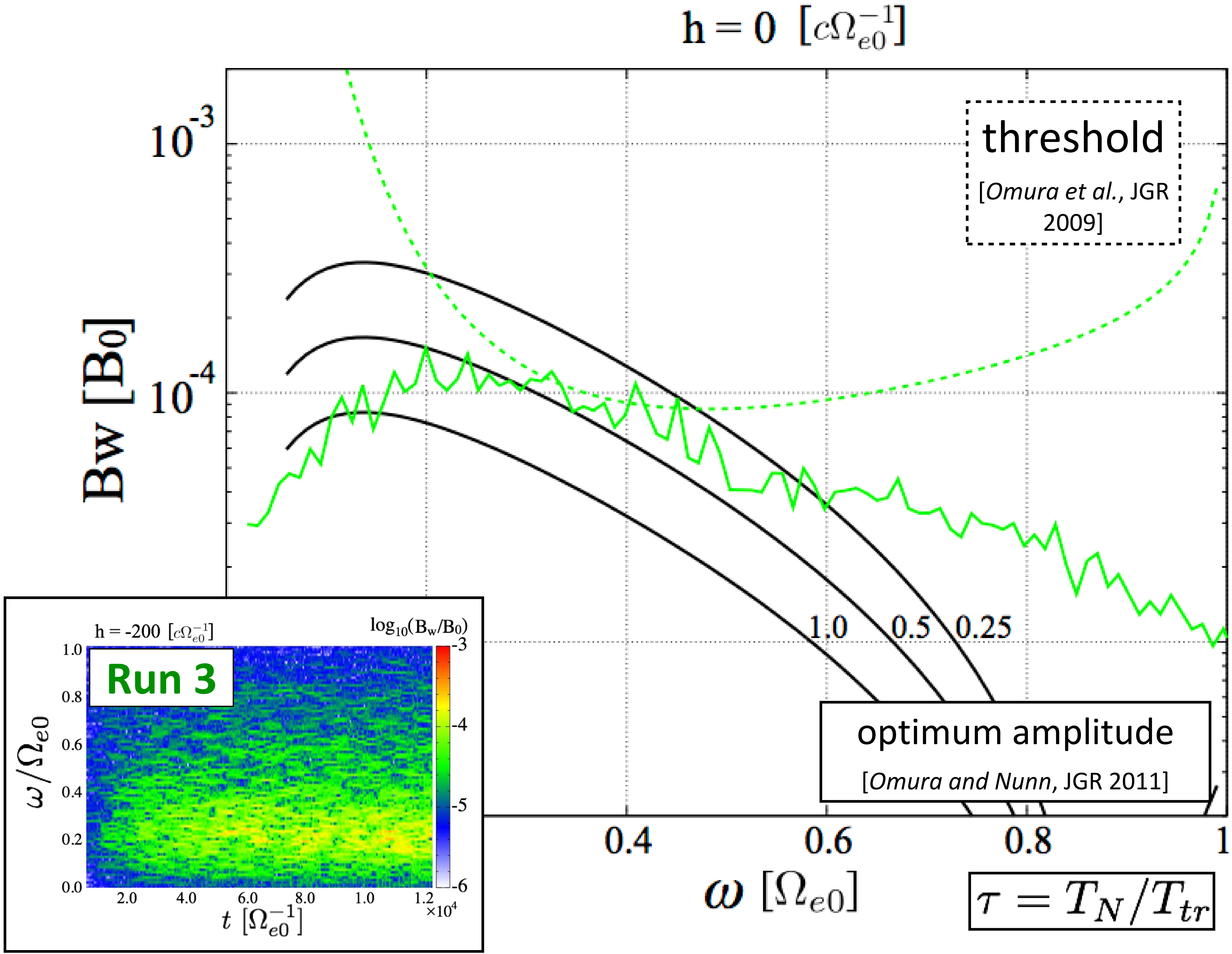




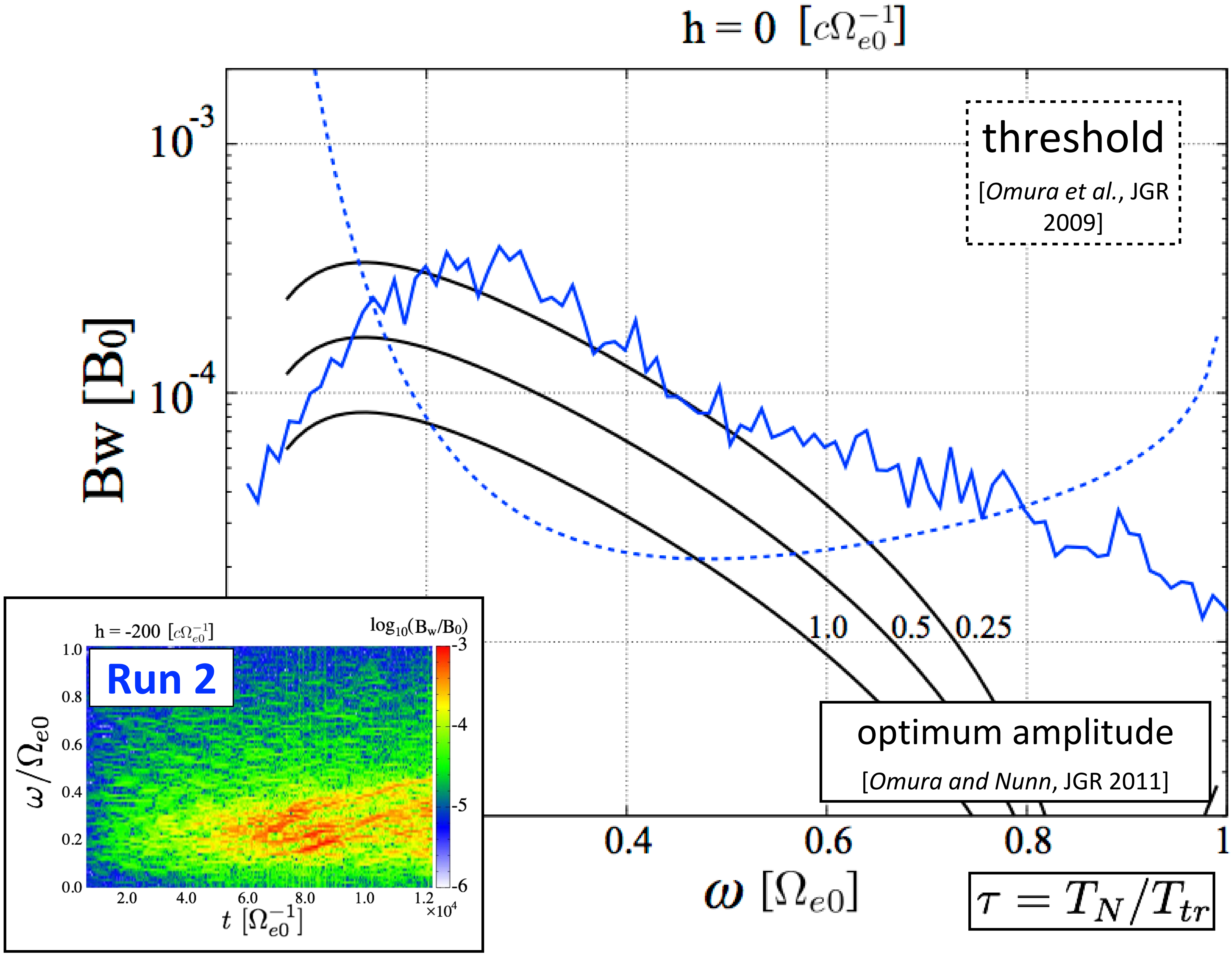
h = 0 $[c\Omega_{e0}^{-1}]$
Bw [B0]
10^{-3}
10^{-4}
10^{-5}
0
0.2
0.4
0.6
0.8
1
ω $[\Omega_{e0}]$
threshold
[*Omura et al.*, JGR 2009]
1.0
0.5
0.25
Ω_{opt}
Run 1
Run 2
Run 3
Run 4
optimum amplitude
[*Omura and Nunn*, JGR 2011]
$\tau = T_N/T_{tr}$



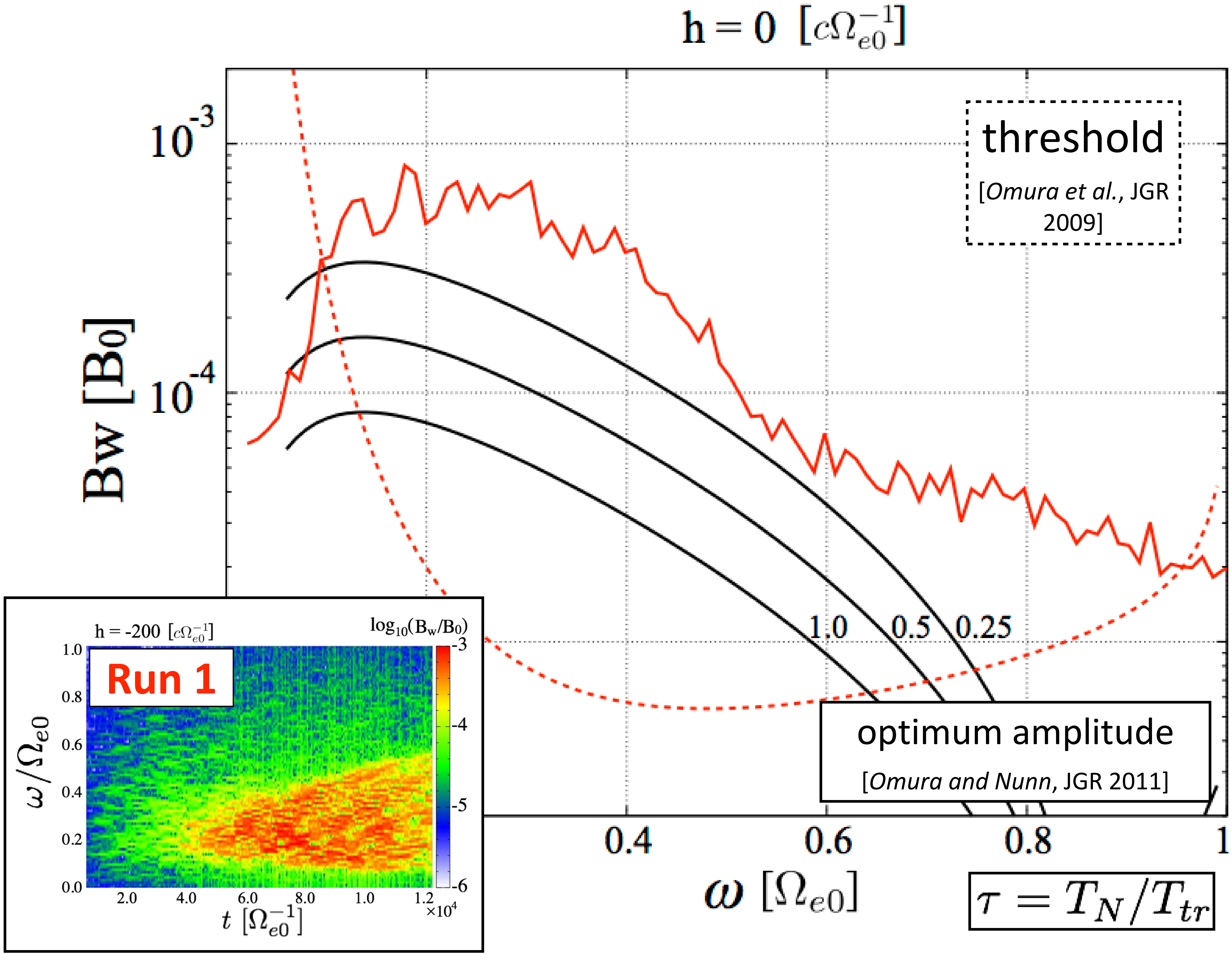
h = 0 [$c\Omega_{e0}^{-1}$]
threshold
[*Omura et al.*, JGR 2009]
optimum amplitude
[*Omura and Nunn*, JGR 2011]
Bw [B0]
10^{-3}
10^{-4}
1.0
0.5
0.25
0.4
0.6
0.8
1
ω [Ω_{e0}]
$\tau = T_N/T_{tr}$
h = -200 [$c\Omega_{e0}^{-1}$]
$\log_{10}(B_w/B_0)$
Run 4
ω/Ω_{e0}
t [Ω_{e0}^{-1}]
$\times 10^4$



h = 0 [cΩ_e0^-1]
threshold
[Omura et al., JGR 2009]
optimum amplitude
[Omura and Nunn, JGR 2011]
Bw [B0]
10^-3
10^-4
1.0
0.5
0.25
0.4
0.6
0.8
1
ω [Ω_e0]
τ = T_N/T_tr
h = -200 [cΩ_e0^-1]
log10(Bw/B0)
Run 3
ω/Ω_e0
t [Ω_e0^-1]
×10^4



h = 0 [cΩe0^-1]
threshold
[Omura et al., JGR 2009]
Bw [B0]
10^-3
10^-4
1.0
0.5
0.25
optimum amplitude
[Omura and Nunn, JGR 2011]
0.4
0.6
0.8
1
ω [Ωe0]
τ = TN/Ttr
Run 2
h = -200 [cΩe0^-1]
log10(Bw/B0)
ω/Ωe0
t [Ωe0^-1]
×10^4



h = 0 $[c\Omega_{e0}^{-1}]$
threshold
[Omura et al., JGR 2009]
optimum amplitude
[Omura and Nunn, JGR 2011]
1.0
0.5
0.25
Bw [B0]
10^{-3}
10^{-4}
0.4
0.6
0.8
1
$\omega\ [\Omega_{e0}]$
$\tau = T_N/T_{tr}$
h = -200 $[c\Omega_{e0}^{-1}]$
$\log_{10}(B_w/B_0)$
Run 1
ω/Ω_{e0}
$t\ [\Omega_{e0}^{-1}]$
$\times10^4$

Summary

- We conducted a series of electron hybrid simulations for the study of dependencies of the generation process of whistler-mode chorus emissions
- We carried out (1) different temperature anisotropy of energetic electrons at the magnetic equator and (2) different gradient of the background magnetic field with the same property of energetic electrons at the magnetic equator
- Simulation results revealed that **nonlinear processes arise when the wave amplitude exceeds the threshold wave amplitude for the chorus generation**
- These dependencies should be tested by *in situ* observations and by observations at ground stations

Threshold of the wave amplitude $B_{w,th}$ for nonlinear wave growth

[*Omura et al.*, JGR 2009]

$$\frac{\boxed{B_{w,th}}}{B_0} = \tilde{\Omega}_{th}$$

$$\boxed{\begin{aligned} \Omega_e(h) &= \Omega_{e0}(1 + ah^2) \\ a &= A \times 4.5/(LR_E)^2 \end{aligned}}$$

$$\tilde{\Omega}_{th} = \frac{100\pi^3\gamma^3\xi}{\tilde{\omega}\tilde{\omega}_{ph}^4 \tilde{V}_{\perp 0}^5 \delta^5} \left(\frac{\boxed{\tilde{a}} s_2 \tilde{U}_{t\parallel}}{Q} \right)^2 \exp\left(\frac{\gamma^2 \tilde{V}_R^2}{\tilde{U}_{t\parallel}^2} \right)$$

$$s_2 = \frac{1}{2\xi\delta} \left\{ \frac{\gamma\omega}{\Omega_e} \left(\frac{V_{\perp 0}}{c} \right)^2 - \left[2 + \Lambda \frac{\delta^2(\Omega_e - \gamma\omega)}{\Omega_e - \omega} \right] \frac{V_R V_p}{c^2} \right\}$$

$$\tilde{\omega}_{ph} = \omega_{ph}/\Omega_{e0}$$

$$\omega_{ph} = \omega_{pe}\sqrt{N_h/N_0}$$

$$\Lambda = 1 \qquad Q = 0.5$$

$$\delta^2 = (1 + \xi^2)^{-1}$$

$$\xi^2 = \omega(\Omega_e - \omega)/\omega_{pe}^2$$