THE PARAMETERIZATION OF WAVE-PARTICLE INTERACTIONS IN THE OUTER RADIATION BELT

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This research was funded in part by the Natural Environment Research Council (NERC) Highlight Topic Grant #NE/P01738X/1 (Rad-Sat).
SUMMARY

• Variability of wave-particle interactions
• Uncertainty
• Parameterisations
• An example
• Future directions
VARIABILITY OF WAVE-PARTICLE INTERACTIONS

- ULF waves - variability relative to parameterizations
  - Jaynes, Monday: “Diffusion rates are highly event-specific”
  - Olifer, Thursday: Sometimes $D_{LL}$ inferred directly from event-specific observations is much larger than our current parameterization, sometimes much less.

- EMIC waves - evaluating effectiveness of wpi
  - Millan, Wednesday: Presence of EMIC waves not always sufficient condition for precipitation – perhaps local plasma conditions are controlling interaction

- Whistler-mode waves – evaluating effectiveness of wpi
  - Blum, Wednesday: nature of wpi depends on local composition, $wpe/wce$ ratio as well as wave properties
TWO MAIN POINTS

• Natural variability in system

• Construction of wave-particle interaction parameterizations
IS SYSTEM DETERMINISTIC?

• Natural system - Chaotic

• Uncertainty might not be due to ignorance, but due to stochastic nature of processes.

• We suggest that the necessary diffusion treatment of Outer Radiation Belt dynamics has an “irreducible uncertainty.” [Palmer and Williams, Proc. Roy. Soc. A., 2008]
SOURCES OF UNCERTAINTY

• Parameterization
• Initial conditions
• Boundary conditions – real and energy space
• Numerical methods
• Underlying physical equations (!!)
INCLUDING VARIANCE IN PARAMETERIZATION

- Numerical Weather Prediction and Climate Modelling now embracing stochastic parameterization [e.g. Berner et al., BAMS, 2017]
  - Need to know underlying distribution of parameters
  - Numerical schemes need stochastic nature built in
    - Can run “ensembles”
    - Can fold in underlying distribution if well-behaved (e.g. Gaussian or log-normal)

PARAMETERIZATIONS

• Usually based upon geomagnetic activity and location

• How do we assess how good they are?
A GOOD PARAMETERIZATION

- A “good” parameterization is one which limits the variance \( \sigma \) in the quantity you are trying to predict, \( X \)
- \( \sigma \) should be small compared to the change in mean/median value with \( P \)

\[
P_1, P_2, P_3
\]
IS THAT THE CASE FOR WAVE PARAMETERIZATIONS?

Murphy et al., [2016]  

Whistler-mode wave power @ GEO in morning sector

Watt et al., [2017]
CONSTRUCTION OF PARAMETERIZATIONS - 1

• Let’s get specific:
  • ELF/VLF momentum-space diffusion

\[
D_{\alpha\alpha}(L^+, p, \alpha, t) = \frac{e^2}{4\pi} \sum_n \left[ \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \frac{d\theta}{\cos\theta} \sum_i \frac{\hat{B}_i^2(\omega_i, t)G(\theta)|\Phi_{n,k}|^2}{|v_{||} - \partial\omega/\partial k_{||}|_{k_{||,j}} \left(\frac{n\Omega_{\text{ce}} - \omega_i\sin^2\alpha}{\cos\alpha}\right)^2}\right]
\]

- wavenormal angle (obs)
- wave intensity (obs)
- Range of integral obtained from obs

Requires Stix parameters \((\omega_{pe}/\Omega_e, \text{composition} - \text{obs})\)
CONSTRUCTION OF PARAMETERIZATIONS - 2

• Inputs usually modelled independently
  • Time-average of observations obtained over many years
  • Semi-empirical models for B, n

• What happens if you construct $D_{\alpha\alpha}$ from individual samples of magnetospheric parameters, then look at distribution/statistical description?
PILOT STUDY

- CRRES data
- Diffusion due to hiss for $2.5 < L < 3.5$
- Any MLT
- 187 points

- CRRES collects simultaneous
  - $f_{pe}/f_{ce}$
  - Wave intensity

\[
\log_{10}(\text{intensity}) = \log_{10}(f_{pe}/f_{ce})
\]
PLASMA/GYRO-FREQUENCY

The diagrams illustrate the occurrence of $f_{pe}/f_{ce}$ in different regions of the $L^*$ and half-orbit number parameters. The histograms show the distribution of $f_{pe}/f_{ce}$ for different AE (Apindex) ranges.
WAVE AMPLITUDE

Wave intensity

$L^*$

Half-orbit number

$150 \rightarrow 200 \rightarrow 250 \rightarrow 300 \rightarrow 350 \rightarrow 400 \rightarrow 450 \rightarrow 500$

$24 \rightarrow 26 \rightarrow 28 \rightarrow 30 \rightarrow 32 \rightarrow 34$

$1 \rightarrow 2 \rightarrow 3 \rightarrow 4$

$15 \rightarrow 100 \rightarrow 300 \rightarrow \infty$

All data

AE<100nT

100<AE<300nT

AE>300nT
PADIE DIFFUSION COEFFICIENTS

- Note, we have insufficient information from CRRES to study effects of spectral shape etc.

- Two inputs:
  - plasma:gyro frequency ratio
  - Wave intensity

- Coefficient of variation:
  - Wave amplitude: 0.98
  - plasma:gyro frequency: 0.25
  - Diffusion coefficient: 2.96
CONCLUSIONS AND FUTURE WORK

• Our parameterizations of diffusion in Outer Radiation Belt due to all wave types could be improved (see S. Bentley – this session)
  • parameterization of $D_{ij}$, not inputs to $D_{ij}$
  • $D_{ij}$ likely to have larger variance than variance due to wave amplitudes alone
  • Parameterization with activity level currently leads to very large variance compared to difference in median values
  • Seek new parameterizations that minimize the variance in the diffusion coefficient

• Diffusion models with stochastic parameterizations