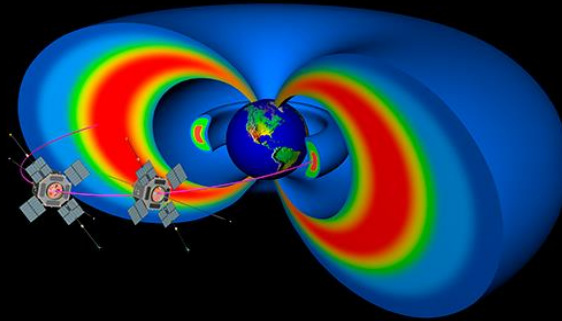


# THE PARAMETERIZATION OF WAVE-PARTICLE INTERACTIONS IN THE OUTER RADIATION BELT



**Rad-Sat**

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# SUMMARY

- Variability of wave-particle interactions
- Uncertainty
- Parameterisations
- An example
- Future directions

# VARIABILITY OF WAVE-PARTICLE INTERACTIONS

- ULF waves - variability relative to parameterizations
  - Jaynes, Monday: “Diffusion rates are highly event-specific”
  - Olfier, Thursday: Sometimes  $D_{LL}$  inferred directly from event-specific observations is much larger than our current parameterization, sometimes much less.
- EMIC waves - evaluating effectiveness of wpi
  - Millan, Wednesday: Presence of EMIC waves not always sufficient condition for precipitation – perhaps local plasma conditions are controlling interaction
- Whistler-mode waves – evaluating effectiveness of wpi
  - Blum, Wednesday: nature of wpi depends on local composition,  $wpe/wce$  ratio as well as wave properties

# TWO MAIN POINTS

- Natural variability in system
- Construction of wave-particle interaction parameterizations

# IS SYSTEM DETERMINISTIC?

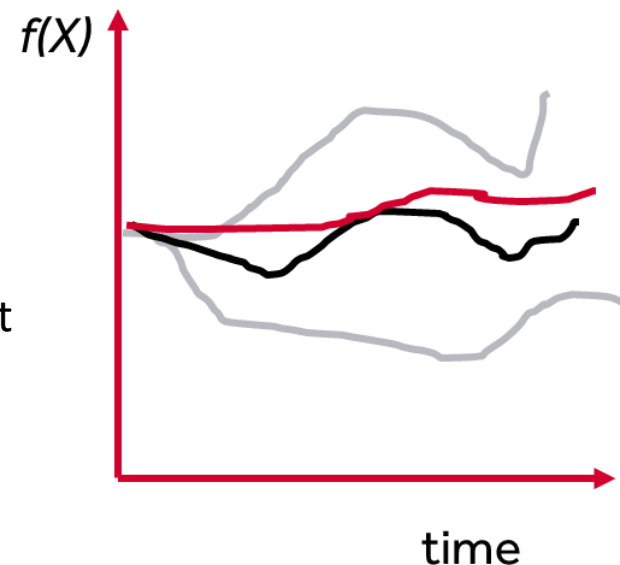
- Natural system - Chaotic
- Uncertainty might not be due to ignorance, but due to stochastic nature of processes.
- We suggest that the necessary diffusion treatment of Outer Radiation Belt dynamics has an “irreducible uncertainty.”  
*[Palmer and Williams, Proc. Roy. Soc. A., 2008]*

# SOURCES OF UNCERTAINTY

- Parameterization
- Initial conditions
- Boundary conditions – real and energy space
- Numerical methods
- Underlying physical equations (!!)

# INCLUDING VARIANCE IN PARAMETERIZATION

- Numerical Weather Prediction and Climate Modelling now embracing stochastic parameterization [e.g. Berner et al., BAMS, 2017]
  - Need to know underlying distribution of parameters
  - Numerical schemes need stochastic nature built in
    - Can run “ensembles”
    - Can fold in underlying distribution if well-behaved (e.g. Gaussian or log-normal)



Berner, J., et al. (2017), *Stochastic parameterization: Toward a new view of weather and climate models*, *Bull. Am. Meteorol. Soc.*, **98**, 565–588, doi:[10.1175/BAMS-D-15-00268.1](https://doi.org/10.1175/BAMS-D-15-00268.1).

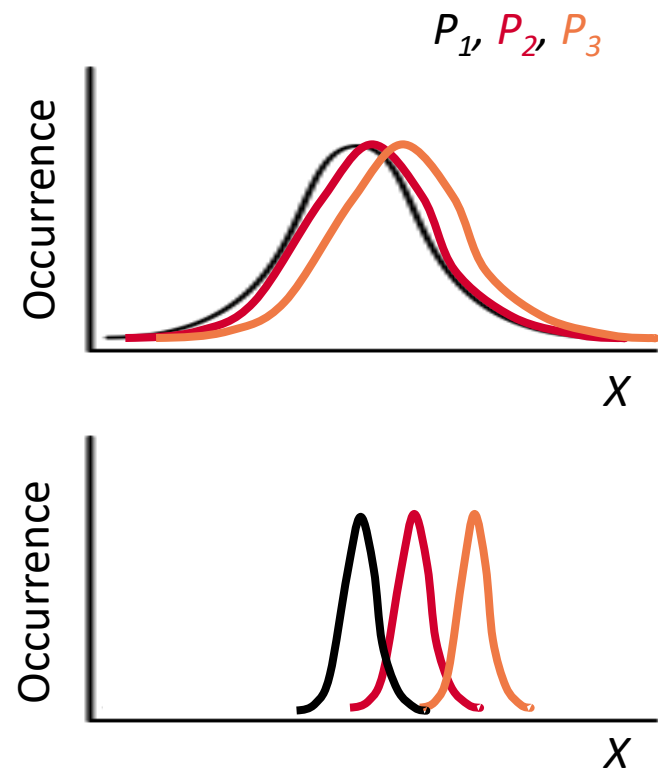
# PARAMETERIZATIONS

- Usually based upon geomagnetic activity and location
- How do we assess how good they are?



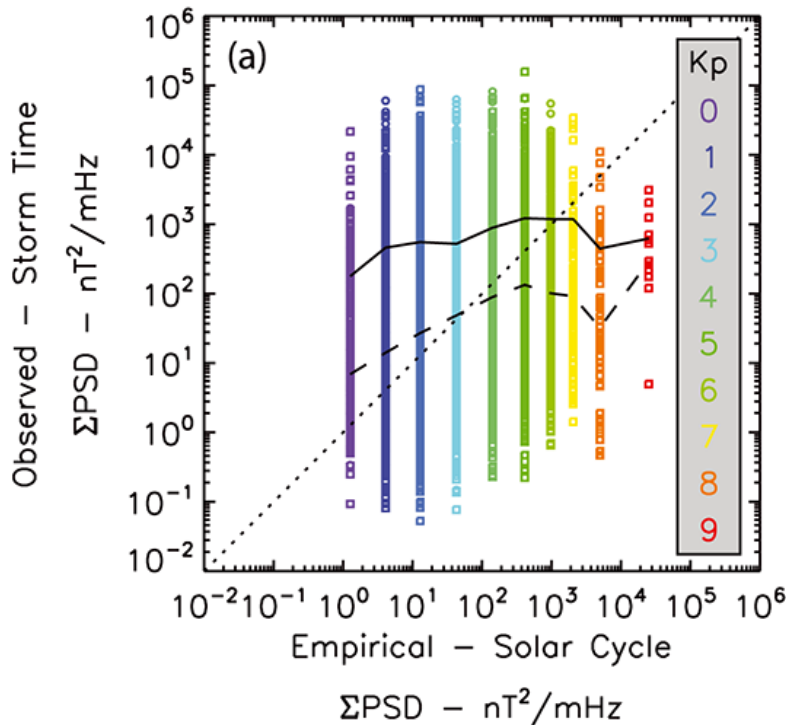
# A GOOD PARAMETERIZATION

- A “good” parameterization is one which limits the variance  $\sigma$  in the quantity you are trying to predict,  $X$
- $\sigma$  should be small compared to the change in mean/median value with  $P$



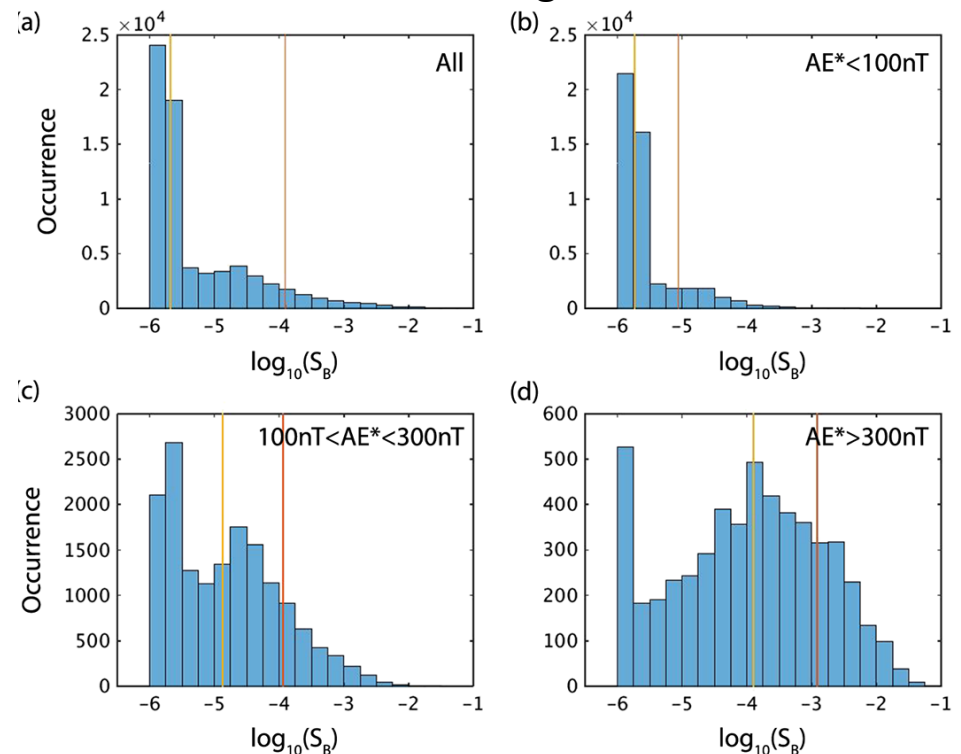
# IS THAT THE CASE FOR WAVE PARAMETERIZATIONS?

ULF Wave Power  
Storm-time vs Empirical



*Murphy et al., [2016]*

Whistler-mode wave power @  
GEO in morning sector



*Watt et al., [2017]*

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# CONSTRUCTION OF PARAMETERIZATIONS - 1

- **Let's get specific:**

- ELF/VLF momentum-space diffusion

Requires Stix parameters  
( $\omega_{pe}/\Omega_e$ , composition - obs)

$$D_{\alpha\alpha}(L^*, p, \alpha, t) = \frac{e^2}{4\pi} \sum_n \int_{\theta_{\min}}^{\theta_{\max}} \frac{d\theta}{\cos\theta} \sum_i \frac{\hat{B}_{\omega}^2(\omega_i, t) G(\theta) |\Phi_{n,k}|^2}{|v_{||} - \partial\omega/\partial k_{||}|_{k_{||,i}}} \left( \frac{\frac{n\Omega_{ce}}{\gamma} - \omega_i \sin^2 \alpha}{\cos\alpha} \right)^2$$

Annotations for the equation:

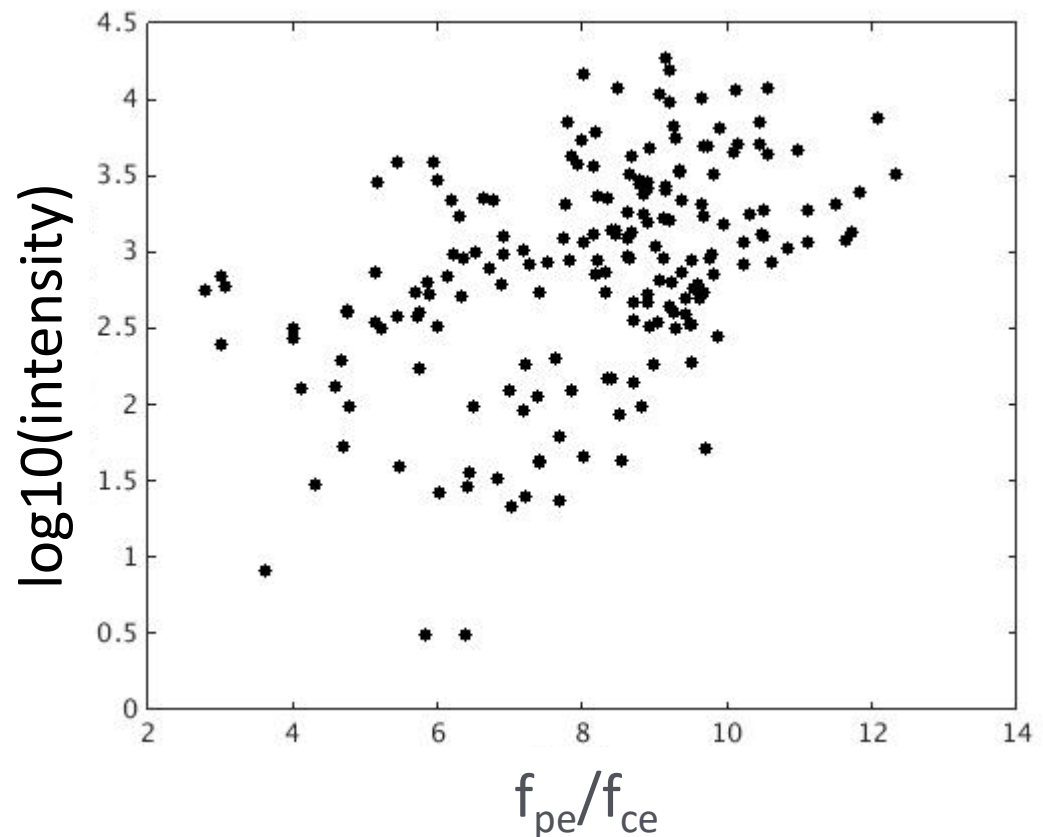
- Wavenormal angle (obs) points to  $\theta$
- Wave intensity (obs) points to  $\hat{B}_{\omega}^2(\omega_i, t)$
- Range of integral obtained from obs points to the integration limits  $\theta_{\min}$  and  $\theta_{\max}$
- Requires Stix parameters ( $\omega_{pe}/\Omega_e$ , composition - obs) points to the term  $\left( \frac{\frac{n\Omega_{ce}}{\gamma} - \omega_i \sin^2 \alpha}{\cos\alpha} \right)^2$

# CONSTRUCTION OF PARAMETERIZATIONS - 2

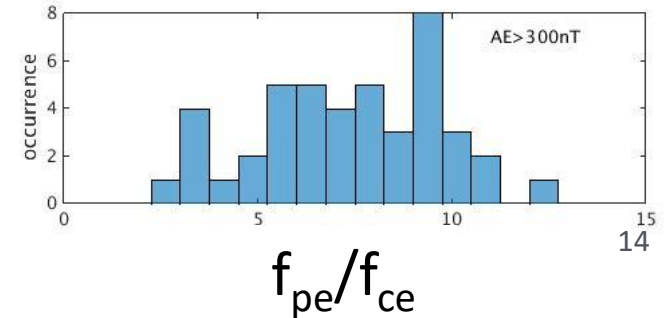
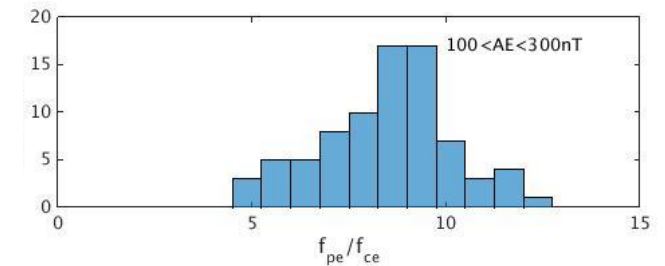
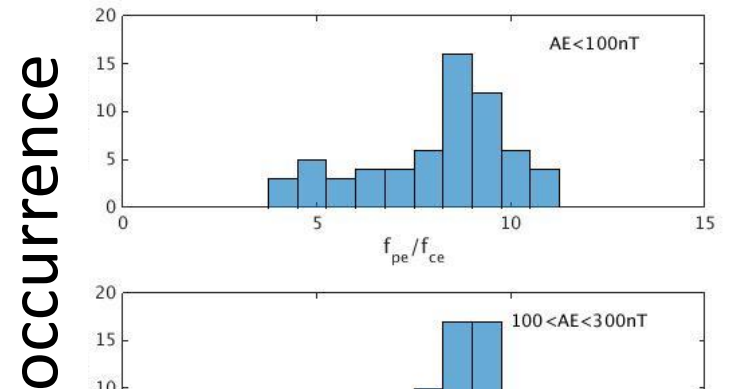
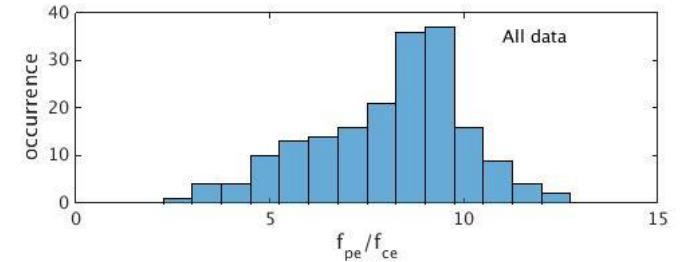
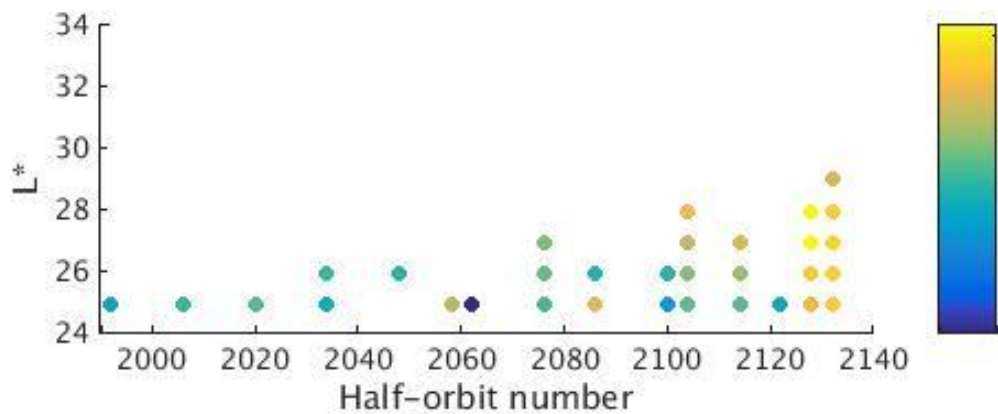
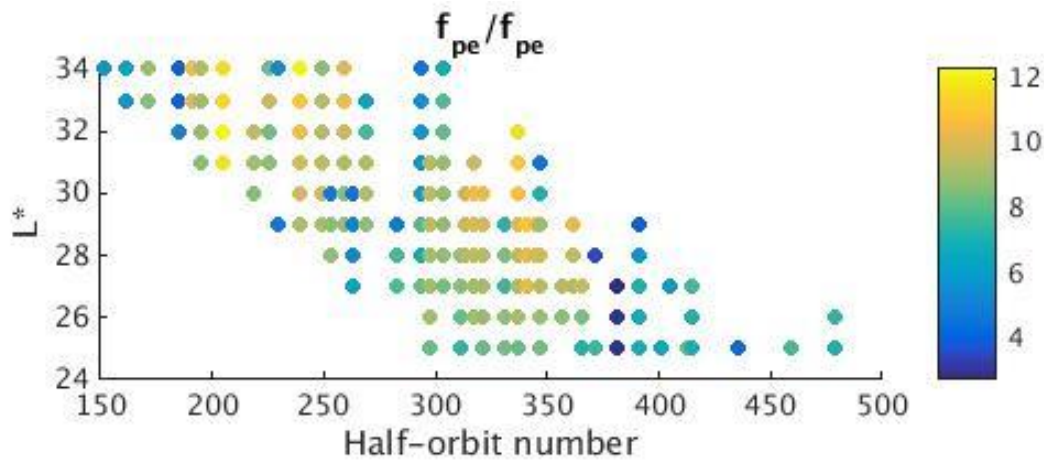
- Inputs usually modelled independently
  - Time-average of observations obtained over many years
  - Semi-empirical models for B, n
- What happens if you construct  $D_{\alpha\alpha}$  from individual samples of magnetospheric parameters, then look at distribution/statistical description?

# PILOT STUDY

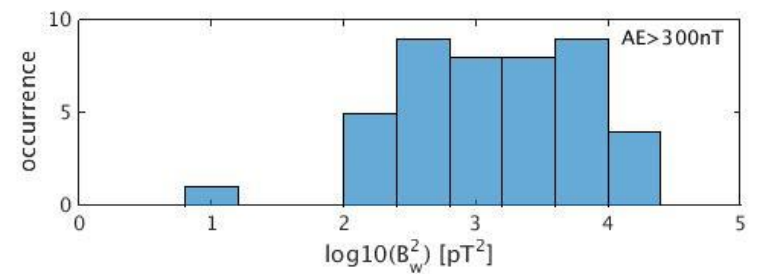
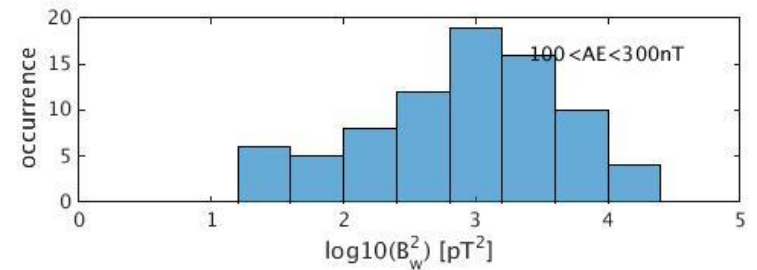
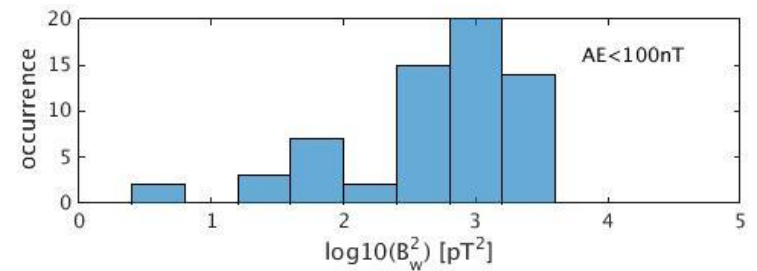
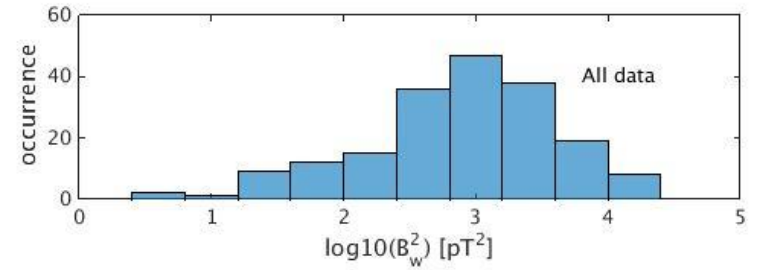
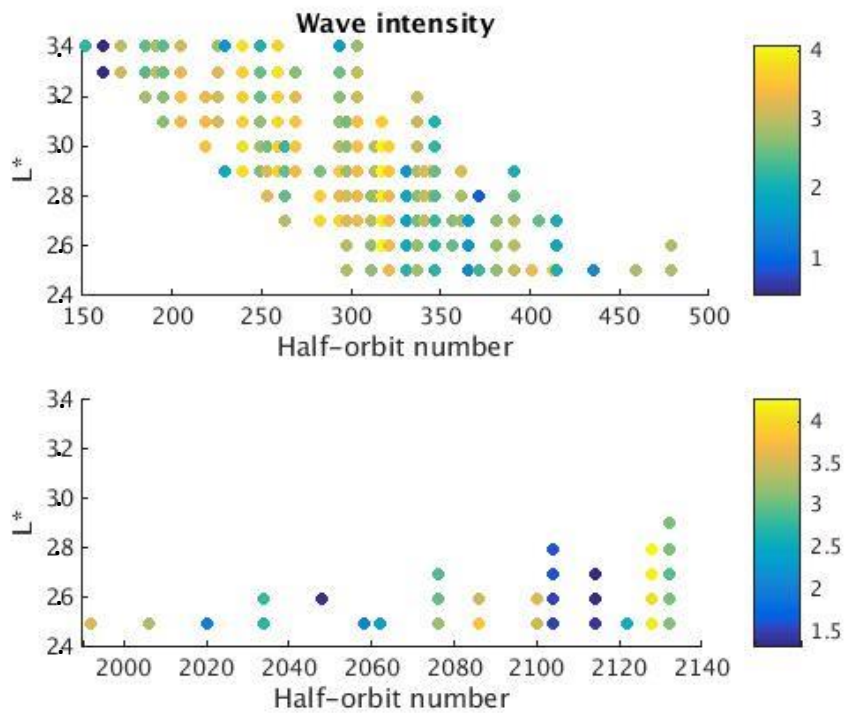
- CRRES data
- Diffusion due to hiss for  $2.5 < L < 3.5$
- Any MLT
- 187 points
  
- CRRES collects simultaneous
  - $f_{pe}/f_{ce}$
  - Wave intensity



# PLASMA/ GYRO-FREQUENCY

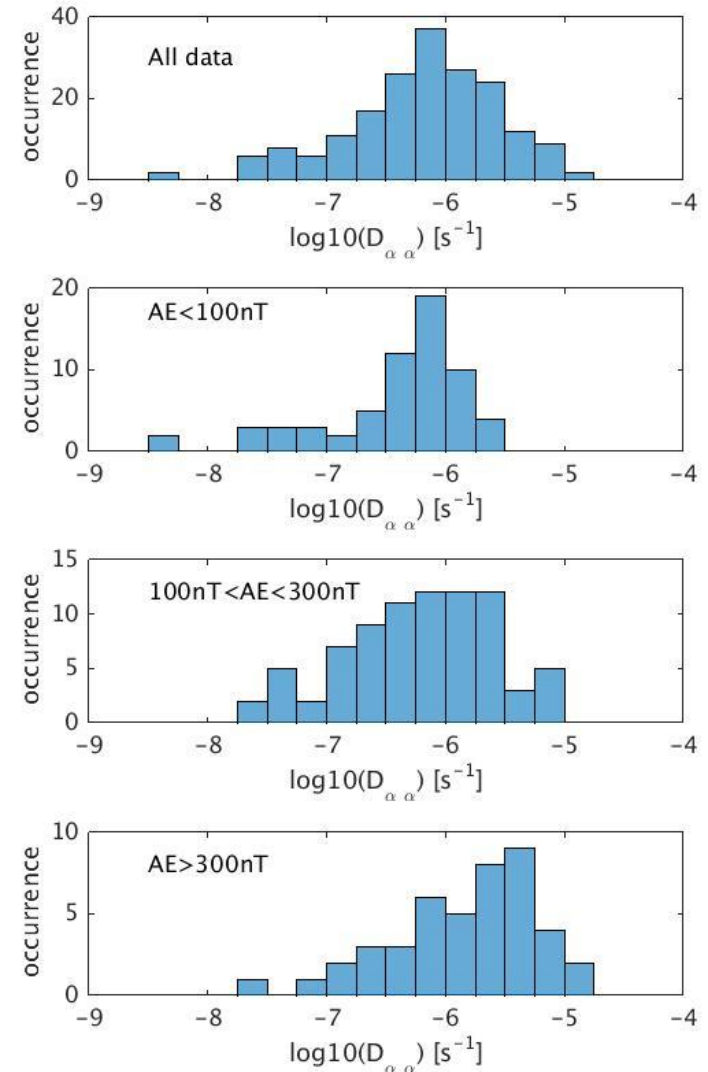


# WAVE AMPLITUDE



# PADIE DIFFUSION COEFFICIENTS

- Note, we have insufficient information from CRRES to study effects of spectral shape etc
- Two inputs:
  - plasma:gyro frequency ratio
  - Wave intensity
- Coefficient of variation:
  - Wave amplitude: 0.98
  - plasma:gyrofrequency: 0.25
  - Diffusion coefficient: 2.96





# CONCLUSIONS AND FUTURE WORK

- Our parameterizations of diffusion in Outer Radiation Belt due to all wave types could be improved (see S. Bentley – this session)
  - parameterization of  $D_{ij}$ , not inputs to  $D_{ij}$
  - $D_{ij}$  likely to have larger variance than variance due to wave amplitudes alone
  - Parameterization with activity level currently leads to very large variance compared to difference in median values
  - Seek new parameterizations that minimize the variance in the diffusion coefficient
- Diffusion models with stochastic parameterizations