Neoclassical and anomalous radial diffusion of trapped electrons in the inner belt

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Motivation and overview: MeV electron lifetimes at very low L (<1.25)

- Early history on MeV electron lifetimes for L<1.25: 1962-1973
  - 1962: Starfish high-altitude nuclear explosion creates new belt
  - 1964-66: Coulomb collisions explain lifetime (Walt, Van Allen)
  - 1967: Lifetimes 10-100x longer than expected (Imhof et al)
  - 1968: Needed D_{LL} estimated (Newkirk and Walt)
  - 1973: D_{LL} could come from “neoclassical diffusion” (Roederer et al)
- Our previous work
  - Build a 3D Fokker-Planck model for neoclassical diffusion
  - Predict lifetimes at L<1.25 and compare to historical data
- Current work: incorporate ground-based VLF transmitters to see if “anomalous diffusion” explains residual differences

Lifetime of MeV electrons at L<1.2 not controlled by Coulomb collisions!

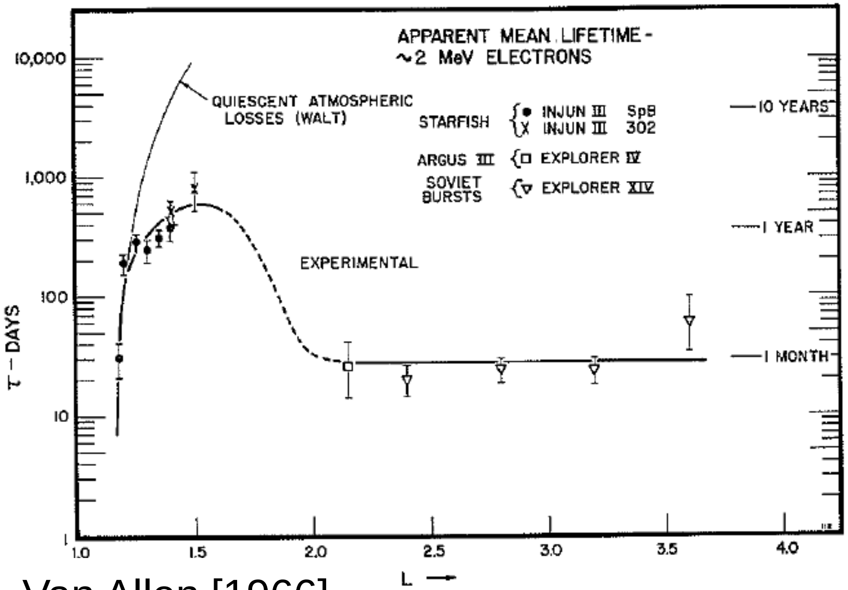
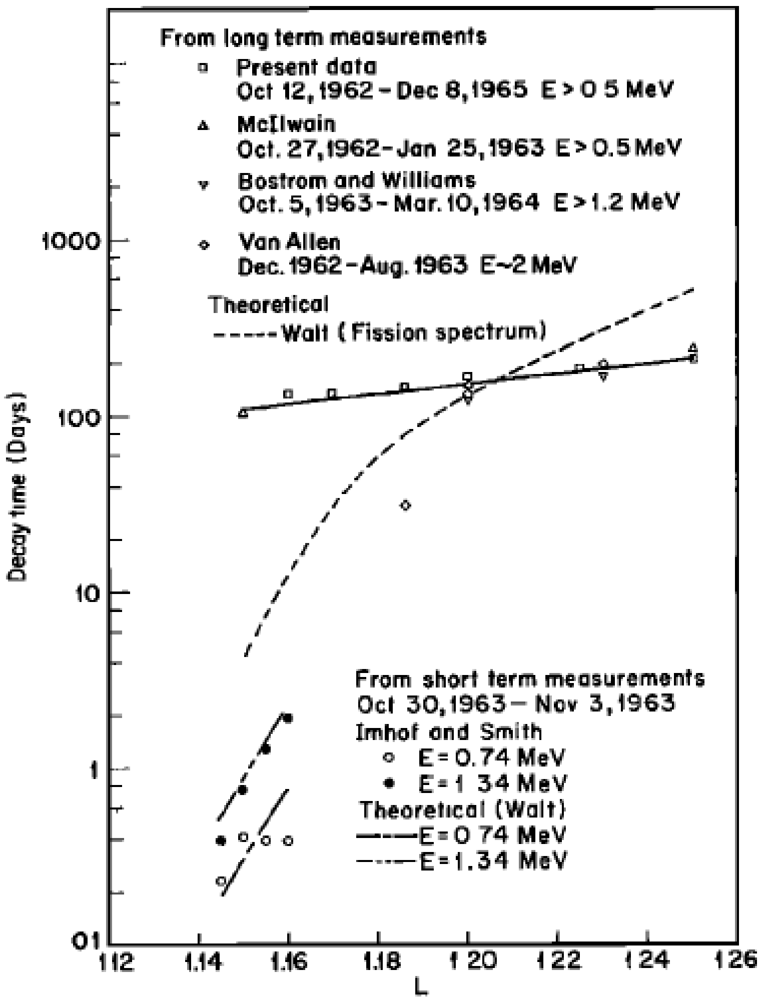


Fig. 18.

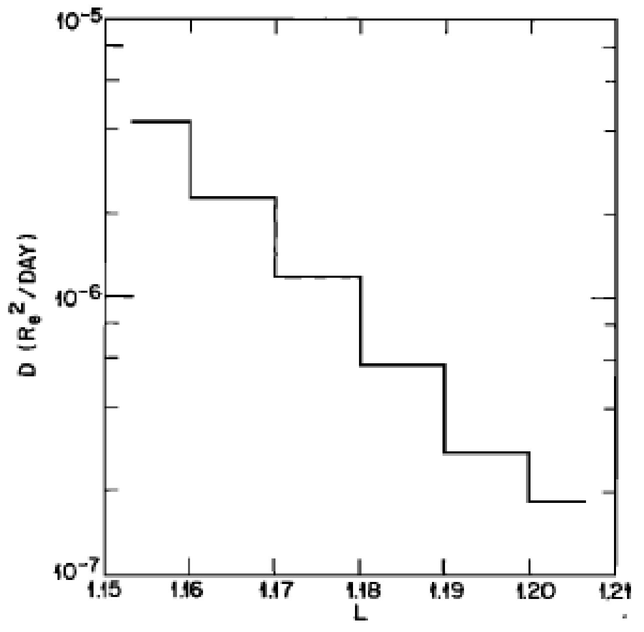
Van Allen [1966]

Lifetime measurements from Van Allen confirm Coulomb collision theory of Walt and MacDonald [1964] for L<1.25. Lifetime measurements from Imhof et al 10-100x longer at L<1.2.



Imhof et al [1967]

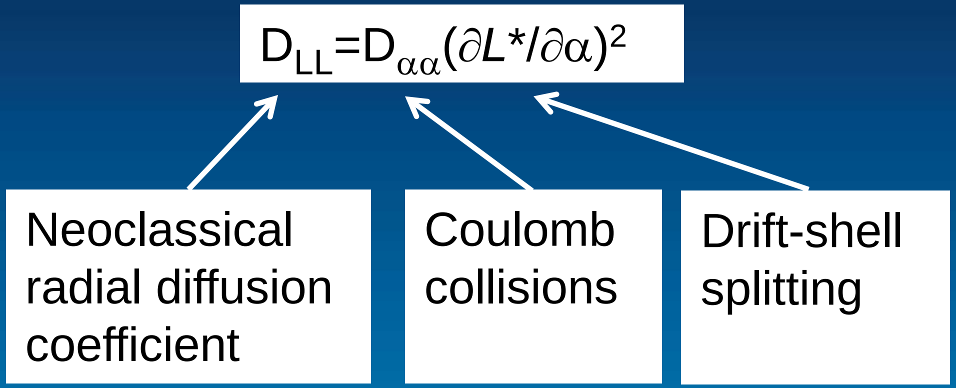
Radial diffusion inward from large L increases apparent lifetime at low L



Newkirk and Walt estimated D_{LL} (at constant μ) needed to produce apparent lifetime plot in Imhof et al.

Decreases with L!

Roederer et al [1973] showed that neoclassical diffusion produces a D_{LL} (at constant energy) of the order estimated by Newkirk and Walt:



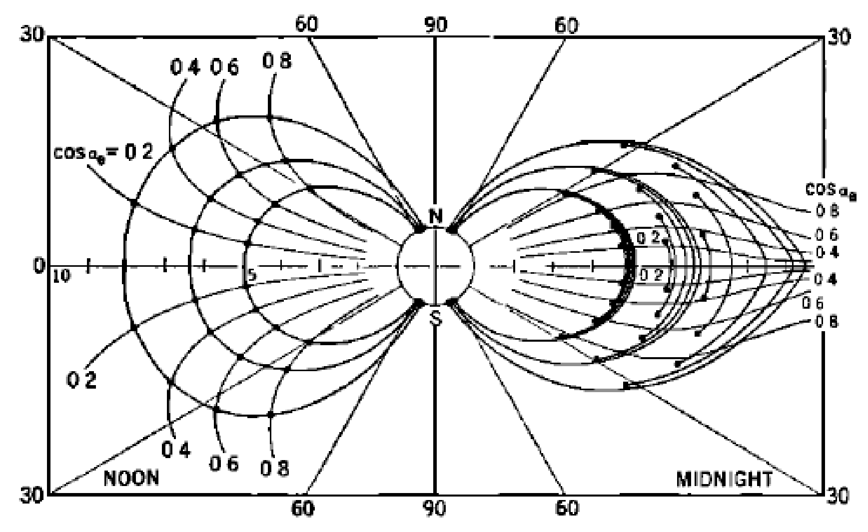
$D_{LL}=D_{\alpha\alpha}(\partial L^*/\partial\alpha)^2$

Neoclassical radial diffusion coefficient

Coulomb collisions

Drift-shell splitting

Multi-pole terms in IGRF magnetic field model cause drift-shell splitting

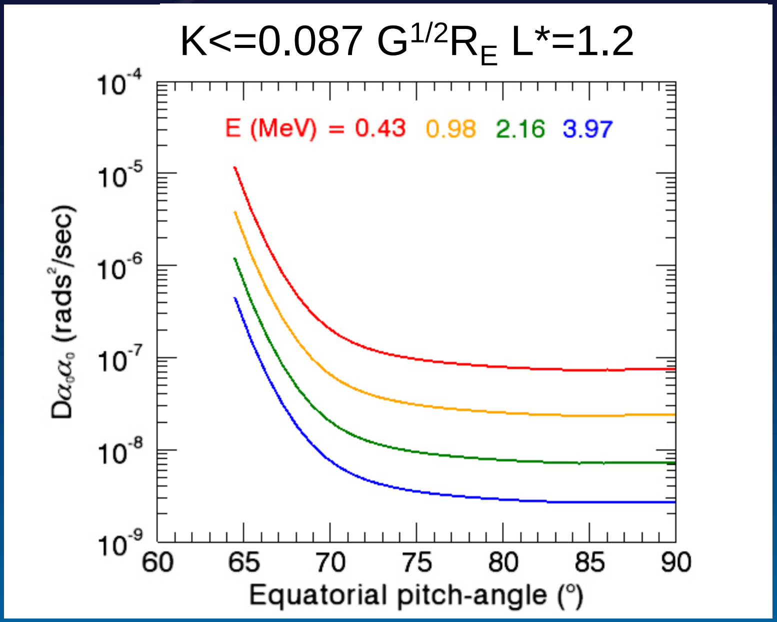


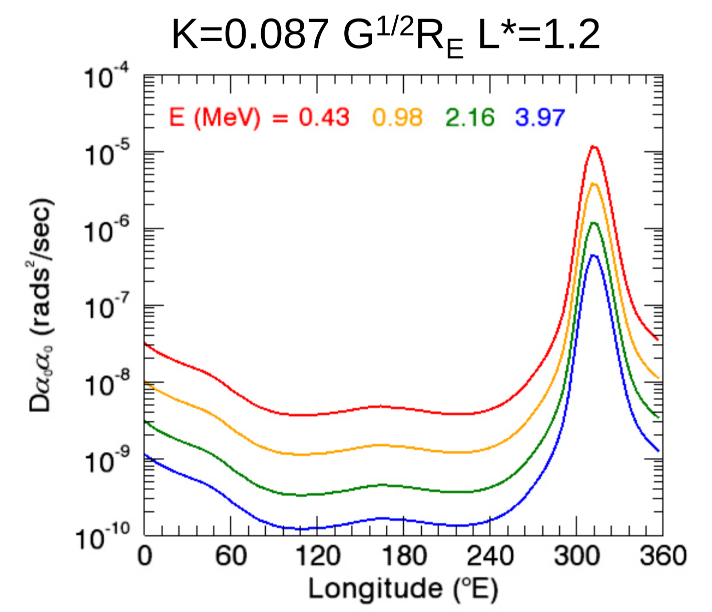
Roederer [1967]

Particles with different pitch-angle cosines that start on the same field line at noon (left) map to different field lines at midnight (right)

If we compute the L* of the full drift-shell for a given field line at two equatorial pitch-angles, α_0, and α_0+dα_0, we can compute dL*/dα_0

$D\alpha_0\alpha_0$ peaks where electron hits densest atmosphere





$D\alpha_0\alpha_0$ peaks for smallest α_0 which mirrors at lowest altitude.

$D\alpha_0\alpha_0$ varies with longitude and peaks near South Atlantic Anomaly where mirror point reaches lowest altitude due to dipole offset.

Neoclassical diffusion tensor

- At each longitude

$$\begin{bmatrix} D_{\alpha_0\alpha_0} & D_{\alpha_0 L*} \\ D_{L*\alpha_0} & D_{L*L*} \end{bmatrix} = D_{\alpha_0\alpha_0}\begin{bmatrix} 1 & \partial L^*/\partial\alpha_0 \\ \partial L^*/\partial\alpha_0 & (\partial L^*/\partial\alpha_0)^2 \end{bmatrix}$$

- Convert to (K,L*)

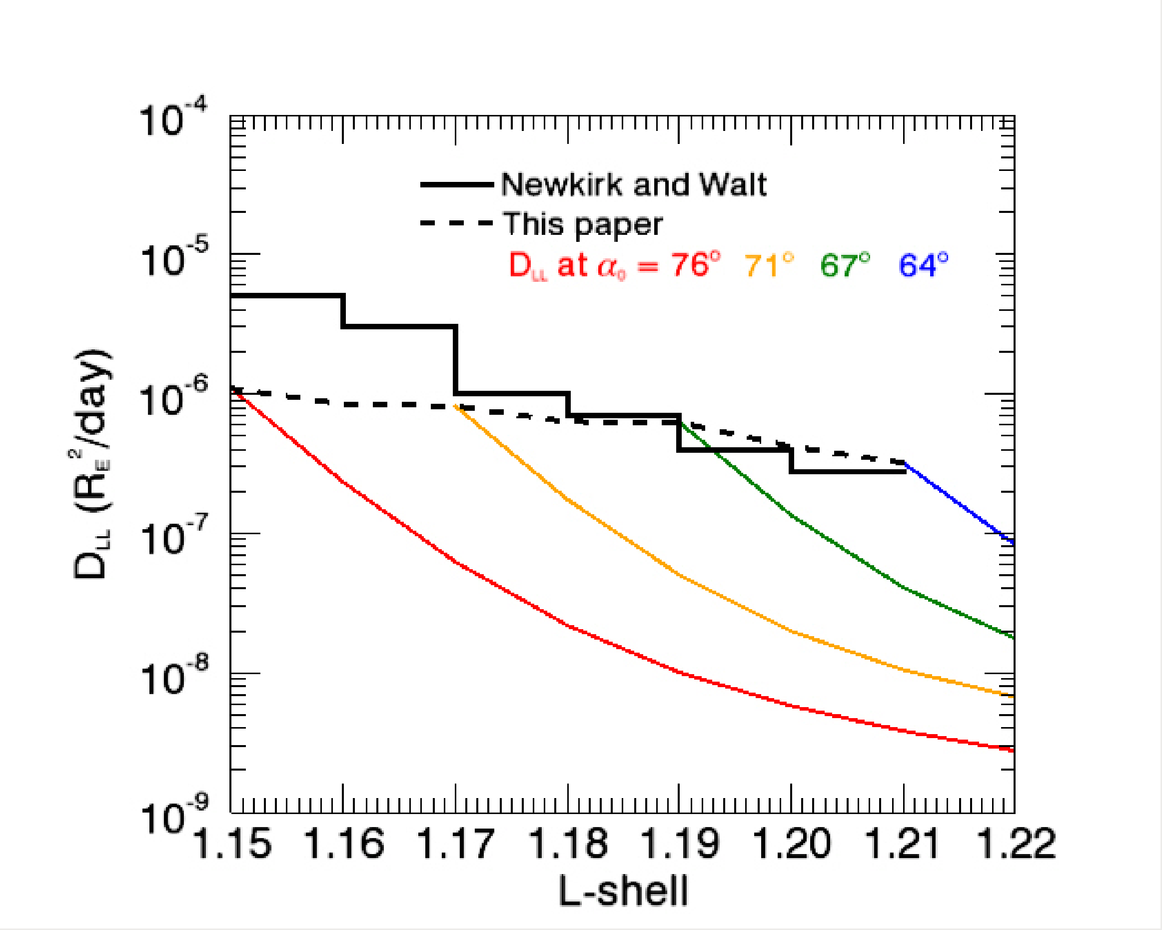
$$\begin{bmatrix} D_{KK} & D_{KL*} \\ D_{L*K} & D_{L*L*} \end{bmatrix} = D_{\alpha_0\alpha_0}\begin{bmatrix} (\partial K/\partial\alpha_0)^2 & (\partial K/\partial\alpha_0)(\partial L^*/\partial\alpha_0) \\ (\partial K/\partial\alpha_0)(\partial L^*/\partial\alpha_0) & (\partial L^*/\partial\alpha_0)^2 \end{bmatrix}$$

- Average over all longitudes, ϕ

$$\begin{bmatrix} D_{KK} & D_{KL*} \\ D_{L*K} & D_{L*L*} \end{bmatrix} = \frac{\Omega_3}{2\pi}\int_0^{2\pi}\left(\frac{\mathrm{d}\phi}{\mathrm{d}t}\right)^{-1} D_{\alpha_0\alpha_0}\begin{bmatrix} (\partial K/\partial\alpha_0)^2 & (\partial K/\partial\alpha_0)(\partial L^*/\partial\alpha_0) \\ (\partial K/\partial\alpha_0)(\partial L^*/\partial\alpha_0) & (\partial L^*/\partial\alpha_0)^2 \end{bmatrix}\mathrm{d}\phi,$$

- Average over 24-hour rotation of Earth

Neoclassical diffusion coefficient compares well to Newkirk and Walt



Evaluate neoclassical D_{LL} at pitch-angle that mirrors at B_m=0.2158 G [Imhof et al, 1967] to get dashed line, which matches N&W [1968] reasonably well.

Neoclassical D_{LL} valid for constant momentum, p, whereas N&W D_{LL} valid at constant μ.

Fokker-Planck model needed to model observed fluxes.

A 3D Fokker-Planck model of the evolution of the Starfish artificial belt

- Convert diffusion tensor in (K,L*) into (α_0,L) under dipole assumption to use DREAM3D
- Solve 2D diffusion equation for each momentum, p

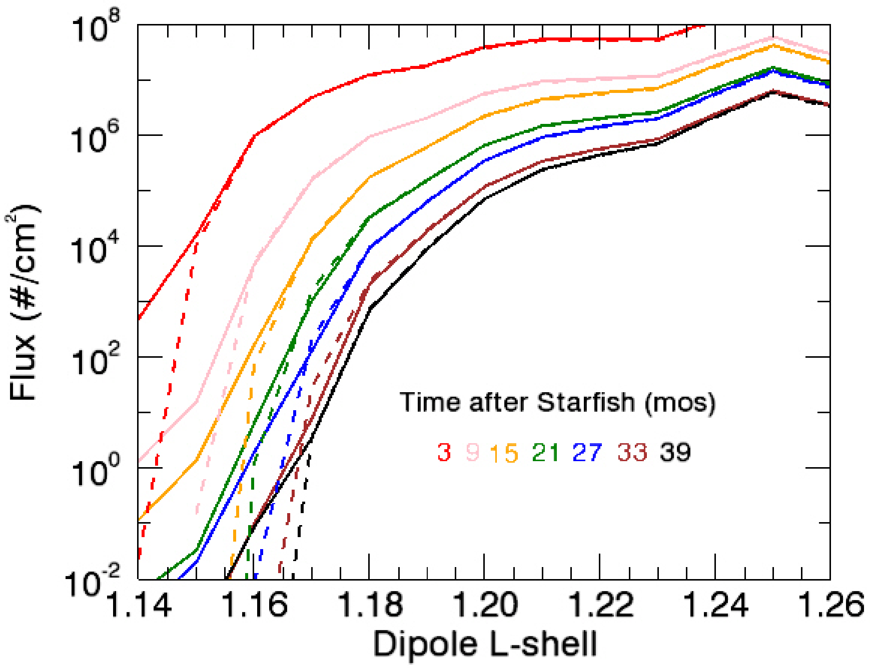
$$\frac{\partial f}{\partial t} = \frac{1}{J_\alpha}\frac{\partial}{\partial \alpha} J_\alpha \hat{D}_{\alpha_0\alpha_0} \frac{\partial f}{\partial \alpha_0} + \frac{1}{J_\alpha}\frac{\partial}{\partial \alpha_0} J_\alpha \hat{D}_{\alpha_0 L} \frac{\partial f}{\partial L}$$
$$+ \frac{1}{J_L}\frac{\partial}{\partial L} J_L \hat{D}_{L\alpha_0} \frac{\partial f}{\partial \alpha_0} + \frac{1}{J_L}\frac{\partial}{\partial L} J_L \hat{D}_{LL} \frac{\partial f}{\partial L}$$

- Model energy loss due to inelastic collisions

$$\frac{\partial f}{\partial t} = -\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 f \frac{dp}{dt}\right)$$

- Simple initial and boundary conditions. Simulate for 39 months after Starfish

Simulation shows increased flux at lowest L-shells

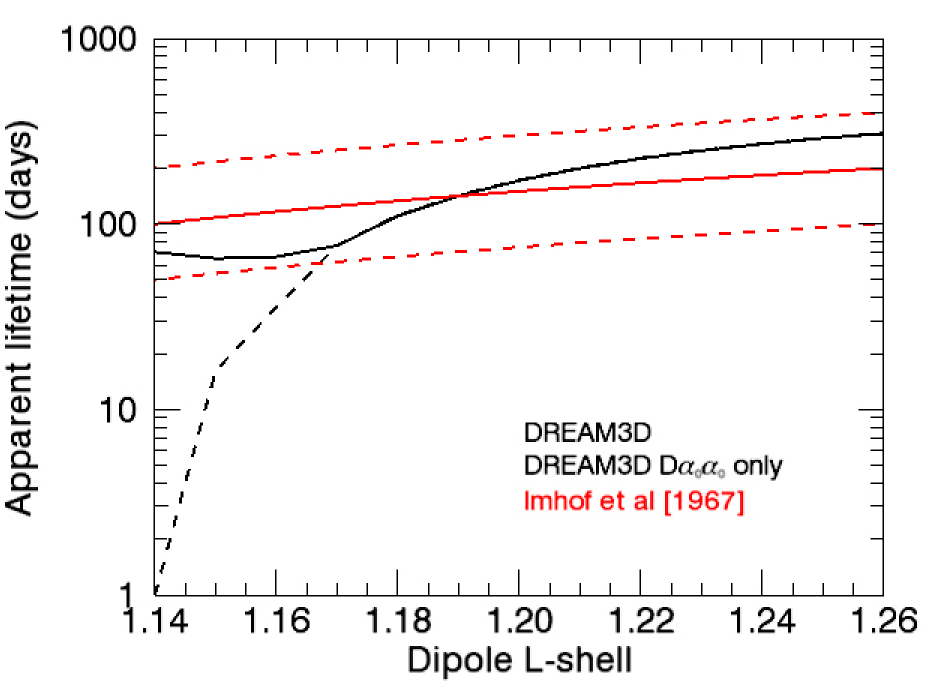


Omnidirectional, integral (>0.5 MeV) flux at B_m=0.2158 G for Coulomb collisions only (dashed lines) and with neoclassical diffusion (solid lines).

Simulated flux at lowest L-shells increases by 4 orders of magnitude using neoclassical diffusion.

Interesting seasonal effect at largest L-shells. No change from 9-15, 21-27, 33-39 months. All change occurs during 3-9, 15-21, 27-33 month intervals.

Neoclassical diffusion increases lifetimes at lowest L by 10-100x



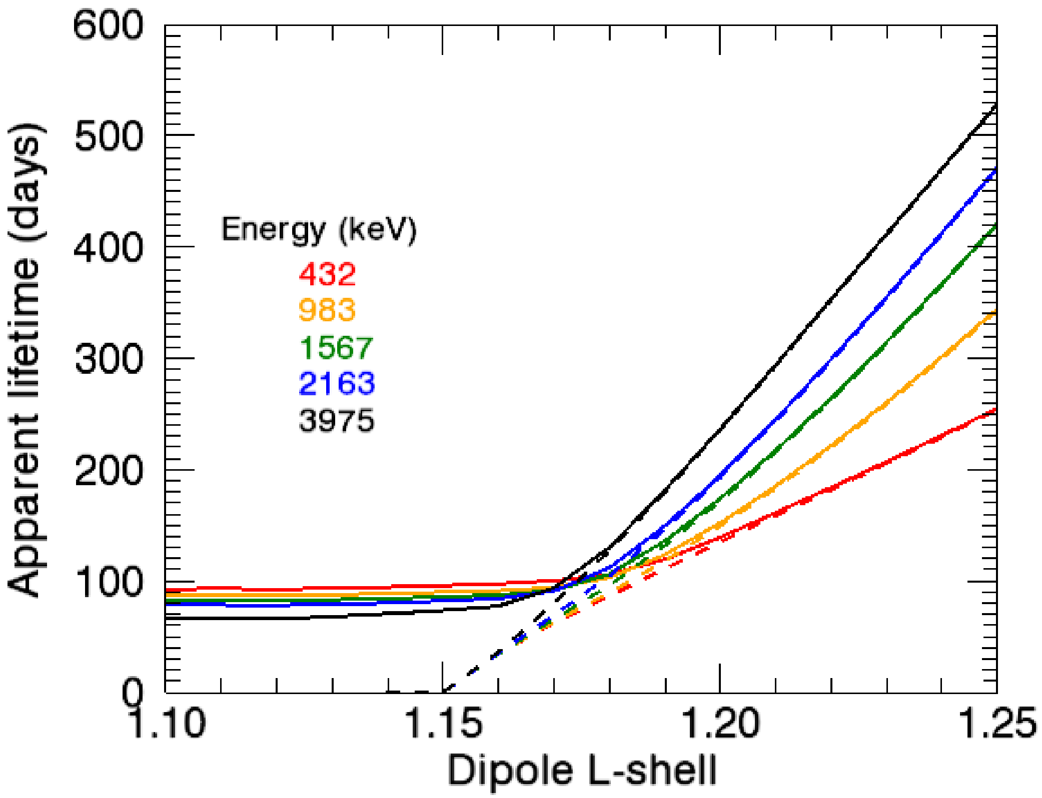
Apparent lifetime of MeV electrons using omnidirectional integral flux evaluated over interval [3,39] months using neoclassical diffusion (solid black) and Coulomb collisions only (dashed black).

Apparent lifetimes computed using neoclassical diffusion are 10-100x larger than with Coulomb collisions alone for L<1.2 .

Simulation results match data (solid red) well and fall within error bounds from Imhof et al [1967] (dashed red).

There is room for improvement!

Dependence on energy and equatorial pitch-angle



Lifetimes for omni flux at the magnetic equator show that entire pitch-angle dstribution is affected by neoclassical diffusion (not just near mirror point). Lifetimes at different enegies converge and cross over, showing importance of energy loss.

Incorporation of ground-based transmitters

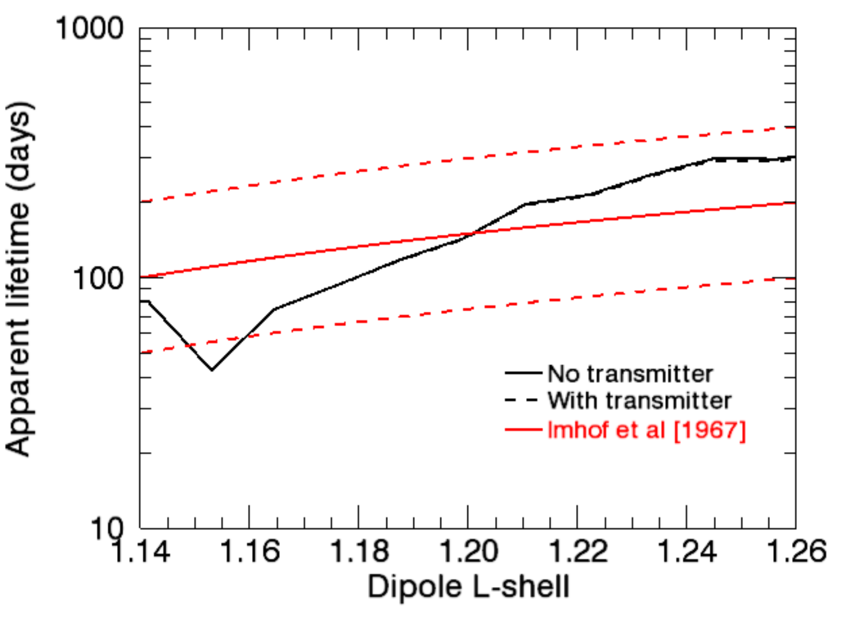
- AFRL has computed
  - ionospheric transmission of many current ground-based VLF transmitters
  - drift-averaged pitch-angle diffusion coefficients
  - as a function of
    - local time (or **LT-averaged**)
    - month (**January**)
    - plasmaspheric density model (**Abel**)
    - ducted or **non-ducted**

Use of current transmitter Daa's to represent post-Starfish era

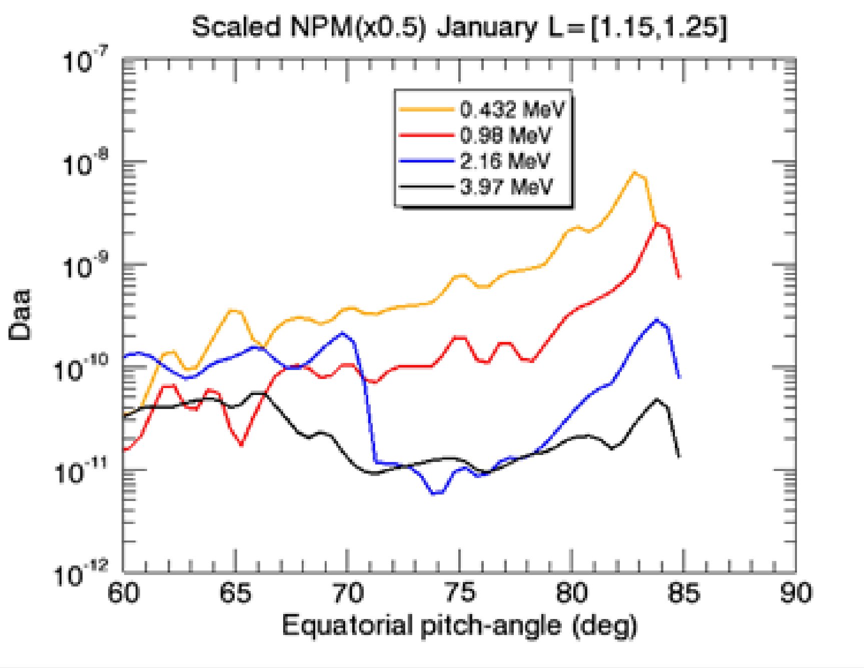
Starfish era				Current –day representative			
	L	kW	kHz		L	kW	kHz
NPM	1.155	65	23.4	NPM	1.155	423	21.4
NBA	1.15	150	18	NPM	1.155	423	21.4
RPS	1.52	315	17.1	NAU	1.38	100	40.75
				HWU	1.83	200	15.1

Used current-day calculations for NPM to represent Starfish-era NPM and NBA (with scaled power, ignoring frequency differences). Averaged current-day NAU and HWU to represent Starfish-era RPS (with scaled power, ignoring frequency differences). Assumed NPM longitude for all three; assumed all power in 7.5 longitudinal section.

Ground-based transmitters play no role at very low L (<1.25)

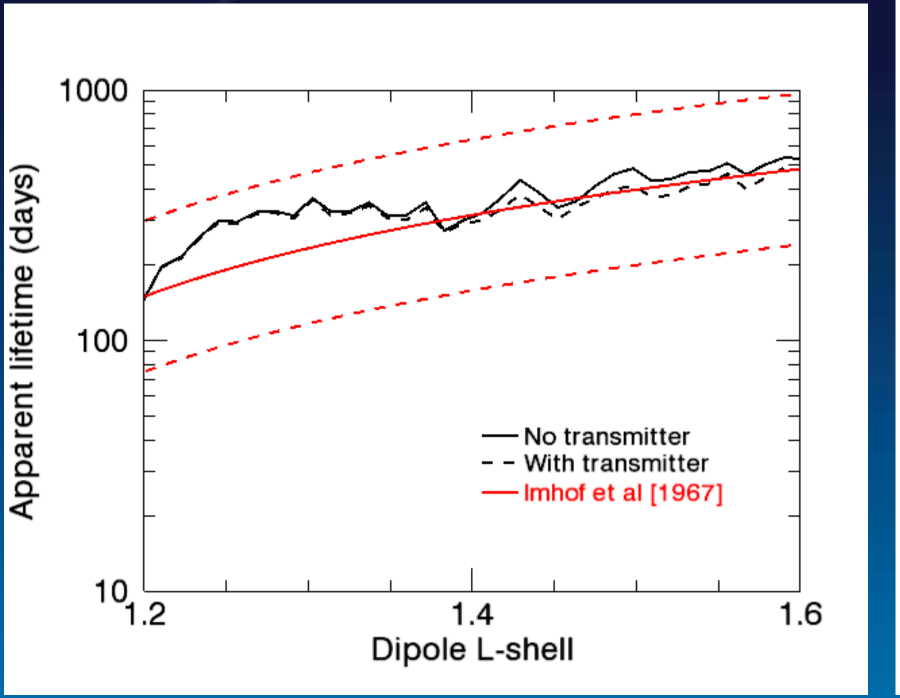


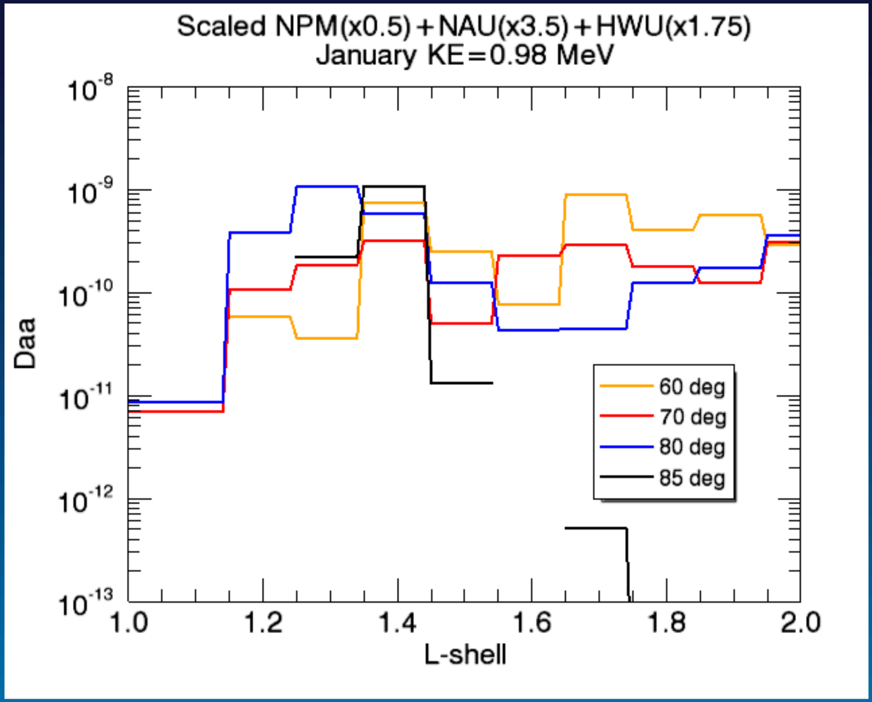
Only a few percent difference in lifetimes below L=1.25



Diffusion coeffficients at low L are smaller than from Coulomb collisions

At larger L (1.3-1.6) transmitters do reduce lifetime but no DLL efffect





Lifetime reduced by ~20% at larger L but no increased DLL effect

Diffusion coeffficients at larger L are comparable to Coulomb collisions

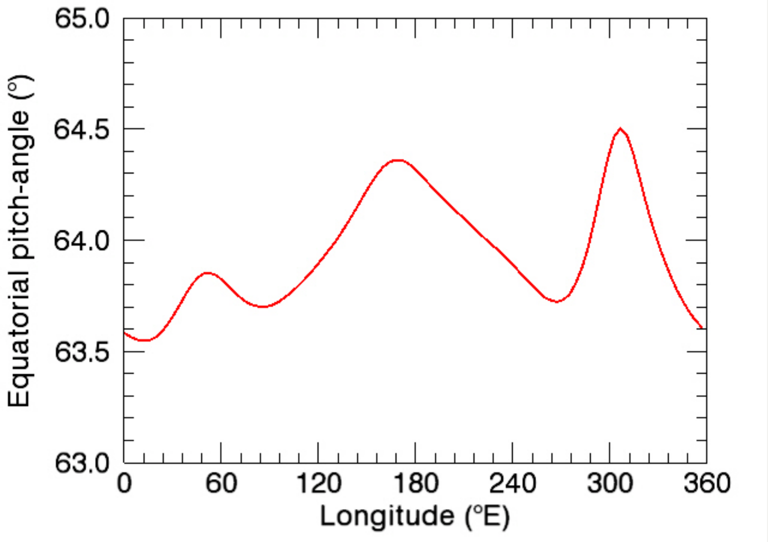
Conclusions and future work

- MeV electron lifetimes at L<1.2 due purely to Coulomb collisions for first few months after Starfish
- Apparent lifetimes 10-100x greater than expected over next 3 years, inward radial diffusion needed
- Neoclassical diffusion from drift-shell splitting in IGRF field and Coulomb collisions explains increased lifetimes
- VLF waves from ground-based transmitters
  - do not influence behavior at very low L (<1.25)
  - affect lifetimes at larger L (1.3-1.6)
  - no effect seen from ‘anomalous’ diffusion
- Use full suite of current transmitters to study region of interest to Selesnick

Extra slides

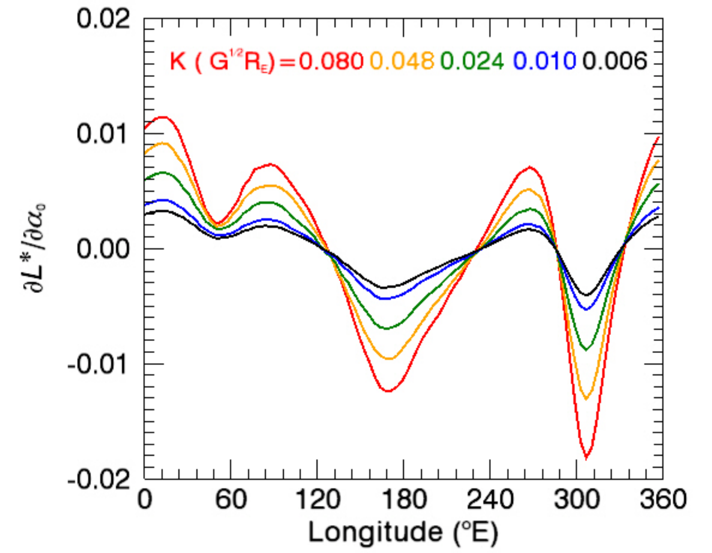
Multi-pole terms in IGRF magnetic field model cause drift-shell splitting

K=0.08 $G^{1/2}R_E$ L*=1.2



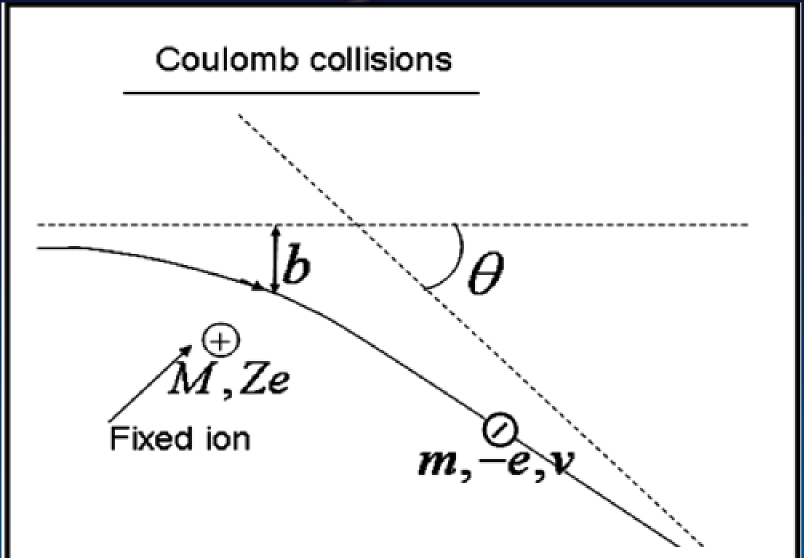
Equatorial pitch-angle varies with longitude for a drift-shell in the IGRF field.

L*=1.2



Drift-shell splitting (evaluated at the equatorial pitch-angle that conserves K) varies with longitude for a drift-shell in the IGRF field.

Pitch-angle diffusion from Coulomb collisions



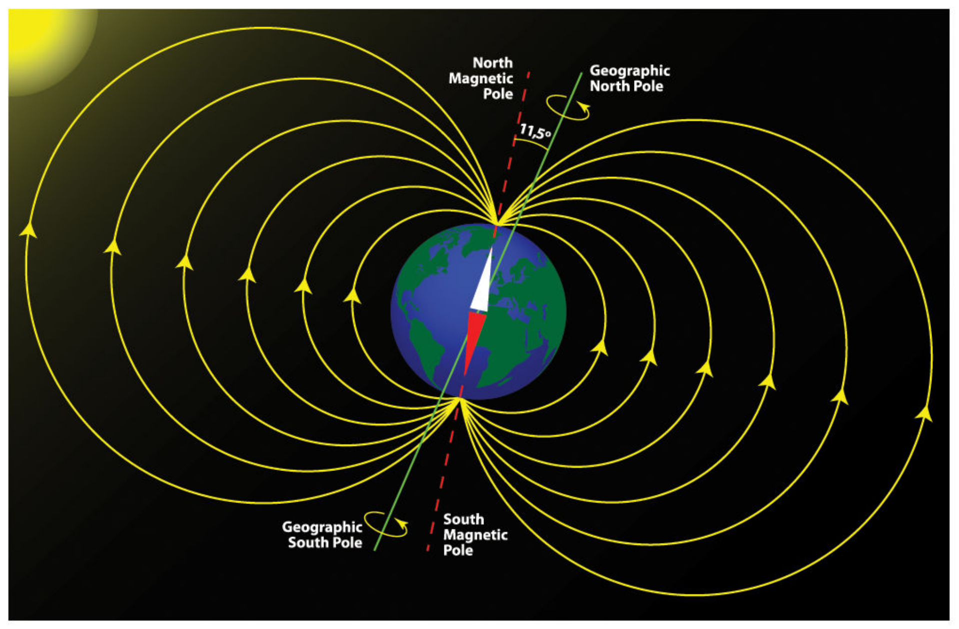
From physics.stackexchange.com

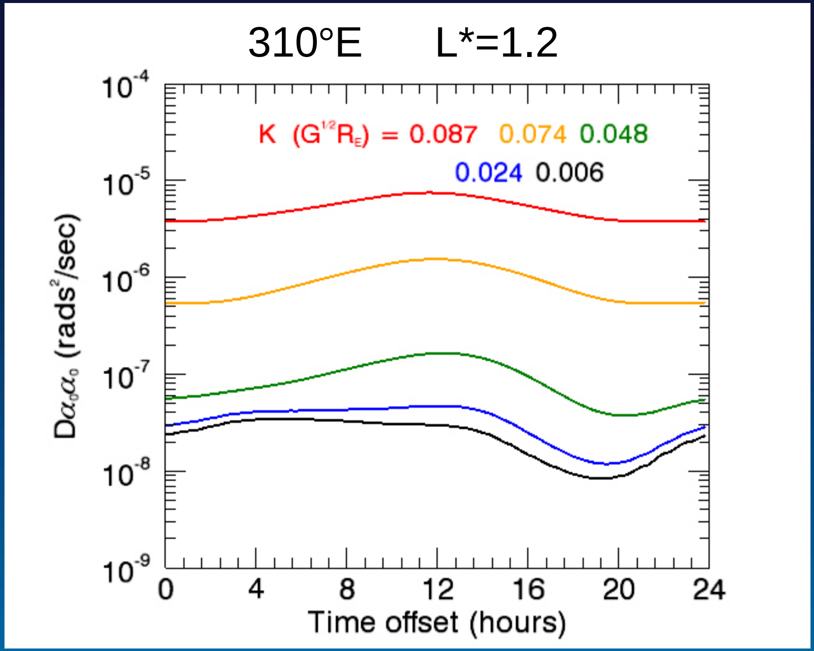
A Coulomb ‘collision’ with a neutral or ion causes the MeV electron to change its angle but not its momentum (nearly).

$$D_{\alpha_0\alpha_0} = \frac{\gamma^2 m_0^3 c^4 r_e^2 \Omega_2}{(1-y)^2 p^4} \oint \left[\frac{B_0}{B(s)} - y^2 \right] \times \left[N_e \lambda_e + \sum_i N_i Q_i^2 \left(\lambda_p - \lambda_{ni} \right) + \sum_j N_j Z_j \lambda_{nj} \right] \frac{ds}{\cos\alpha},$$

- diffusion coefficient, $D\alpha_0\alpha_0$, in equatorial pitch-angle, α_0
- bounce-averaged
- less diffusion for higher momentum, p
- more diffusion for higher density, N

D$\alpha_0\alpha_0$ varies with local time, season and geomagnetic activity





Field lines at fixed longitude rotate about Earth's geographic axis from dayside to nightside. Atmospheric/ionospheric density changes with local time, season and geomagnetic activity.

D$\alpha_0\alpha_0$ for a field line at fixed longitude varies with local time due to background density.