Global Simulations of Wave-Particle Interactions in the Radiation Belts: March 17, 2018 Acceleration Event

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AGU Chapman Conference
Particle Dynamics in the Radiation Belts
Cascais, Portugal
March 9, 2018

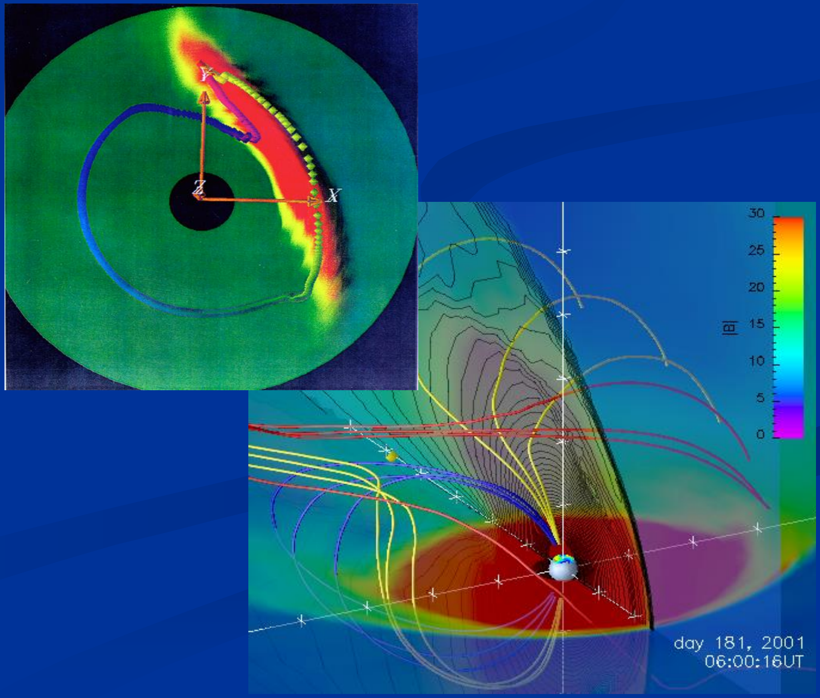
Global simulations of the radiation belts

- Fokker-Planck simulations

$$\frac{df}{dt} = \frac{1}{\Im} \sum_{i,j} \frac{\partial}{\partial J_i} \left(\Im D_{ij} \frac{\partial f}{\partial J_j} \right) - \frac{f}{\tau} + S$$

  - Requires empirical specification of stochastic transport coefficients based on theory and observations
  - Generally cannot model nondiffusive effects (e.g. advection/injection)

- MHD/particle simulations
  - Global MHD model provides time-evolving electric and magnetic fields.
  - Handles radial transport self-consistently
  - Generally cannot model high frequency wave effects, e.g. energy and pitch angle scattering due to chorus, EMIC, etc.



Non-MHD effects via SDE methods

- Every diffusion equation is mathematically equivalent to a set of stochastic differential equations (SDEs; e.g., *Tao, Chan, and Albert,* [*JGR*, 2008]):

$$dX = b\,dt + \sigma\,dW$$

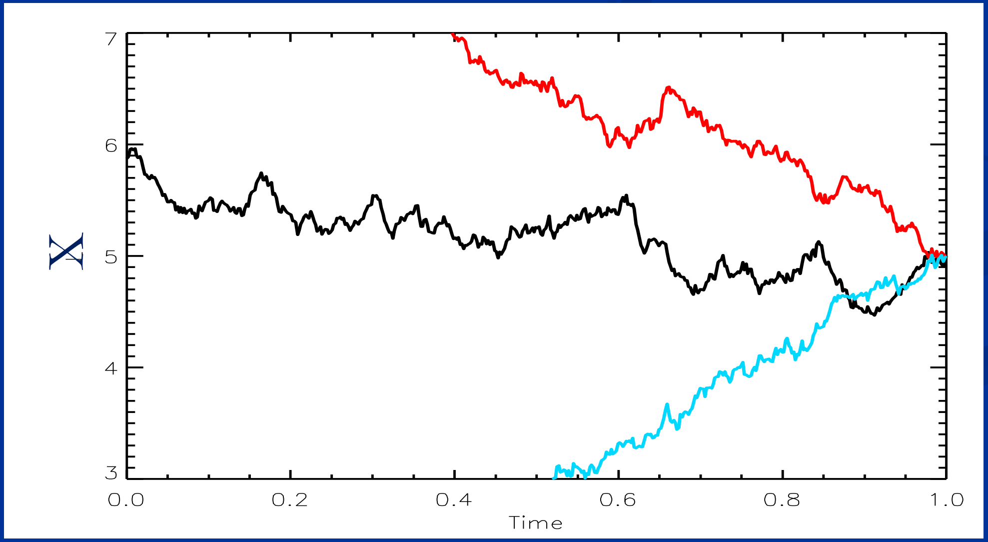
  - *dX* is a change in a stochastic variable *X* over a time *dt* (e.g. *X* may be a pitch angle, energy, or an adiabatic invariant).
  - dW=sqrt(t)N(0,1), where N is a Gaussian random variable $\in$[0,1].
  - $b(X,t)$ and $\sigma(X,t)$ are coefficient functions. e.g., for a 1-dimensional diffusion equation

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial f}{\partial x}\right) = D\frac{\partial^2 f}{\partial x^2} + \frac{\partial D}{\partial x}\frac{\partial f}{\partial x}$$

$$b = \partial D / \partial x$$

$$\sigma = \sqrt{2D}$$

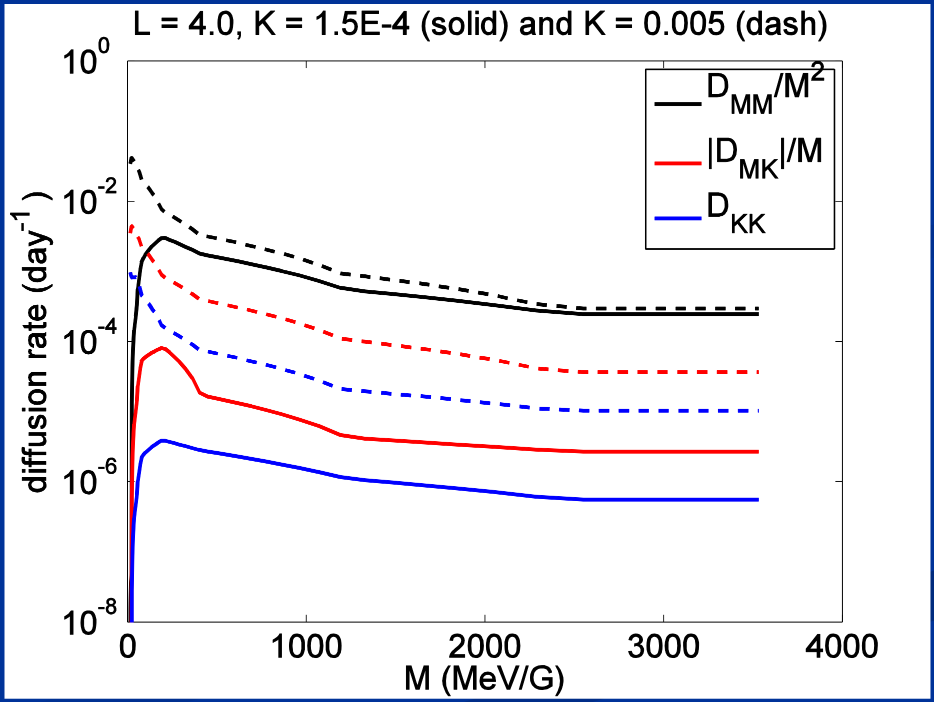
  - Monte Carlo solution of the SDE yields random-walk trajectories in *X*.



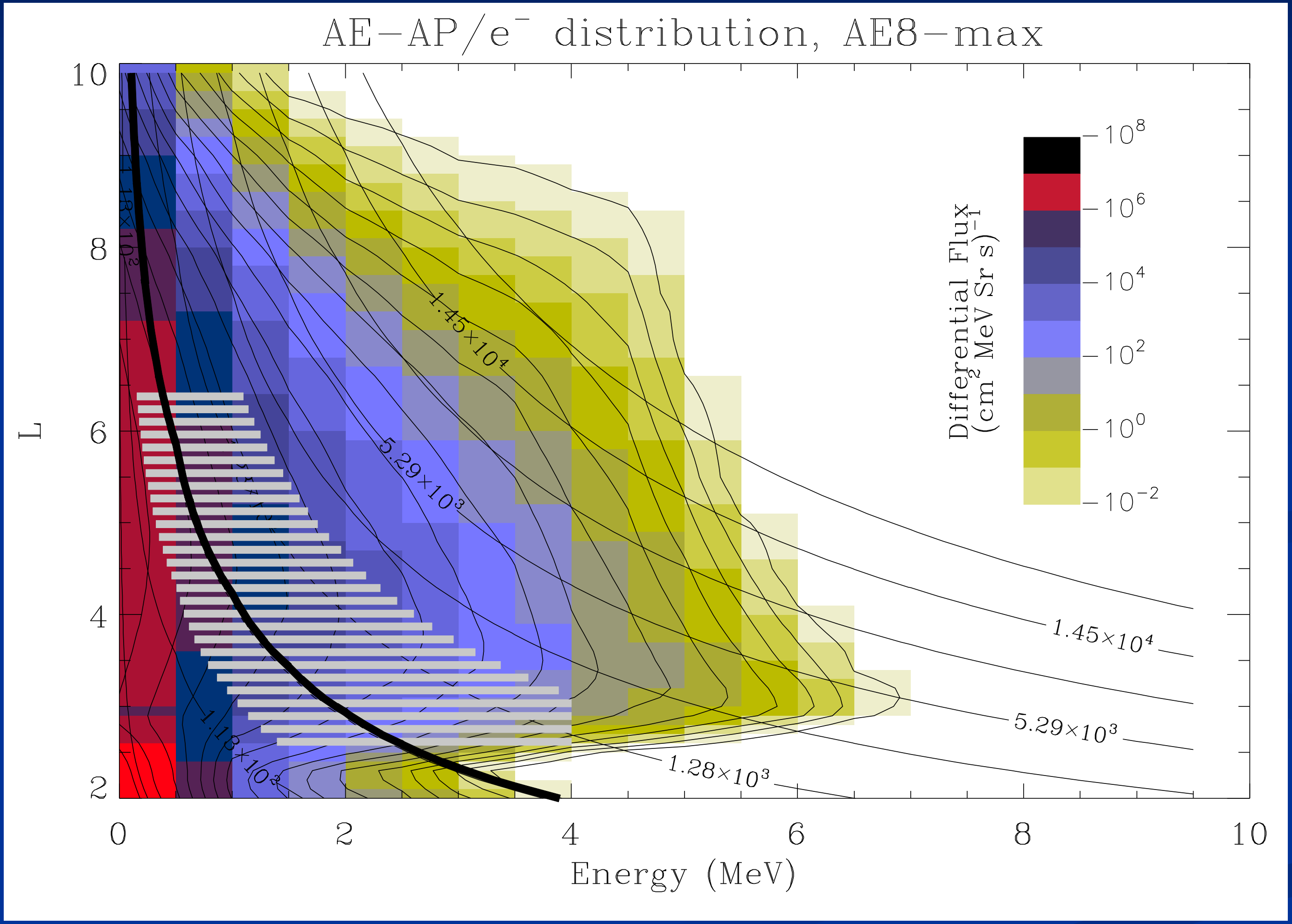
2d sims: Energy diffusion coefficients

We use diffusion coefficients calculated by the PADIE code (R. Horne, S. Glauert, BAS; *J. Geophys. Res.* 110, 2005).

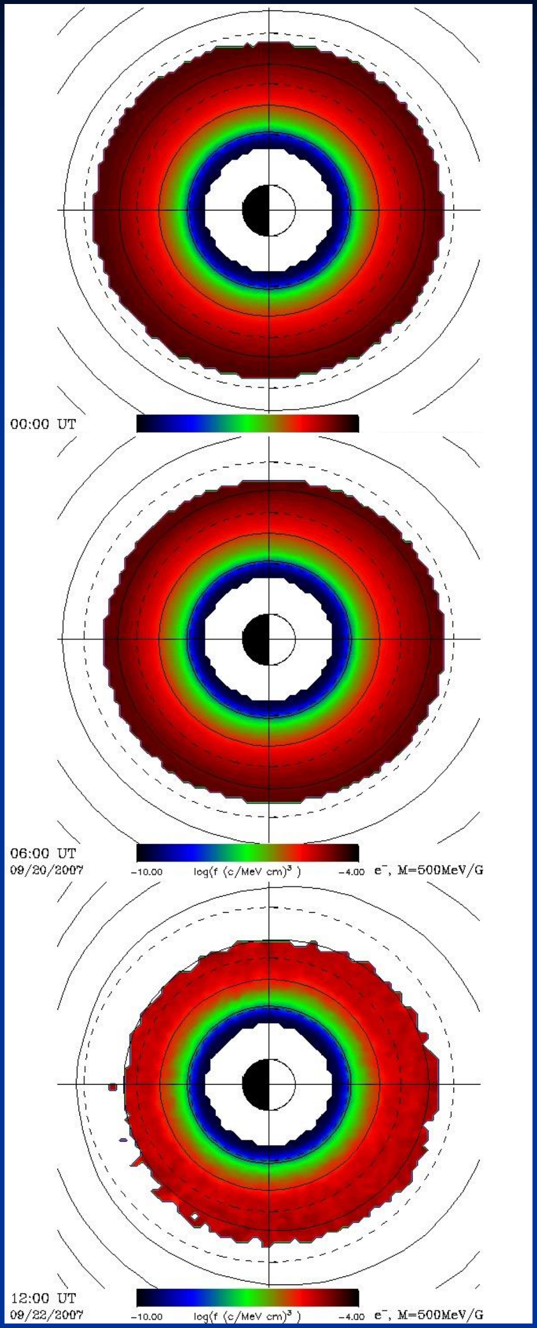
- Diffusion in energy and pitch angle, including cross terms.
- Converted to diffusion in *M-K* space assuming dipole background field.
- Covers the effect of magnetospheric chorus waves on energetic particles
- *L* and *Kp*-dependent: *Kp*<2, 2<*Kp*<4, and *Kp*>6



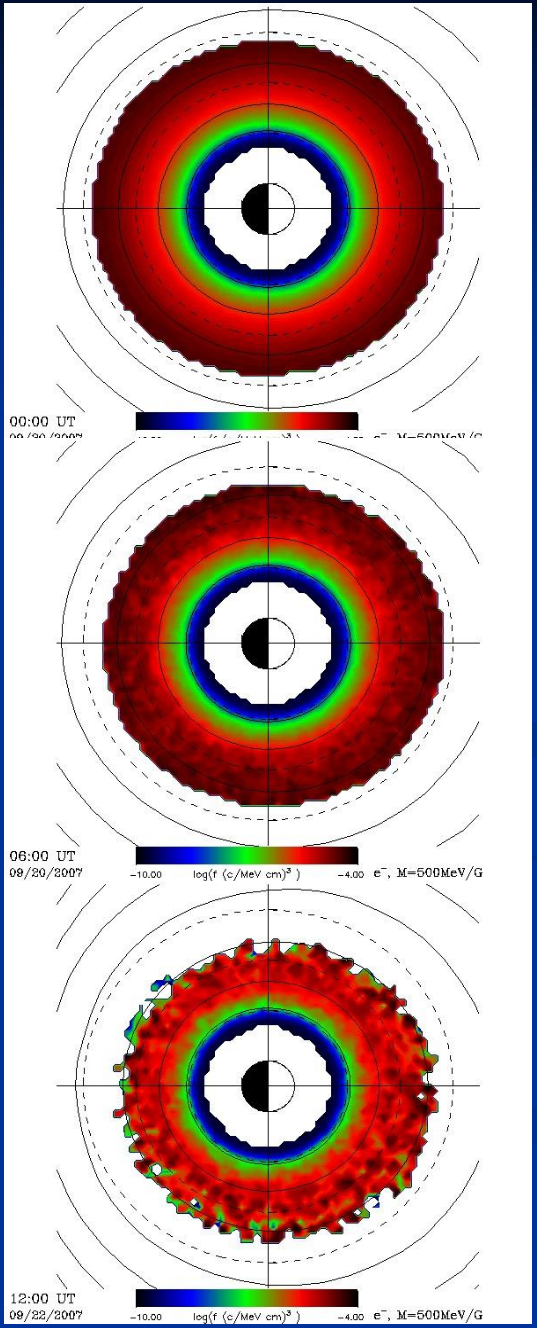
2d simulation domain



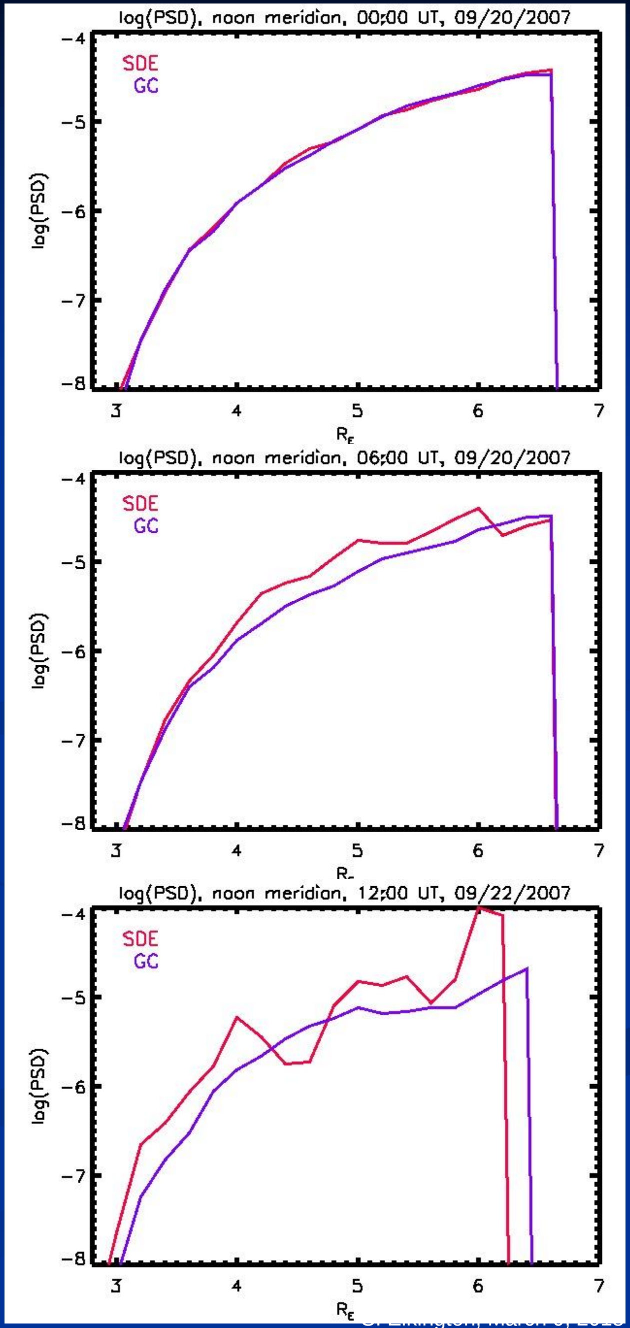
D_{MM} =0



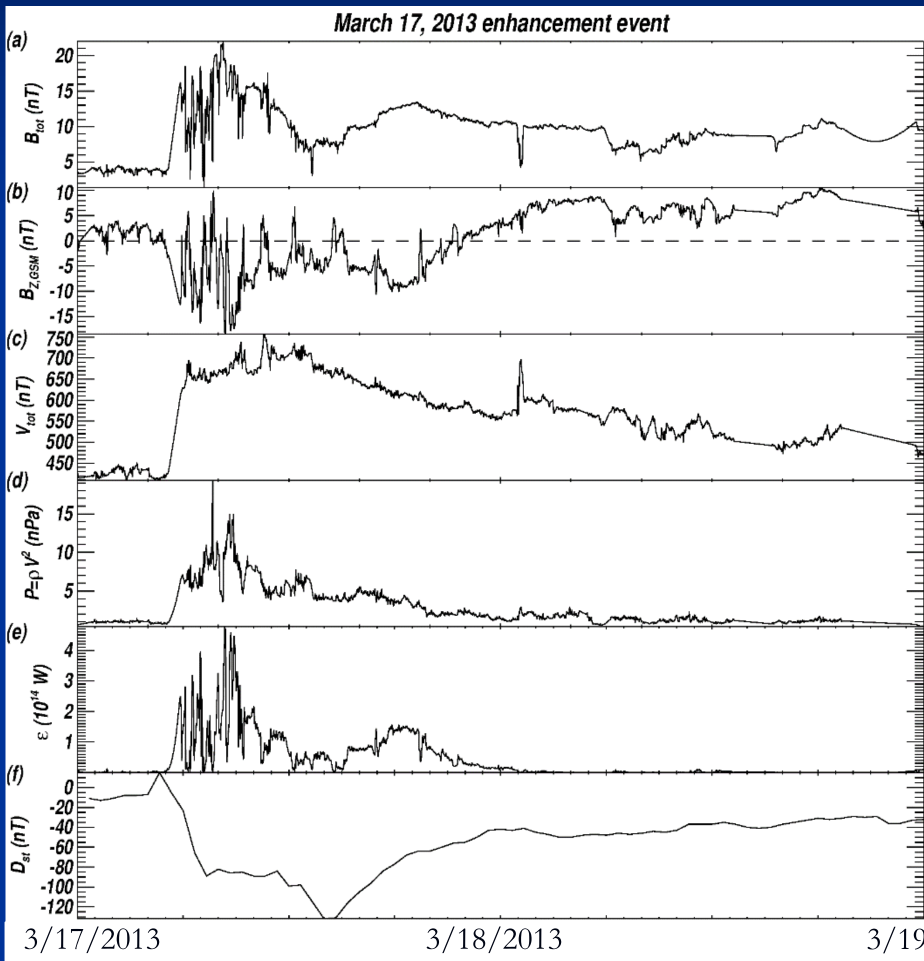
D_{MM} ≠0

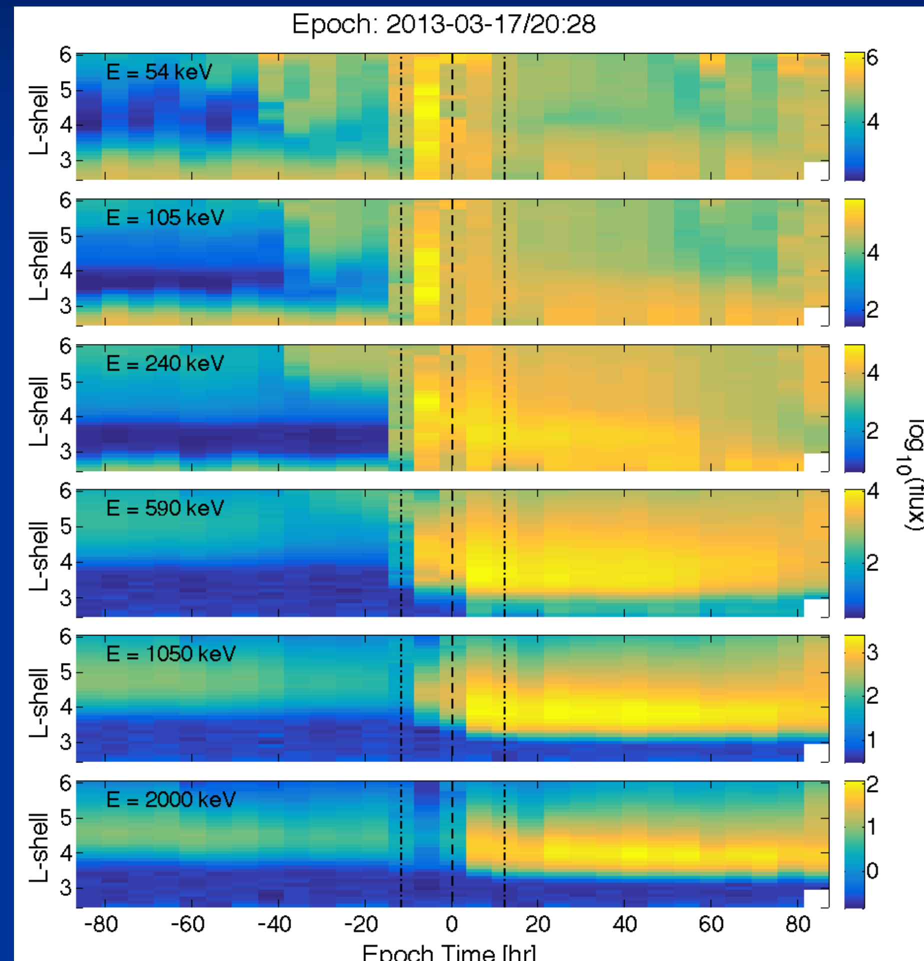


Radial profile



3d Sims: 3/17/2013 QARBM 'Storm Enhancement Event'





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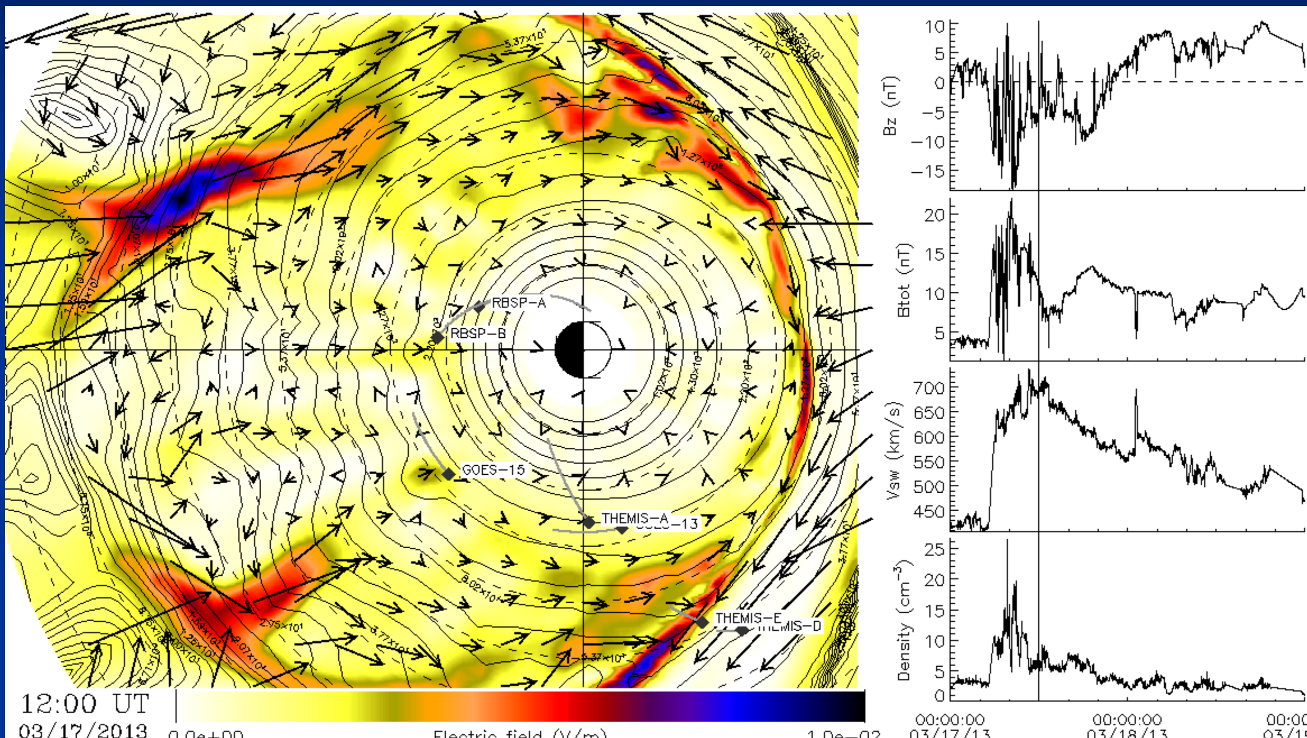
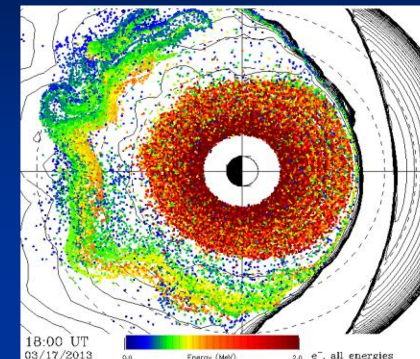
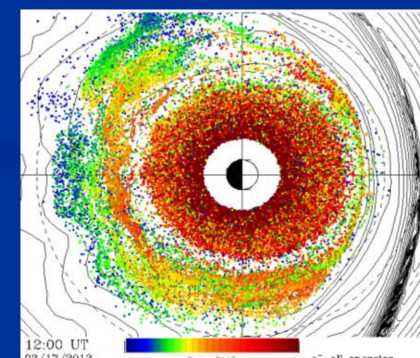
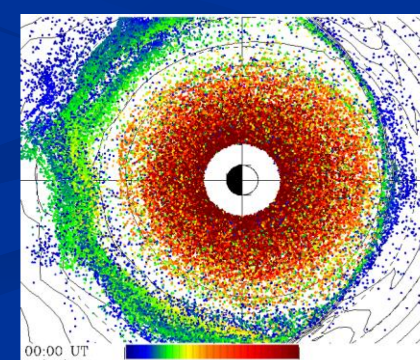




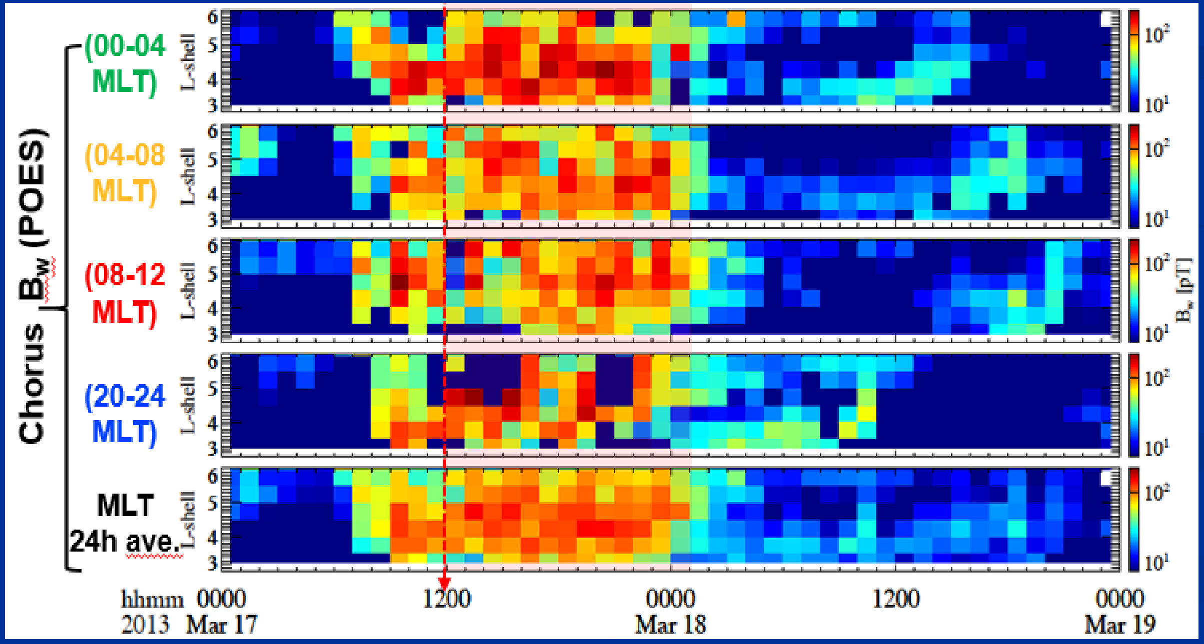


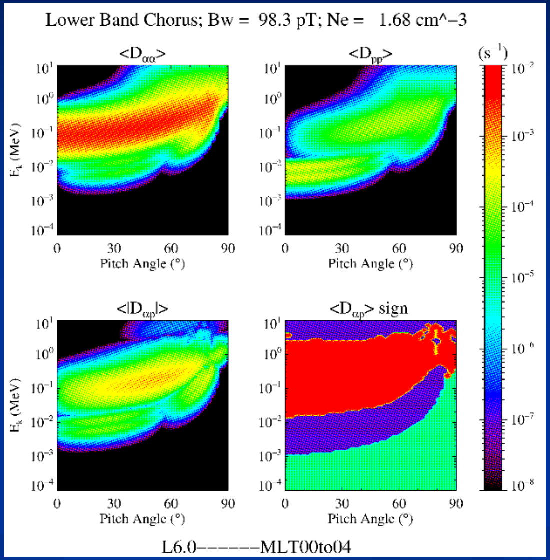


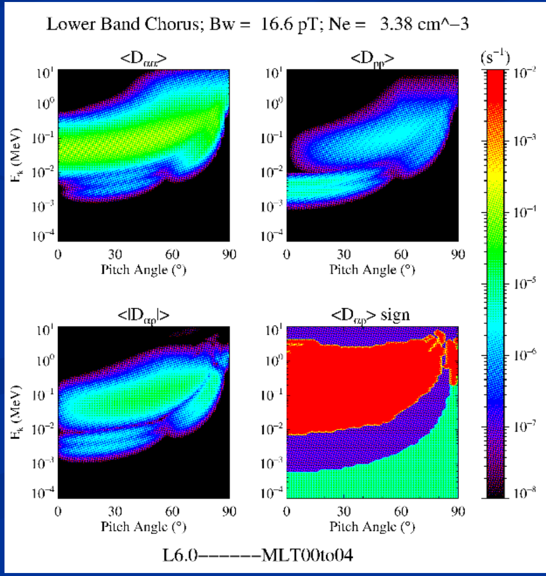
We have undertaken global 3d LFM-MHD simulations of the 3/17/2013 storm period, and used these results to drive test particle simulations of energetic particle populations in the inner magnetosphere and plasmasheet.

3d Sims: 3/17/2013 QARBM 'Storm Enhancement Event'

- Provided by Wen Li and Qianli Ma (Boston U.)
- Upper and Lower band chorus waves as a function of L, MLT, and UT inferred from POES electron measurements.
- $D_{\alpha\alpha}$, D_{pp}, and $D_{\alpha p}$ calculated as a function of particle energy and pitch angle.
- Coefficients converted to M and $p_{\|}$ for particle code.







3d simulations: K2-GC

- K2-GC is a 3d guiding-center test particle code adapted to include diffusive effects of VLF activity (chorus, hiss, etc).
- Each particle is periodically "kicked" in M and $p_\parallel$ in the B-min surface in accordance with specified diffusion coefficients.

$$M_{j+1} = M_j + b_M \delta t + \sigma_{MM}\sqrt{\delta t} N_M + \sigma_{Mp_\parallel}\sqrt{\delta t} N_{p_\parallel}$$

$$p_{\parallel\,j+1} = p_{\parallel\,j} + b_{p_\parallel}\delta t + \sigma_{p_\parallel M}\sqrt{\delta t} N_M + \sigma_{p_\parallel p_\parallel}\sqrt{\delta t} N_{p_\parallel}$$

with

$$\sigma_{MM} = \sqrt{2D_{MM}}, \qquad \sigma_{Mp_\parallel} = \sqrt{2} D_{p_\parallel M}/\sqrt{D_{p_\parallel p_\parallel}}$$

$$\sigma_{p_\parallel M} = \sqrt{2} D_{Mp_\parallel}/\sqrt{D_{MM}}, \qquad \sigma_{p_\parallel p_\parallel} = \sqrt{2D_{p_\parallel p_\parallel}}$$

and

$$b_M(t, M, p_\parallel) = \frac{1}{G}\frac{\partial}{\partial M}(GD_{MM}) + \frac{1}{G}\frac{\partial}{\partial p_\parallel}\left(GD_{Mp_\parallel}\right)$$

$$b_{p_\parallel}(t, M, p_\parallel) = \frac{1}{G}\frac{\partial}{\partial M}\left(GD_{p_\parallel M}\right) + \frac{1}{G}\frac{\partial}{\partial p_\parallel}(GD_{MM})$$

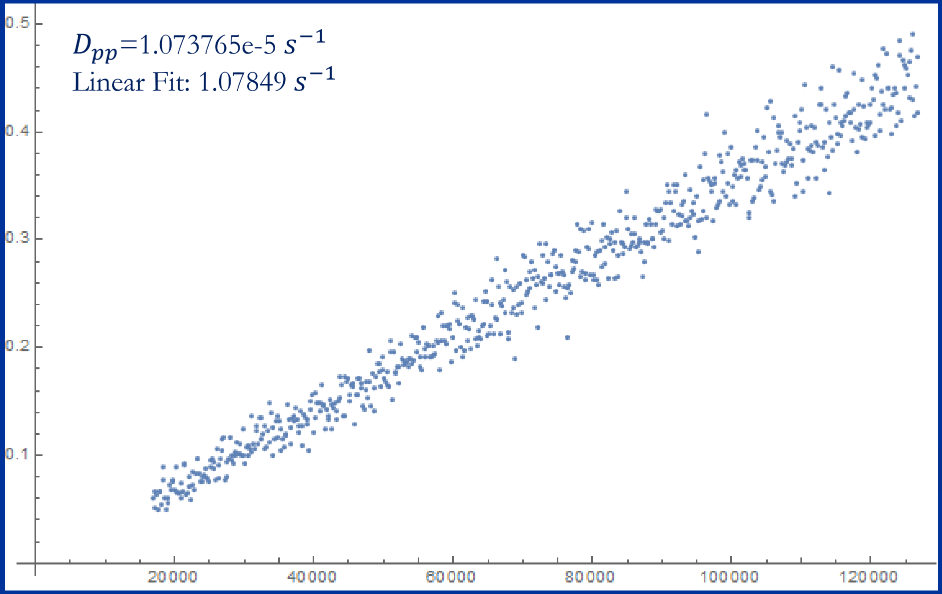
where G is the Jacobian scale factor that results from the conversion of $D_{pp}, D_{\alpha\alpha}$ to $D_{MM}, D_{p_\parallel p_\parallel}$ (dipole approximation).

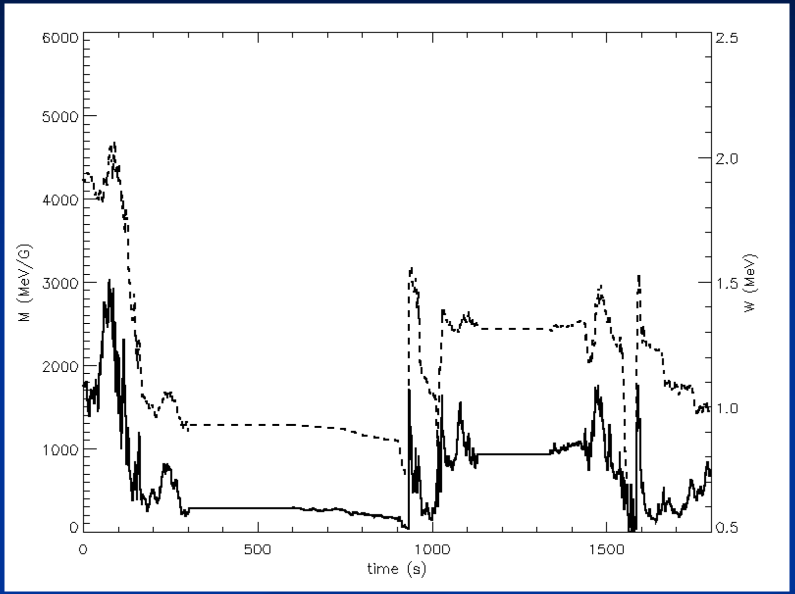
D_{xx} validation, single-particle tests

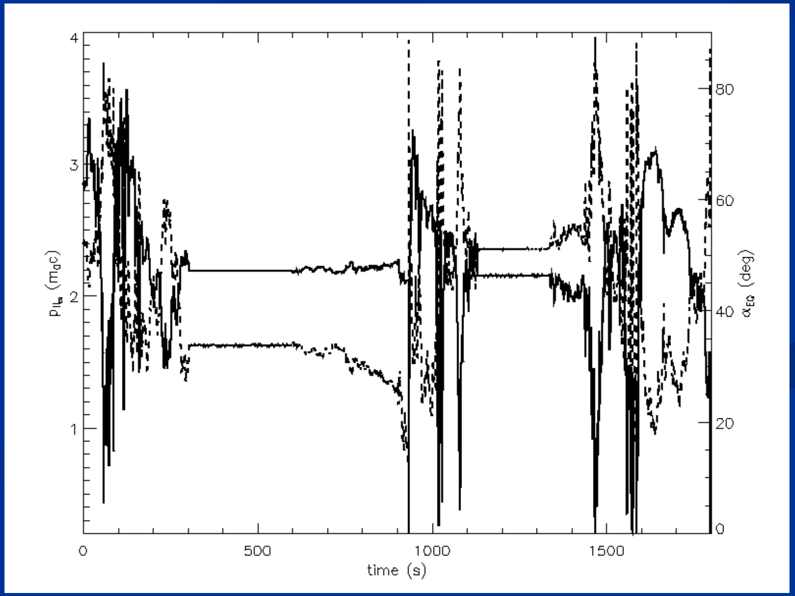
SDE results were validated against given diffusion coefficients.

- Kick multiple particles, calculate $\langle(\Delta x)^2\rangle$ as a function of time.
- D_{xx} inferred from slope of fit.

Test particle simulations were then conducted to validate correct behavior as function of *L*, *W*, and *MLT*.



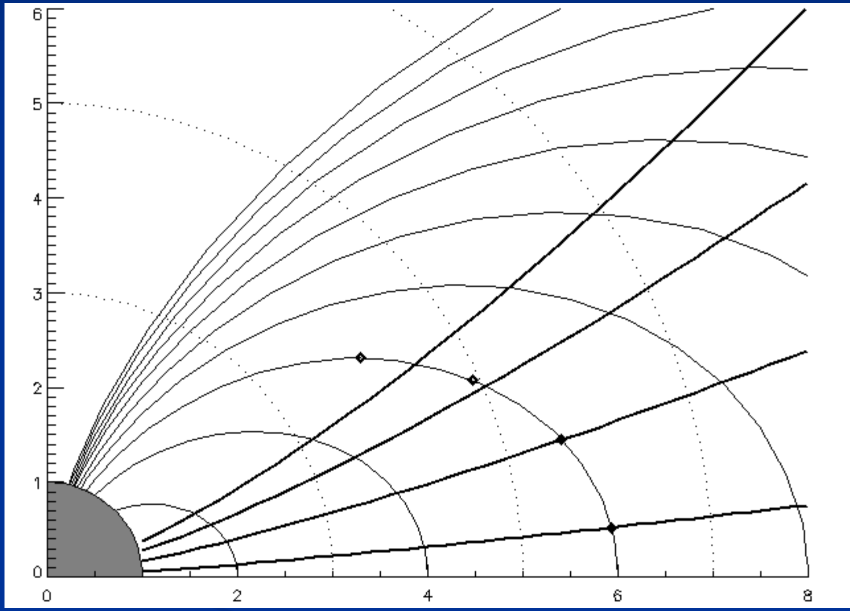




Proof of concept: test particle simulations at constant M, K

Test simulations were undertaken for the March 17-18, 2013 GEM "Storm-time acceleration event" *(http://bit.ly/28UnLpw)*.

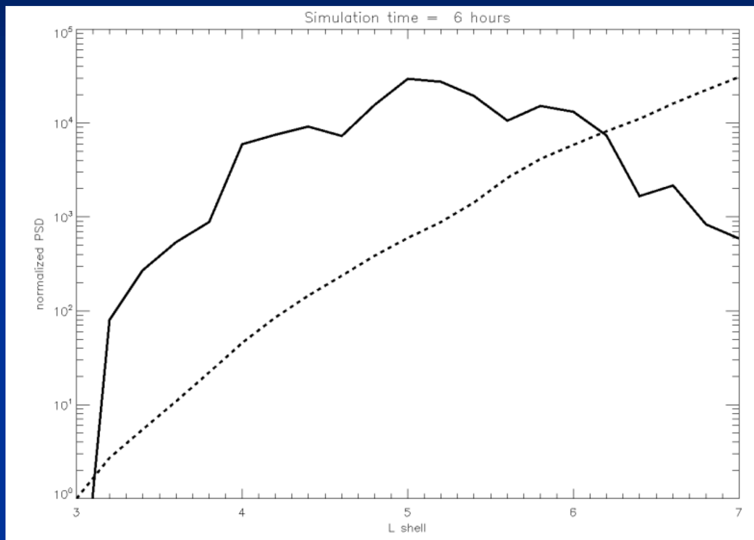
- Constant-*K* surface calculation, a la *Schulz and Lanzerotti (1974)*.
  - Reference field line and mirror latitude selected.
  - *K* calculated for reference point
  - B_m calculated for other field lines base on calculated *K*.
  - Latitude on other *L* shells calculated for given B_m.



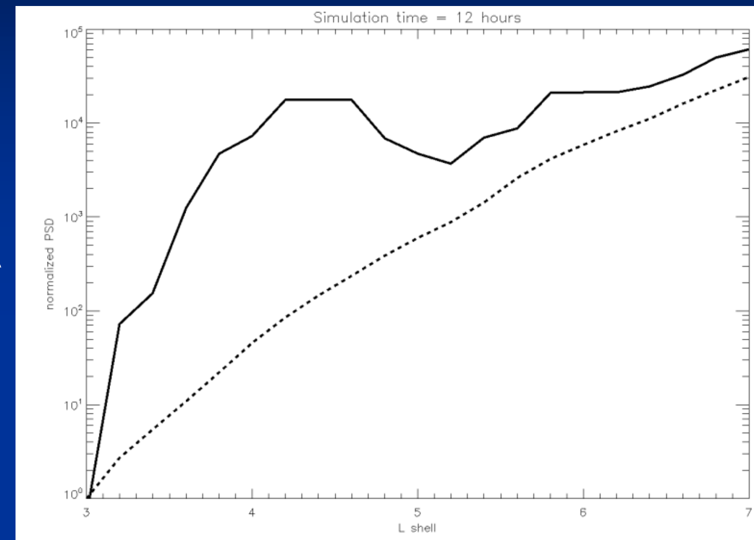
- Time-backwards test particle/SDE simulations
  - Particles of constant *M* distributed at points along constant *K* surface
  - Each particle run backwards until it encounters a boundary or initial condition
  - Phase space density inferred from AE-8 fluxes at IC/BC.
  - PSD and Liouville's theorem used to construct snapshots of evolving PSD profiles as function of time.

Initial results: 3/17/2013 event-specific diffusion coefficients

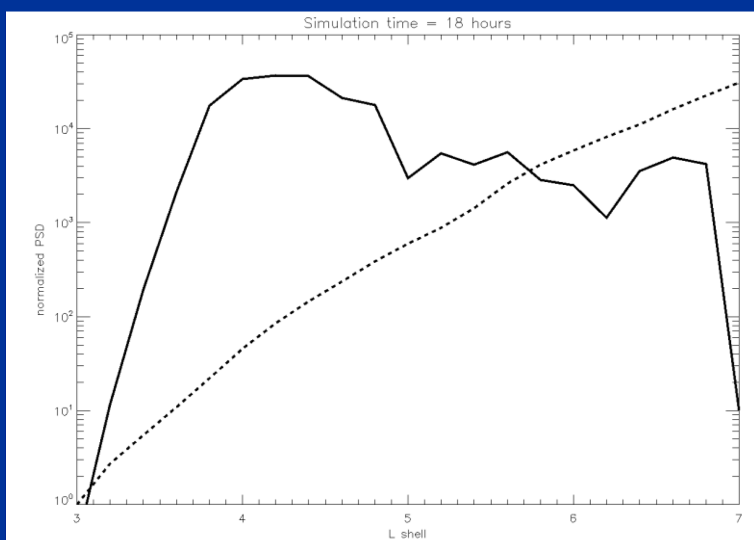
3/17/2018, 6UT



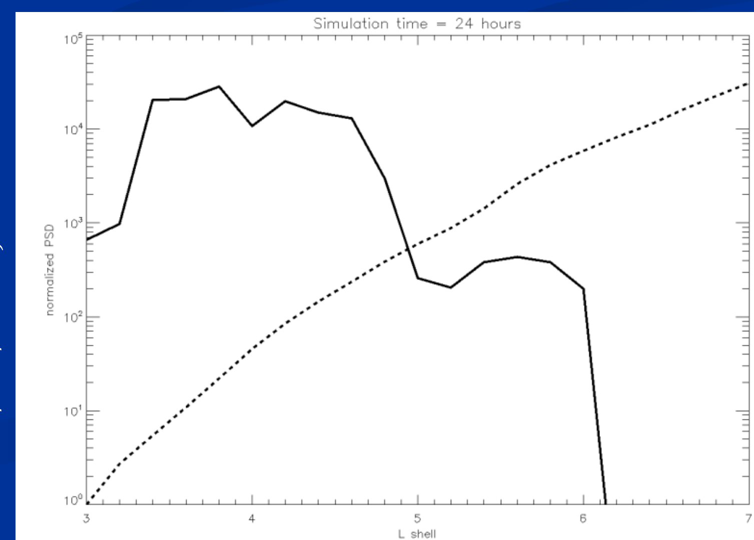
3/17/2013, 12UT



3/17/2013, 18UT

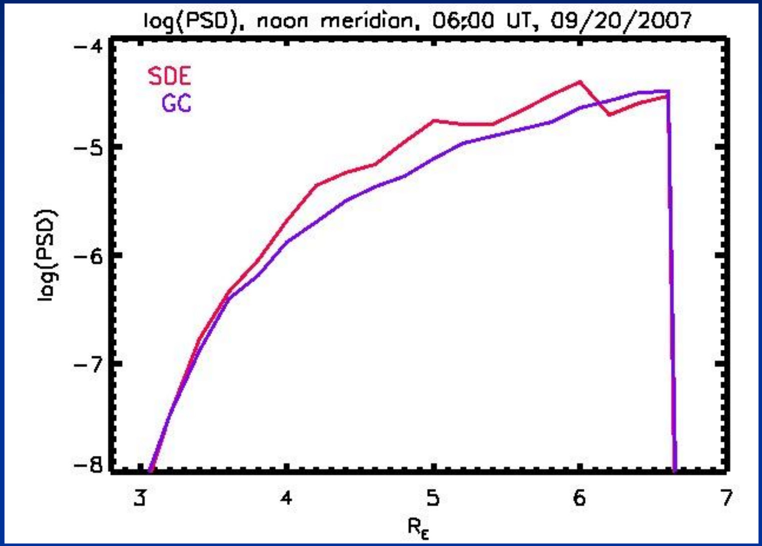


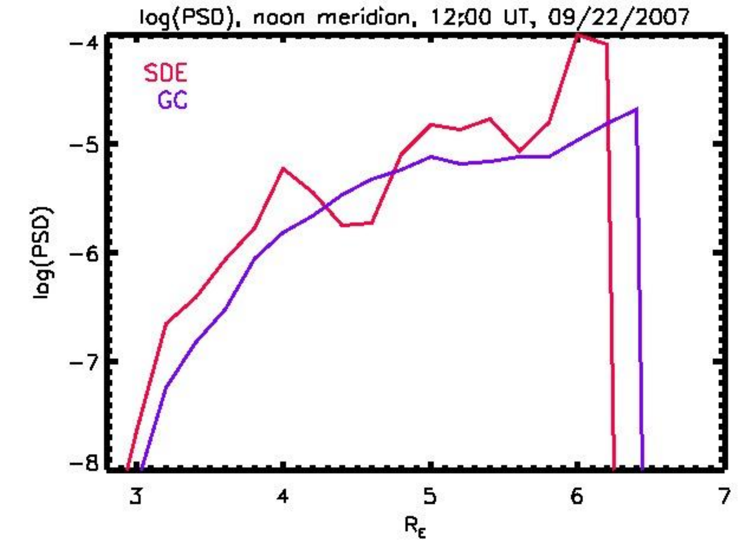
3/17/2013, 24UT



9/20/2007 ISSI event

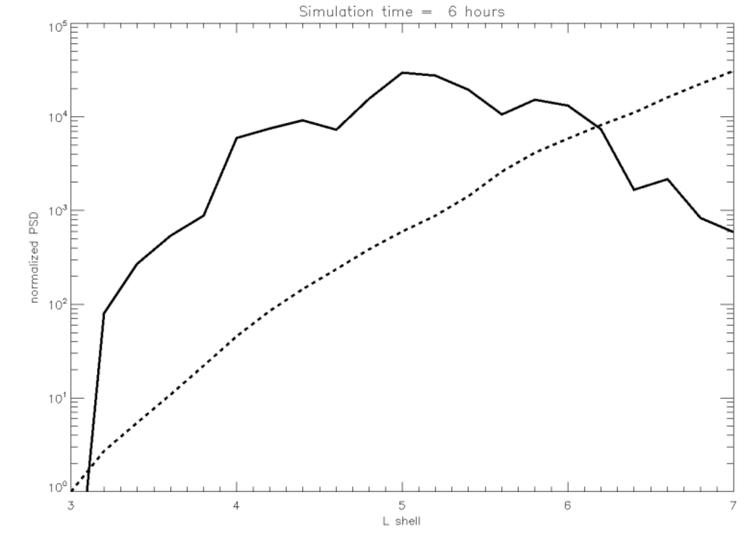
- 2d simulation
- Diffusion in *M* only
- BAS (averaged) diffusion coefficients

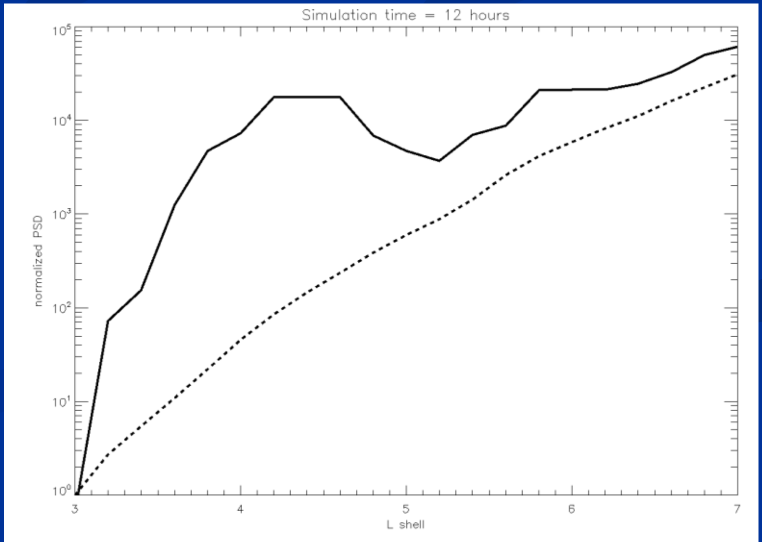




3/17/2013 ISSI event

- 3d simulation
- Diffusion in *M* and *K*
- Event-specific diffusion coefficients





Remarks/Conclusions

- K2-GC is a framework for comprehensive simulations of energetic particle dynamics in the inner magnetosphere
  - Radial transport (diffusion, advection) handled self-consistently via Test Particle/Global MHD simulations
  - Energy and pitch angle diffusion via empirical or event-specific coefficients (ESCARGOT!)
- Remarks:
  - Thus far, relatively computationally-expensive.
    - Obtaining event-specific coefficients is fairly involved.
    - Best-suited to time-backward simulations from a grid or point of interest (e.g. Van Allen probes)
    - On the other hand, its an embarrassingly-parallel problem
  - Long-time simulations suffer from reduced statistics due to bounce cone losses
- To Do (Too Due?):
  - Implementing existing 3d MHD/Test particle approaches to K2-GC
  - Optimizing computational efficiency
  - Quantitative Assessment of (this) Radiation Belt Model
  - More events! (e.g. QARBM storm-time loss, non-storm acceleration, etc)

Thank you.

Stuff. And also Things.

K2-BA: a comprehensive radiation belt simulation method

•Global transport simulations accomplished via an efficient bounce-averaged test particle code (*Roederer 1970*):

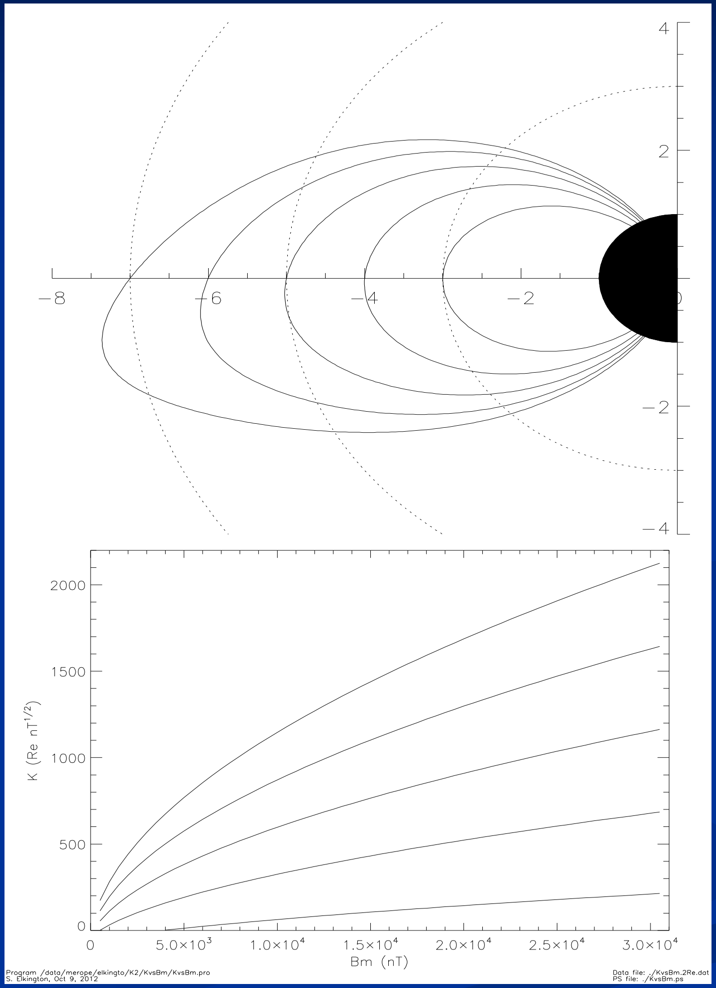
$$\langle \mathbf{v}_0 \rangle = \frac{\vec{E}_0 \times \vec{B}_0}{B_0^2} + \frac{2p}{qT_B B_0^2} \nabla_0 I \times \vec{B}_0,$$

$$I = \int_{S_m}^{S_m'} \sqrt{1 - \frac{B(s)}{B_m}}\, ds \;,$$

$$T_B = \frac{2}{v} \int_{S_m}^{S_m'} \left(1 - \frac{B(s)}{B_m}\right)^{-1/2} ds$$

$$K = \sqrt{B_m}\, I = \int_{S_m}^{S_m'} \sqrt{B_m - B(s)}\, ds$$
$$= J / 2\sqrt{2m_0 M}\,,$$

$$p = \sqrt{2mMB_m} = \gamma m v$$

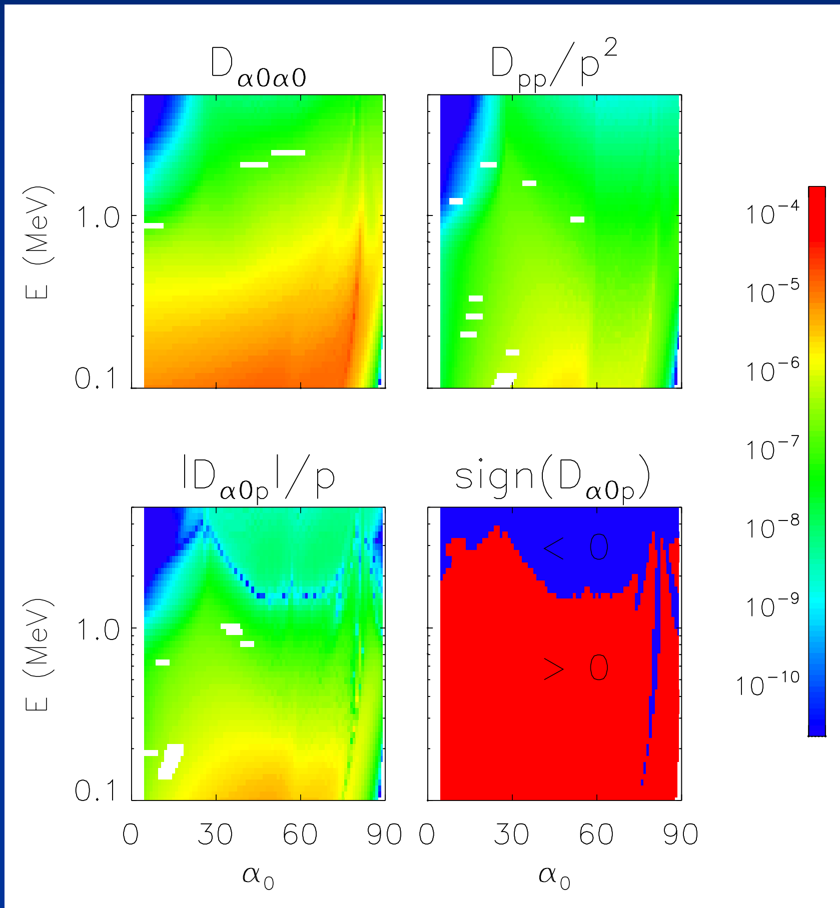


Procedure: pick first and second invariants and equatorial crossing point, calculate field geometry integrals and bounce period. From results, interpolate to find *V* for arbitrary *M*, *K*, *x*, *y* and solve for time-evolving position.

Advantages of SDE methods

- Unlike the finite-difference-like methods, the SDE method does not need a grid. Complicated boundary conditions are handled easily.
- The SDE method is very efficient when solutions are only required at a limited number of points in the phase space.
- For solutions at many points, SDE codes can be parallelized very efficiently.
- The SDE method easily handles off-diagonal diffusion terms in 2D and 3D (full 3D).
- The SDE method is very robust: it can tolerate several orders of magnitude difference in the solution.

Simulation of a simplified HSS storm (Oct. 2002): SDE + F/P



- D_{LL} uses *Brautigam and Albert* [*2000*]

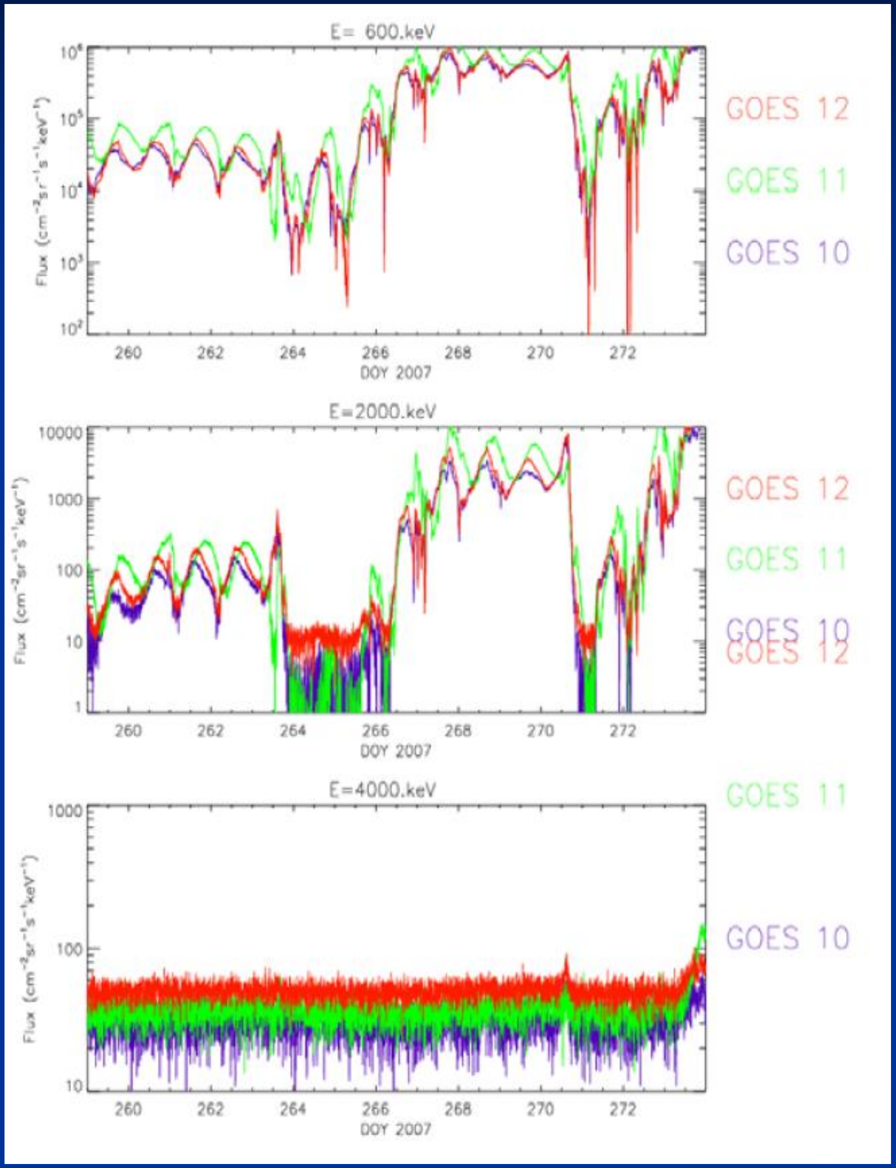
  $$D_{LL} = 10^{0.506Kp-9.325} L^{10}, \quad Kp = 3$$

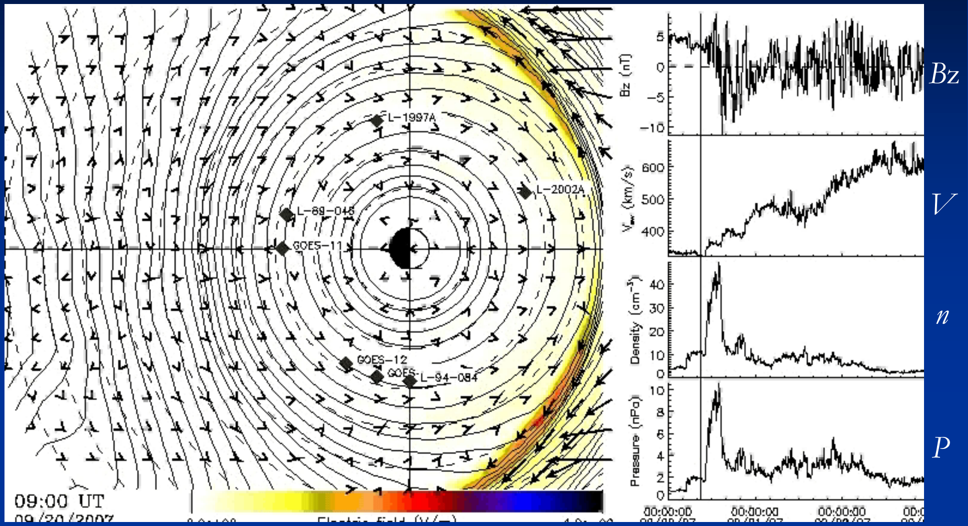
  which is comparable in magnitude to D_{LL} from LFM simulations.

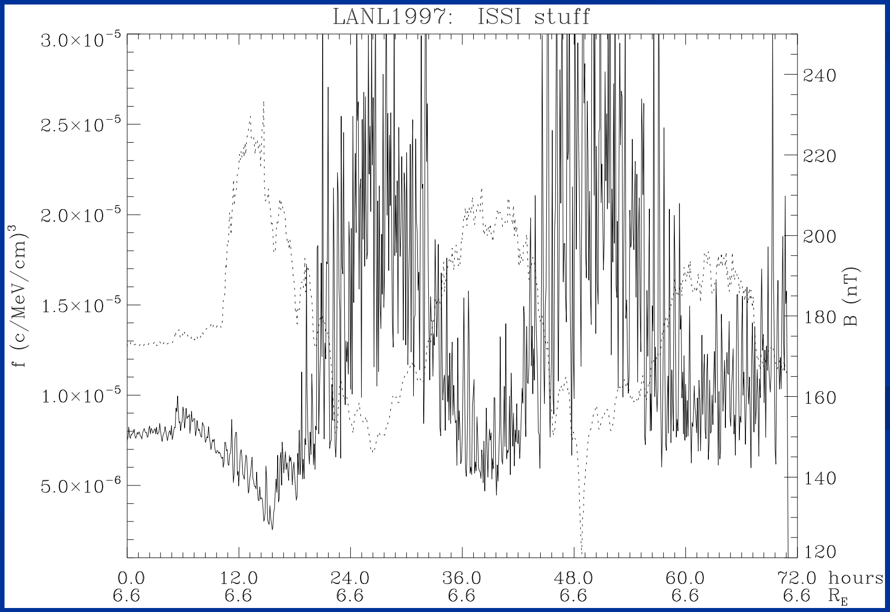
- Chorus wave diffusion coefficients are converted from α_0-p diffusion coefficients assuming no *L*-dependence.

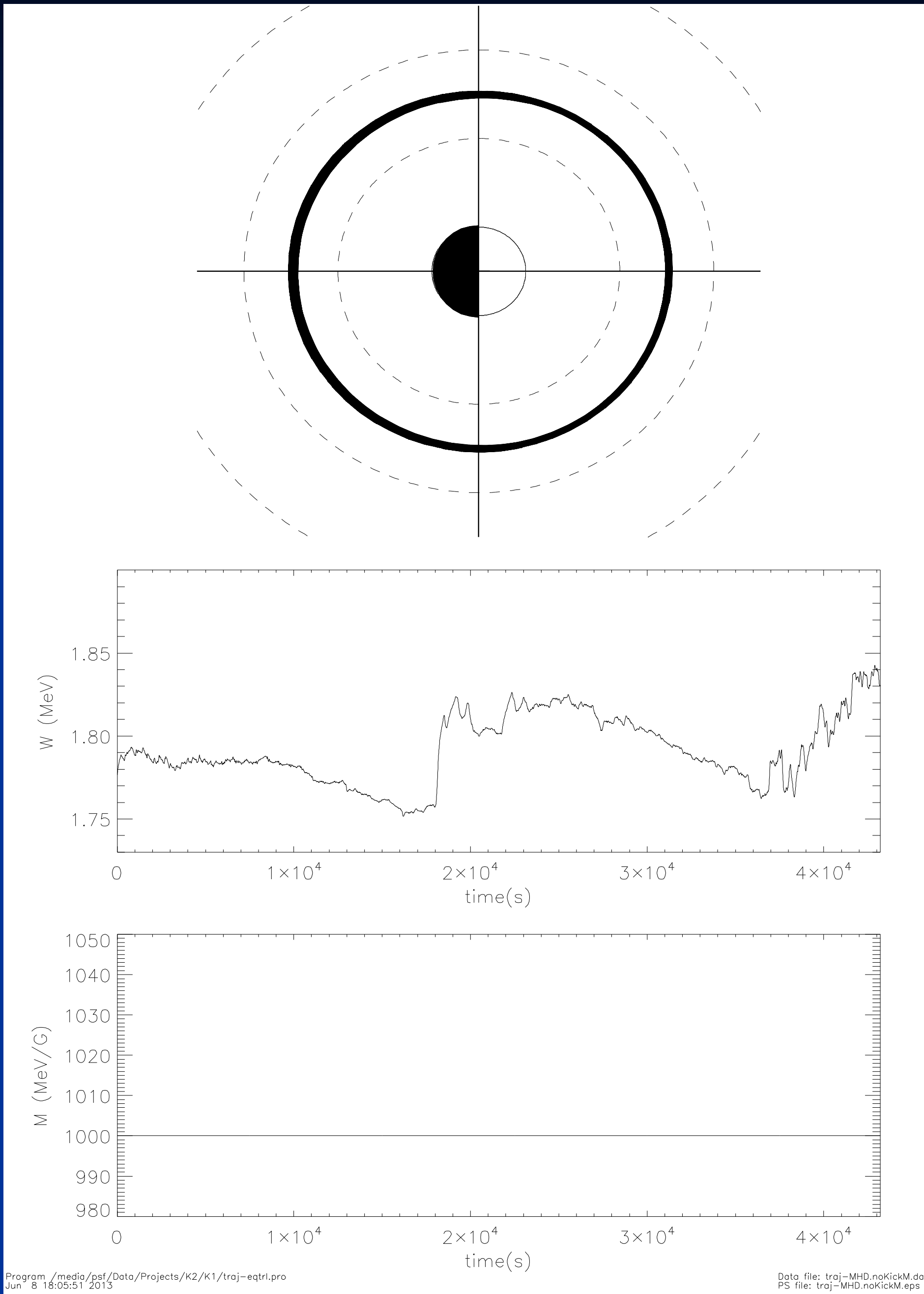
- D_{uL} and D_{KL} are set as zero.

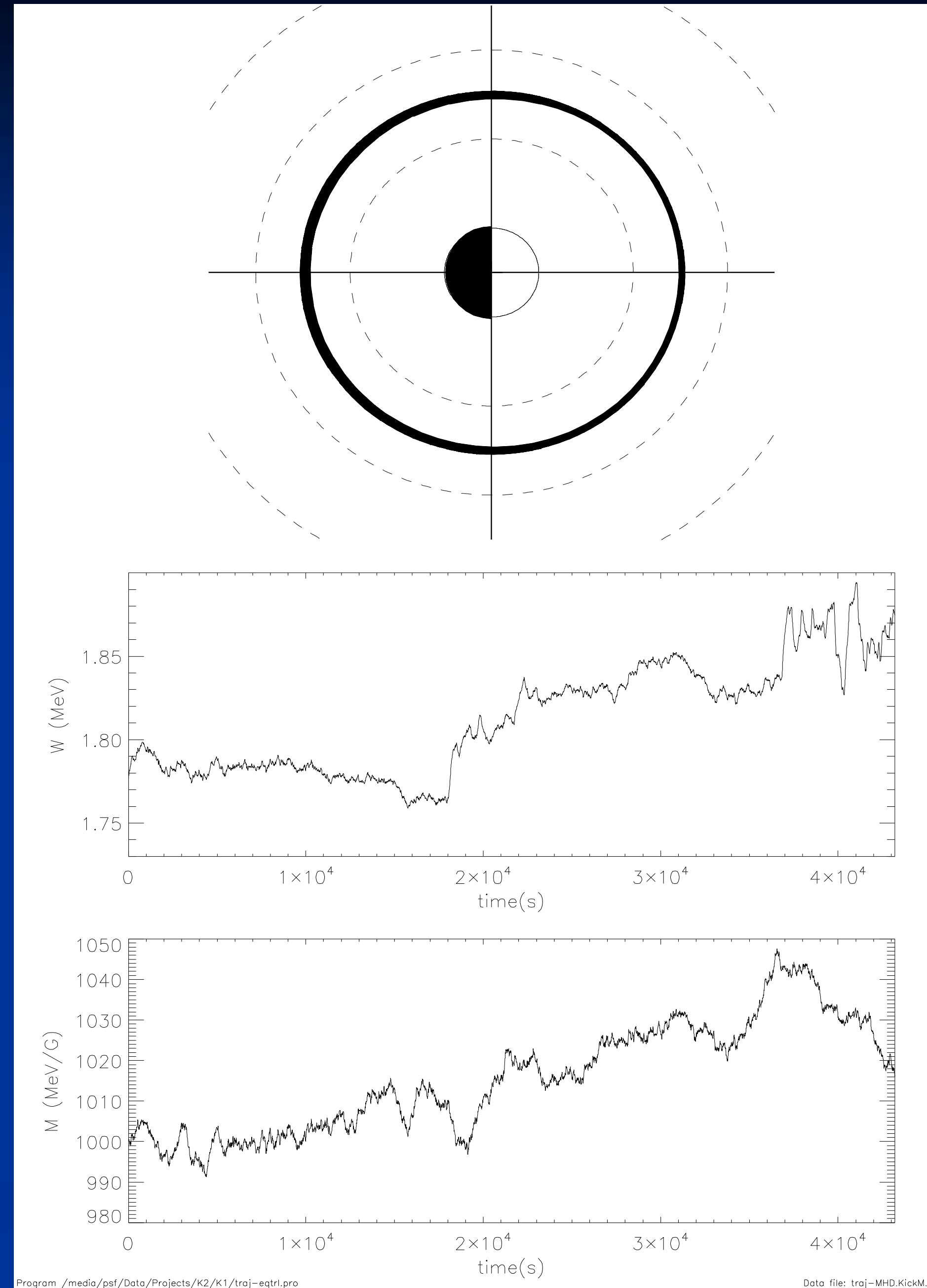
9/20/2007 flux dropout/recovery








W (MeV)
1.85
1.80
1.75
0
1×10^4
2×10^4
3×10^4
4×10^4
time(s)
M (MeV/G)
1050
1040
1030
1020
1010
1000
990
980
0
1×10^4
2×10^4
3×10^4
4×10^4
time(s)
Program /media/psf/Data/Projects/K2/K1/traj-eqtrl.pro
Jun 8 18:05:51 2013
Data file: traj-MHD.noKickM.dat
PS file: traj-MHD.noKickM.eps


W (MeV)
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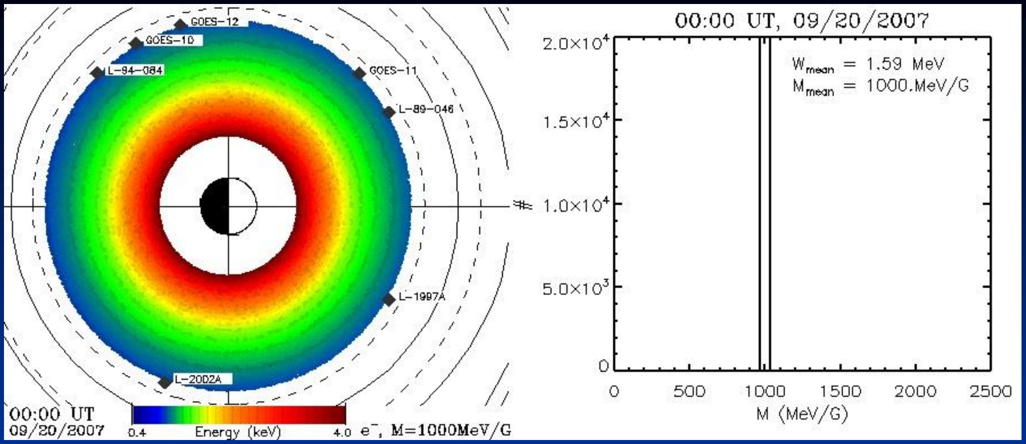
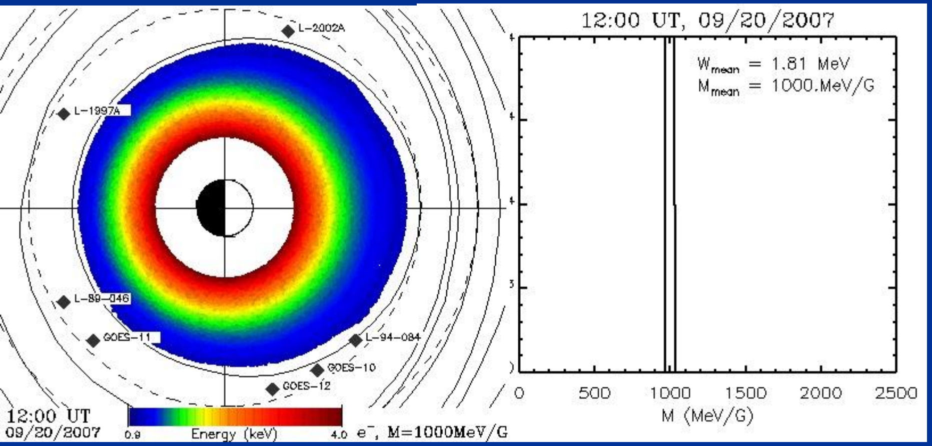
4×10^4
time(s)
M (MeV/G)
1050
1040
1030
1020
1010
1000
990
980
0
1×10^4
2×10^4
3×10^4
4×10^4
time(s)
Program /media/psf/Data/Projects/K2/K1/traj-eqtrl.pro
Jun 8 18:05:57 2013
Data file: traj-MHD.KickM.dat
PS file: traj-MHD.KickM.eps

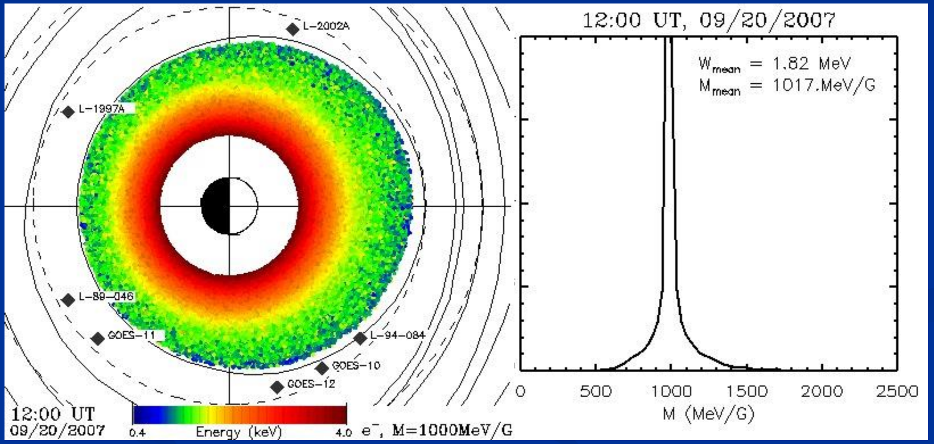
FIRST RESULTS:
HSSW storm
9/20/2007;
1000 MeV/G

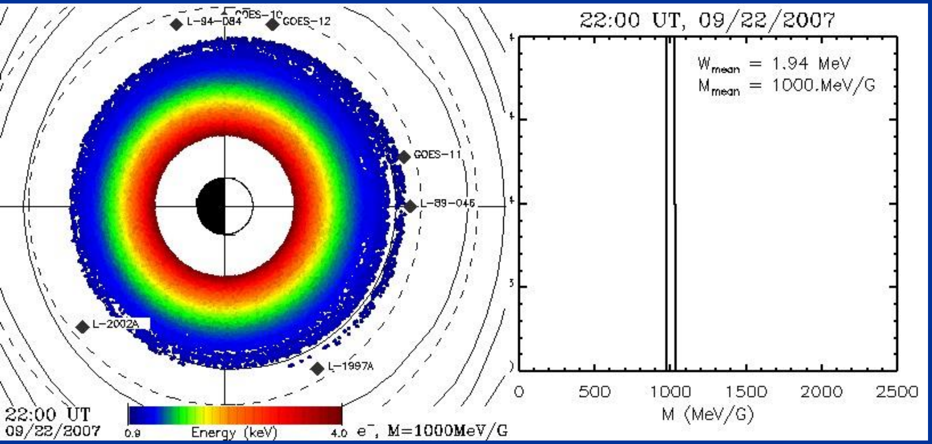


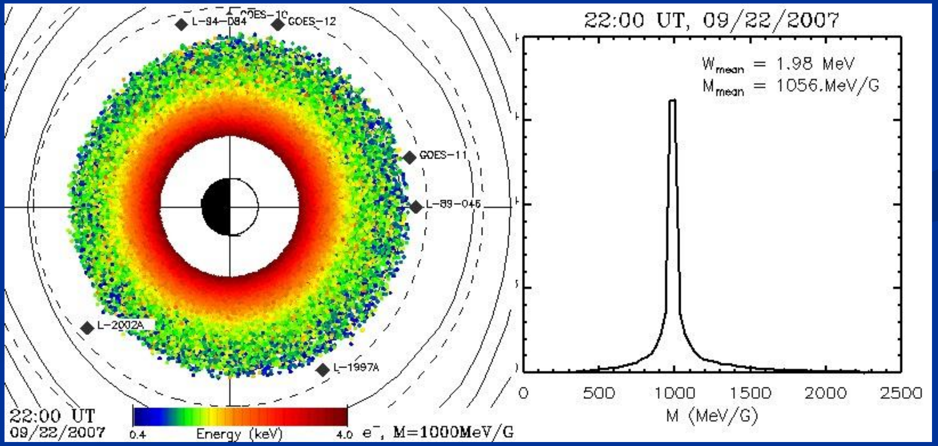
BAS Chorus
Diffusion

Conserving M

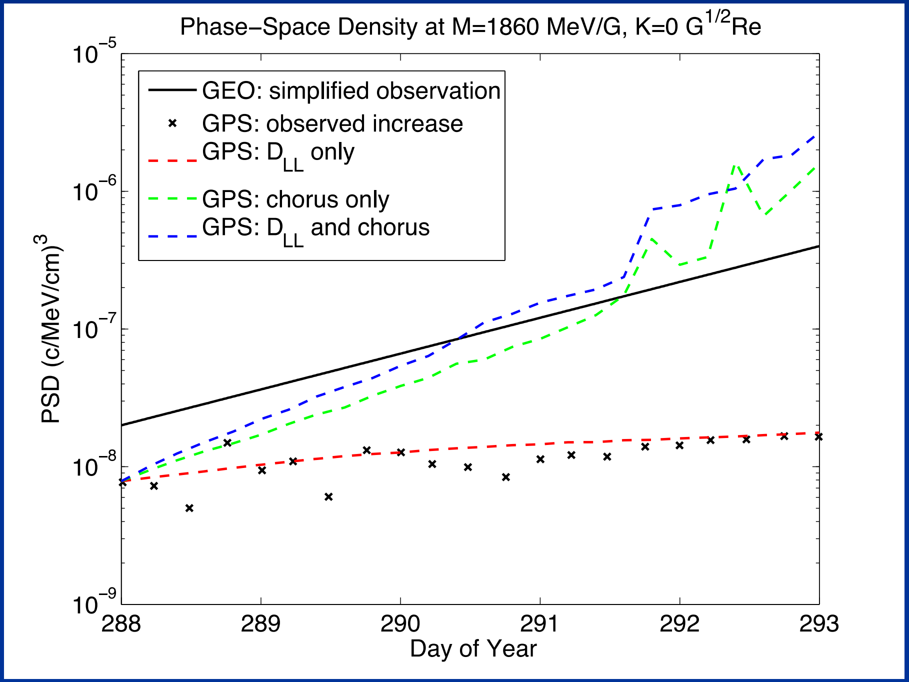








Simulation of a simplified HSS storm (Oct. 2002): SDE + F/P



- Three simulations were made:
  - radial diffusion only;
  - chorus wave diffusion only;
  - both.
- Though with simplified initial and boundary conditions, radial diffusion reproduces the observed increase.
- Chorus wave diffusion is too strong to explain the PSD change in this HSS at this *M* and *K*.