Global Simulations of Wave-Particle Interactions in the Radiation Belts: March 17, 2018 Acceleration Event

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Global simulations of the radiation belts

• Fokker-Planck simulations

\[ \frac{df}{dt} = \frac{1}{\tau} \sum_{i,j} \frac{\partial}{\partial J_i} \left( \mathcal{D}_{ij} \frac{\partial f}{\partial J_j} \right) - \frac{f}{\tau} + S \]

• Requires empirical specification of stochastic transport coefficients based on theory and observations
• Generally cannot model nondiffusive effects (e.g. advection/injection)

• MHD/particle simulations

• Global MHD model provides time-evolving electric and magnetic fields.
• Handles radial transport self-consistently
• Generally cannot model high frequency wave effects, e.g. energy and pitch angle scattering due to chorus, EMIC, etc.
Non-MHD effects via SDE methods

• Every diffusion equation is mathematically equivalent to a set of stochastic differential equations (SDEs; e.g., Tao, Chan, and Albert, [JGR, 2008]):

\[ dX = b \, dt + \sigma \, dW \]

• \( dX \) is a change in a stochastic variable \( X \) over a time \( dt \) (e.g. \( X \) may be a pitch angle, energy, or an adiabatic invariant).

• \( dW = \sqrt{t}N(0,1) \), where \( N \) is a Gaussian random variable \( \in [0,1] \).

• \( b(X,t) \) and \( \sigma(X,t) \) are coefficient functions. e.g., for a 1-dimensional diffusion equation

\[ \frac{\partial f}{\partial t} = \frac{\partial}{\partial x} \left( D \frac{\partial f}{\partial x} \right) = D \frac{\partial^2 f}{\partial x^2} + \frac{\partial D}{\partial x} \frac{\partial f}{\partial x} \]

\[ b = \frac{\partial D}{\partial x} \]

\[ \sigma = \sqrt{2D} \]

• Monte Carlo solution of the SDE yields random-walk trajectories in \( X \).
We use diffusion coefficients calculated by the PADIE code (R. Horne, S. Glauert, BAS; *J. Geophys. Res.* 110, 2005).

- Diffusion in energy and pitch angle, including cross terms.
- Converted to diffusion in $M$-$K$ space assuming dipole background field.
- Covers the effect of magnetospheric chorus waves on energetic particles
- $L$ and $K_p$-dependent: $K_p < 2$, $2 < K_p < 4$, and $K_p > 6$
2d simulation domain
$D_{MM} = 0$

$D_{MM} \neq 0$

Radial profile
3d Sims: 3/17/2013 QARBM ‘Storm Enhancement Event’

March 17, 2013 enhancement event

Epoch: 2013-03-17/20:28

- $E = 54$ keV
- $E = 105$ keV
- $E = 240$ keV
- $E = 590$ keV
- $E = 1050$ keV
- $E = 2000$ keV

Epoch Time [hr]
We have undertaken global 3d LFM-MHD simulations of the 3/17/2013 storm period, and used these results to drive test particle simulations of energetic particle populations in the inner magnetosphere and plasmasheet.
3d Sims: 3/17/2013 QARBM ‘Storm Enhancement Event’

- Provided by Wen Li and Qianli Ma (Boston U.)
- Upper and Lower band chorus waves as a function of L, MLT, and UT inferred from POES electron measurements.
- $D_{\alpha\alpha}$, $D_{pp}$, and $D_{\alpha p}$ calculated as a function of particle energy and pitch angle.
- Coefficients converted to $M$ and $p_\parallel$ for particle code.
3d simulations: K2-GC

- K2-GC is a 3d guiding-center test particle code adapted to include diffusive effects of VLF activity (chorus, hiss, etc).
- Each particle is periodically “kicked” in $M$ and $p_\parallel$ in the B-min surface in accordance with specified diffusion coefficients.

\[
M_{j+1} = M_j + b_M \delta t + \sigma_{MM} \sqrt{\delta t} N_M + \sigma_{Mp_\parallel} \sqrt{\delta t} N_{p_\parallel}
\]
\[
p_{\parallel j+1} = p_{\parallel j} + b_{p_\parallel} \delta t + \sigma_{p_\parallel M} \sqrt{\delta t} N_M + \sigma_{p_\parallel p_\parallel} \sqrt{\delta t} N_{p_\parallel}
\]

with

\[
\sigma_{MM} = \sqrt{2 D_{MM}}, \quad \sigma_{Mp_\parallel} = \sqrt{2 D_{p_\parallel M} / D_{p_\parallel p_\parallel}}
\]
\[
\sigma_{p_\parallel M} = \sqrt{2 D_{p_\parallel M} / D_{MM}}, \quad \sigma_{p_\parallel p_\parallel} = \sqrt{2 D_{p_\parallel p_\parallel}}
\]

and

\[
b_M(t, M, p_\parallel) = \frac{1}{G} \frac{\partial}{\partial M} (GD_{MM}) + \frac{1}{G} \frac{\partial}{\partial p_\parallel} (GD_{M p_\parallel})
\]
\[
b_{p_\parallel}(t, M, p_\parallel) = \frac{1}{G} \frac{\partial}{\partial M} (GD_{p_\parallel M}) + \frac{1}{G} \frac{\partial}{\partial p_\parallel} (GD_{MM})
\]

where $G$ is the Jacobian scale factor that results from the conversion of $D_{pp}, D_{\alpha \alpha}$ to $D_{MM}, D_{p_\parallel p_\parallel}$ (dipole approximation).
$D_{xx}$ validation, single-particle tests

SDE results were validated against given diffusion coefficients.

- Kick multiple particles, calculate $\langle (\Delta x)^2 \rangle$ as a function of time.
- $D_{xx}$ inferred from slope of fit.

Test particle simulations were then conducted to validate correct behavior as function of $L$, $W$, and $MLT$.

\[ D_{pp} = 1.073765 \times 10^{-5} \text{ s}^{-1} \]

Linear Fit: $1.07849 \text{ s}^{-1}$
Proof of concept: test particle simulations at constant M, K

Test simulations were undertaken for the March 17-18, 2013 GEM “Storm-time acceleration event” (http://bit.ly/28UnLpw).

- Constant-K surface calculation, a la Schulz and Lanzerotti (1974).
  - Reference field line and mirror latitude selected.
  - K calculated for reference point
  - $B_m$ calculated for other field lines base on calculated $K$.
  - Latitude on other L shells calculated for given $B_m$.

- Time-backwards test particle/SDE simulations
  - Particles of constant $M$ distributed at points along constant $K$ surface
  - Each particle run backwards until it encounters a boundary or initial condition
  - Phase space density inferred from AE-8 fluxes at IC/BC.
  - PSD and Liouville’s theorem used to construct snapshots of evolving PSD profiles as function of time.
Initial results: 3/17/2013 event-specific diffusion coefficients

3/17/2013, 6UT

3/17/2018, 6UT

3/17/2013, 12UT

3/17/2013, 18UT

3/17/2013, 24UT

3/17/2018, 24UT
9/20/2007 ISSI event
- 2d simulation
- Diffusion in $M$ only
- BAS (averaged) diffusion coefficients

3/17/2013 ISSI event
- 3d simulation
- Diffusion in $M$ and $K$
- Event-specific diffusion coefficients
Remarks/Conclusions

- K2-GC is a framework for comprehensive simulations of energetic particle dynamics in the inner magnetosphere
  - Radial transport (diffusion, advection) handled self-consistently via Test Particle/Global MHD simulations
  - Energy and pitch angle diffusion via empirical or event-specific coefficients (ESCARGOT!)
- Remarks:
  - Thus far, relatively computationally-expensive.
    - Obtaining event-specific coefficients is fairly involved.
    - Best-suited to time-backward simulations from a grid or point of interest (e.g. Van Allen probes)
    - On the other hand, its an embarrassingly-parallel problem
  - Long-time simulations suffer from reduced statistics due to bounce cone losses
- To Do (Too Due?):
  - Implementing existing 3d MHD/Test particle approaches to K2-GC
  - Optimizing computational efficiency
  - Quantitative Assessment of (this) Radiation Belt Model
  - More events! (e.g. QARBM storm-time loss, non-storm acceleration, etc)
Thank you.
Stuff. And also Things.
K2-BA: a comprehensive radiation belt simulation method

- Global transport simulations accomplished via an efficient bounce-averaged test particle code (Roederer 1970):

\[ \langle \mathbf{v}_0 \rangle = \frac{\mathbf{E}_0 \times \mathbf{B}_0}{B_0^2} + \frac{2p}{qT_B B_0^2} \nabla_0 I \times \mathbf{B}_0, \]

\[ I = \int_{s_m}^{s_m'} \sqrt{1 - \frac{B(s)}{B_m}} ds, \]

\[ T_B = \frac{2}{\nu} \int_{s_m}^{s_m'} \left(1 - \frac{B(s)}{B_m}\right)^{-1/2} ds \]

\[ K = \sqrt{B_m} I = \int_{s_m}^{s_m'} \sqrt{B_m - B(s)} ds \]

\[ = J / 2 \sqrt{2m_0 M}, \]

\[ p = \sqrt{2mMB_m} = \gamma mv \]

Procedure: pick first and second invariants and equatorial crossing point, calculate field geometry integrals and bounce period. From results, interpolate to find \( V \) for arbitrary \( M, K, \alpha, \gamma \) and solve for time-evolving position.
Advantages of SDE methods

- Unlike the finite-difference-like methods, the SDE method does not need a grid. Complicated boundary conditions are handled easily.

- The SDE method is very efficient when solutions are only required at a limited number of points in the phase space.

- For solutions at many points, SDE codes can be parallelized very efficiently.

- The SDE method easily handles off-diagonal diffusion terms in 2D and 3D (full 3D).

- The SDE method is very robust: it can tolerate several orders of magnitude difference in the solution.
Simulation of a simplified HSS storm (Oct. 2002): SDE + F/P

- $D_{LL}$ uses Brautigam and Albert [2000]

\[ D_{LL} = 10^{0.506 Kp} \times 9.325 L^{10}, \quad Kp = 3 \]

which is comparable in magnitude to $D_{LL}$ from LFM simulations.

- Chorus wave diffusion coefficients are converted from $\alpha_0 - p$ diffusion coefficients assuming no $L$-dependence.

- $D_{nL}$ and $D_{KL}$ are set as zero.
9/20/2007 flux dropout/recovery
FIRST RESULTS: HSSW storm 9/20/2007; 1000 MeV/G

Conserving M

BAS Chorus Diffusion
Simulation of a simplified HSS storm (Oct. 2002): SDE + F/P

- Three simulations were made:
  - radial diffusion only;
  - chorus wave diffusion only;
  - both.

- Though with simplified initial and boundary conditions, radial diffusion reproduces the observed increase.

- Chorus wave diffusion is too strong to explain the PSD change in this HSS at this $M$ and $K$. 

S. Elkington, March 9, 2018