Data-adaptive harmonic analysis of radiation belt electron fluxes

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UCLA-VERB 2012–2013 reanalysis

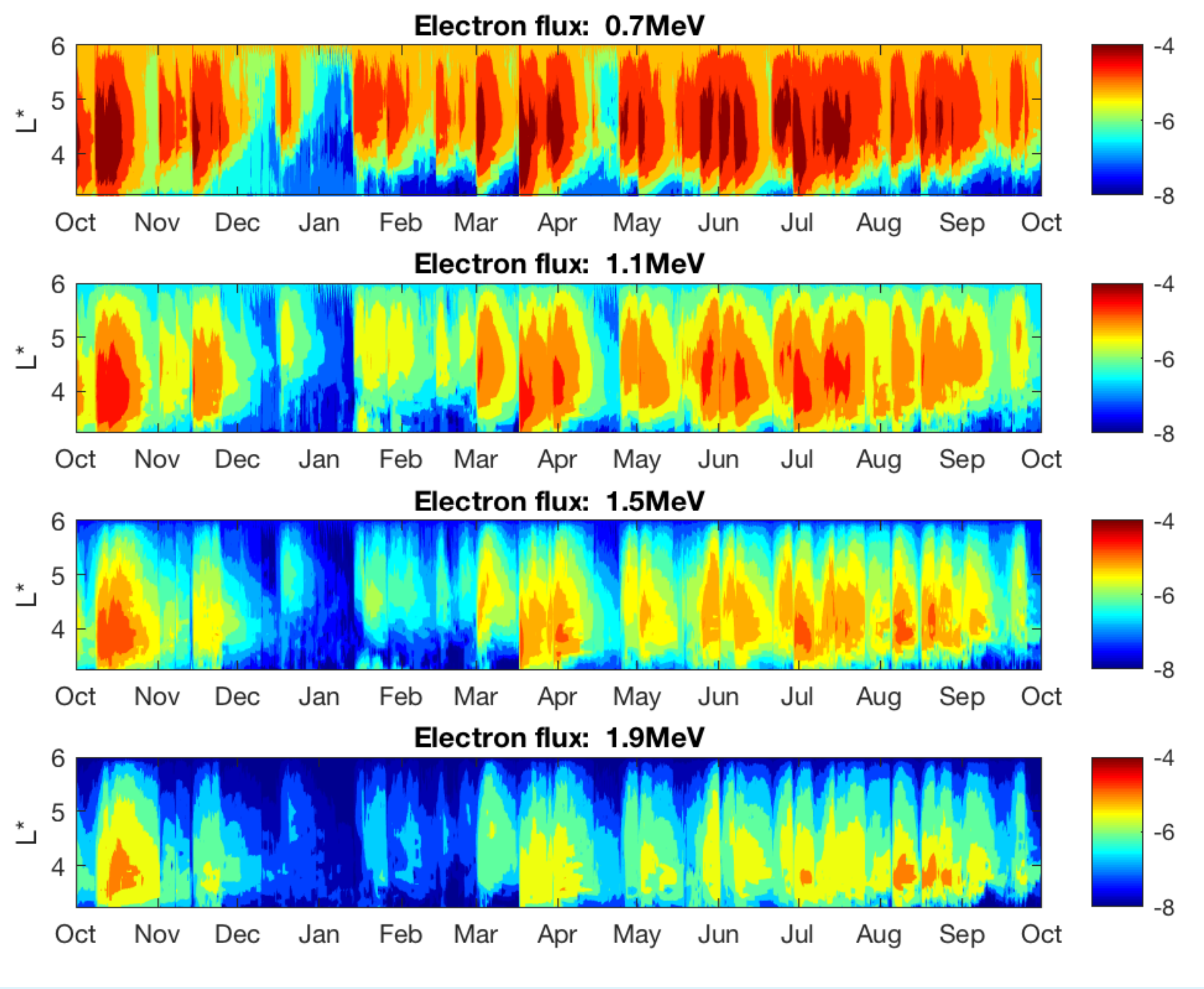

Figure 1: Log(flux) at 80° pitch-angle.

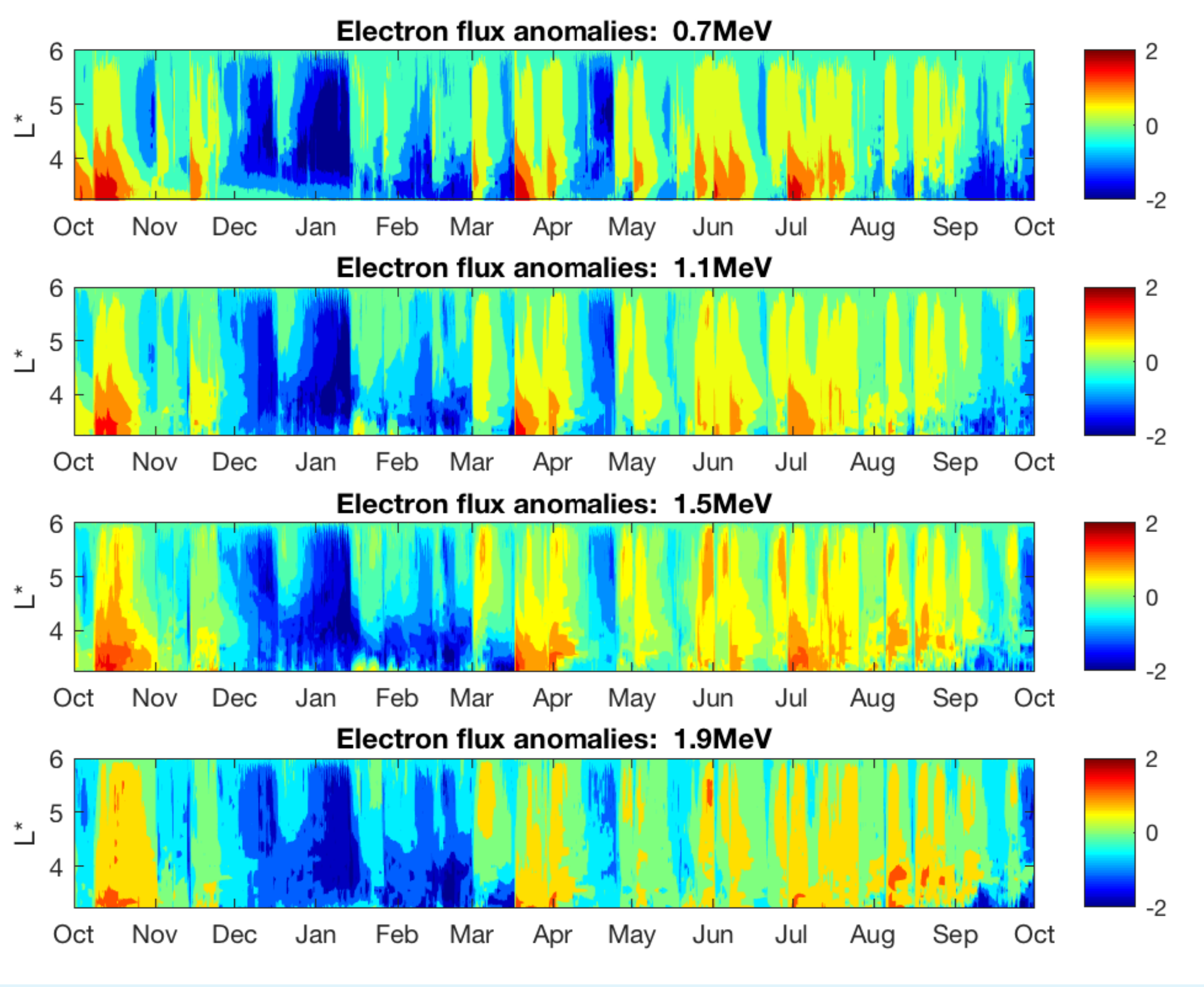

Figure 2: Log(flux) anomalies.

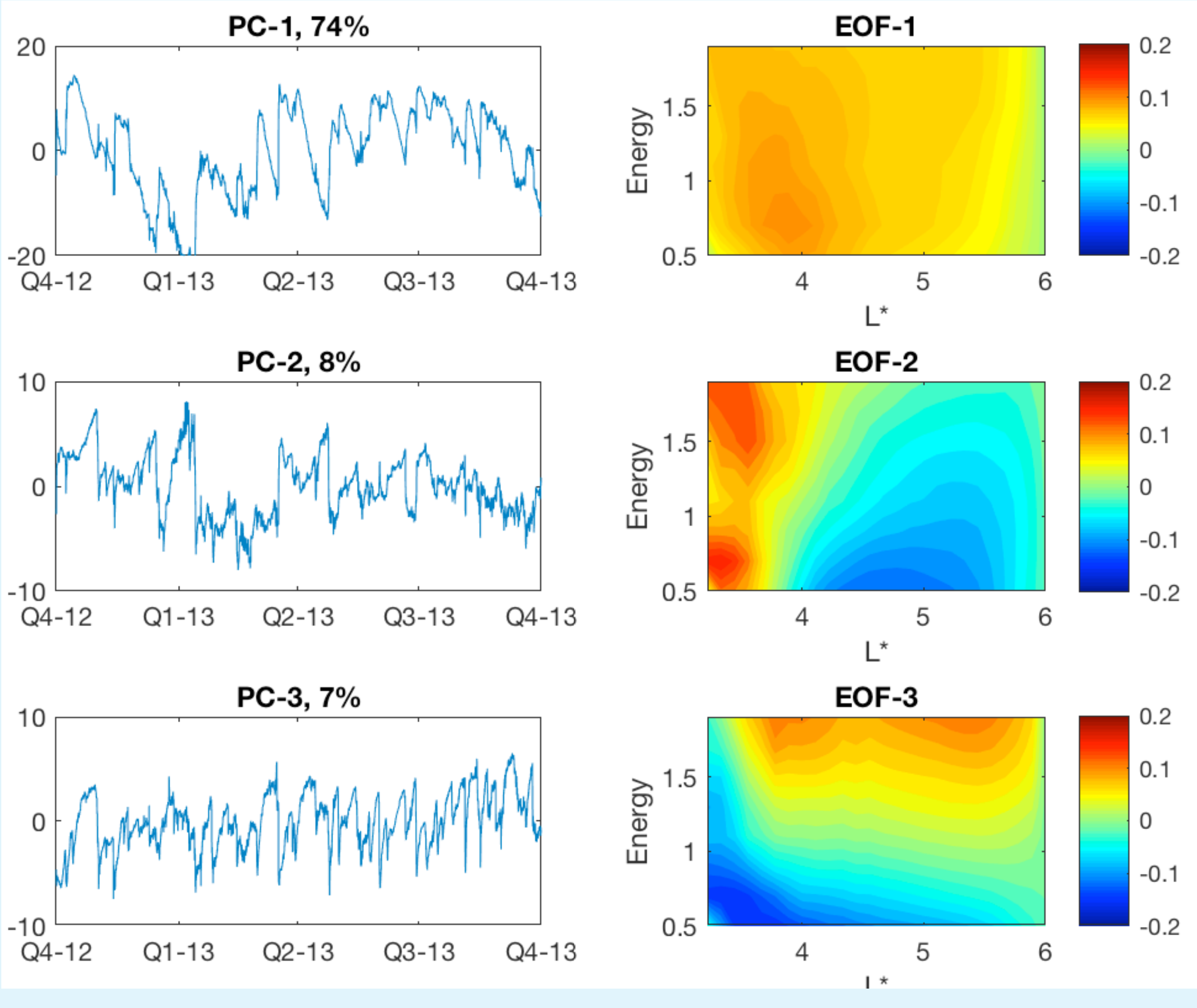

Figure 3: Principal component analysis (PCA) of Log(flux) anomalies: $d = 3$ leading Principal components (PCs) capture 90% variance and reveal complex mixture of temporal scales.

Data-adaptive Harmonic Decomposition (DAHD)

Data-adaptive Harmonic Decomposition (DAHD) [Chekroun and Kondrashov, 2017] is a novel signal processing technique that relies on spectral analysis of integral operators whose kernels are built from correlation functions. DAHD decomposes dataset into data-adaptive narrowband modes associated with distinct temporal frequencies. At a practical level, DAHD is fundamentally different from M-SSA [Ghil et al., 2002] and methods alike, as it leads to eigendecomposition of Hankel matrices built from temporal correlations.

- Given d-channel time series $\mathbf{X}(t_n) = (X_1(t_n), \ldots, X_d(t_n))$, $n = 1, \ldots, N$, compute the cross-correlation coefficients $\rho_\tau^{(p,q)}$ at lag τ between channels p and q up to a maximum lag M, i.e. $-M+1 \le \tau \le M-1$.
- Form Hankel matrix $\mathbf{H}^{(p,q)} = l\text{-circ}(\rho_{-M+1}^{(p,q)}, \cdots, \rho_{-1}^{(p,q)}, \rho_0^{(p,q)}, \rho_1^{(p,q)} \cdots, \rho_{M-1}^{(p,q)})$
- Form symmetric grand block-Hankel matrix $\mathfrak{C}$ with d^2 blocks of size $M' \times M' (M' = 2M-1)$:

$$\mathfrak{C}^{(p,q)} = \mathbf{H}^{(p,q)}, \text{ if } 1 \le p \le q \le d,$$
$$\mathfrak{C}^{(p,q)} = \mathbf{H}^{(q,p)}, \text{ else.}$$

- **DAH modes (DAHMs)** are **eigenvectors of the matrix** $\mathfrak{C} - \{\mathbf{W}^j : 1 \le j \le dM'\}$**, each composed of** d **consecutive** M'**-long segments** – they come out always in pairs with the following properties:
  - it is **orthogonal** set of oscillating functions within the time-embedding window M'.
  - each mode pair is in **exact phase quadrature** in time, a.k.a. *sin* and *cos*.
  - the eigenvalues are paired: $\lambda_j = -\lambda_{dM'-j}$, $1 \le j \le d(M'-1)/2$, and are related to singular values of cross-spectral matrix at each temporal frequency.
  - the associated time-dependent **DAH coefficients** (**DAHCs**), obtained as **projections** of the dataset onto **DAHMs**, are narrowband time series that account for the captured energy at a given frequency.
  - frequency-based reconstruction by convolving **DAHMs** and **DAHCs**.

DAHD spectrum

DAHD is applied to joint $d = 3$ leading log(flux)-PCs with window $M = 56$ days.

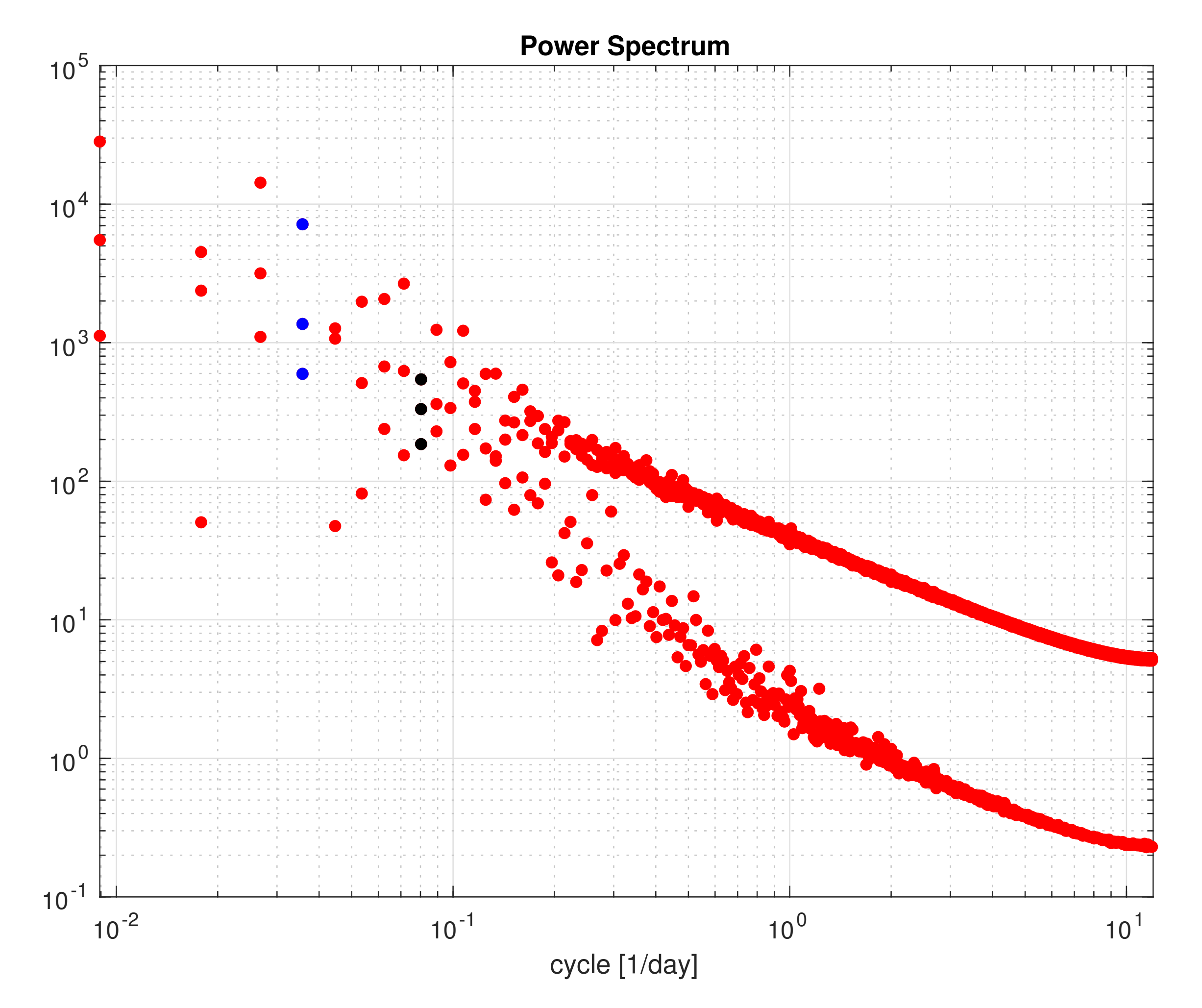

Figure 4: DAHD power spectrum ($|\lambda_j|$) for combined dataset of $d = 3$ leading Log(flux)-PCs. It is evenly spaced in frequency f; each dot is associated with a pair of DAHMs (at $f = 0$ the modes are unpaired). There are $3(\equiv d)$ DAH pairs at a given $f \neq 0$.

Conclusions

- DAHD characterizes variability of radiation belt electron fluxes by a sum of frequency-based oscillators.
- Future work – stochastic modeling and prediction by a network of coupled stochastic oscillators including solar-wind forcing [Kondrashov and Chekroun, 2017].
- DAHD Matlab toolbox under development: http://research.atmos.ucla.edu/tcd/dkondras/Software.html.

References

M. D. Chekroun and D. Kondrashov. Data-adaptive harmonic spectra and multilayer Stuart-Landau models. *Chaos*, 27(9):093110, 2017. doi:10.1063/1.4989400.

M. Ghil, M. R. Allen, M. D. Dettinger, K. Ide, D. Kondrashov, M. E. Mann, A. W. Robertson, A. Saunders, Y. Tian, F. Varadi, and P. Yiou. Advanced spectral methods for climatic time series. *Rev. Geophys.*, 40, 2002.

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Numerical Results

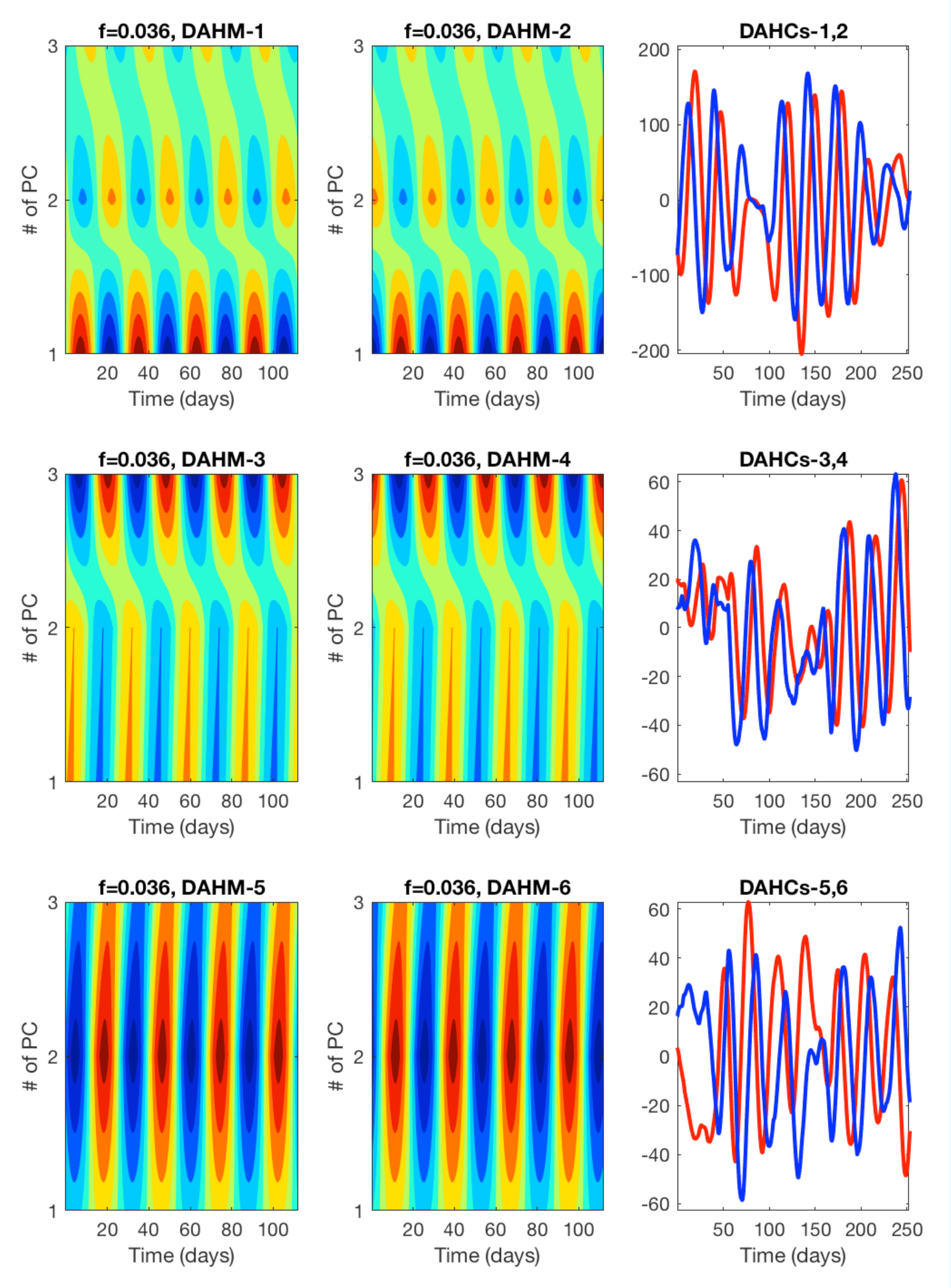

Figure 5: Spatio-temporal patterns of DAHMs and time series of associated DAHCs for top-to-bottom spectral pairs at selected frequency (blue dots in Fig. 4).

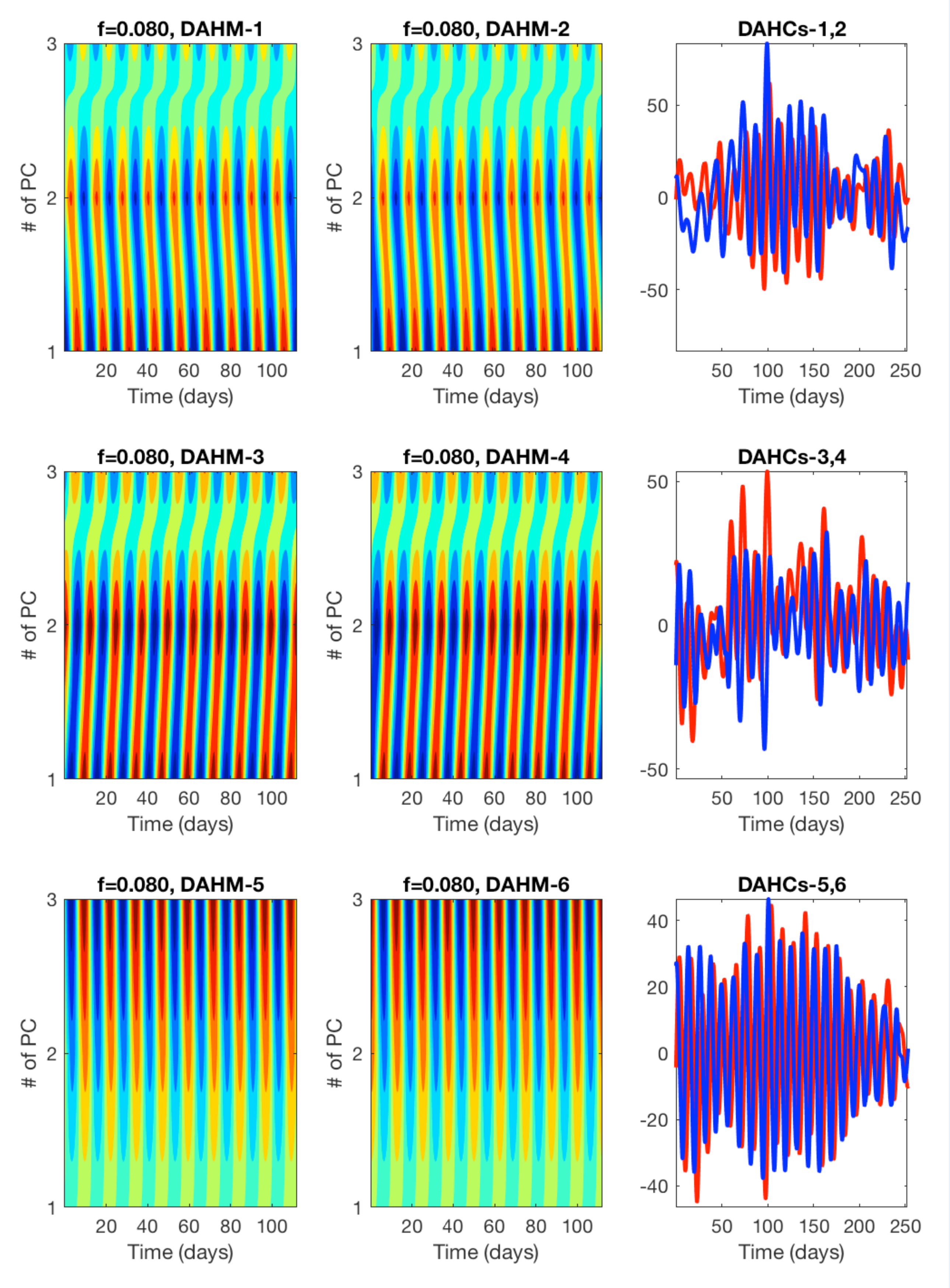

Figure 6: Same as in Fig. 5 but at another frequency (black dots in Fig. 4).

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