Nonlinear/quasilinear bounce resonance interaction between equatorial noises and equatorially mirroring electrons in the magnetosphere

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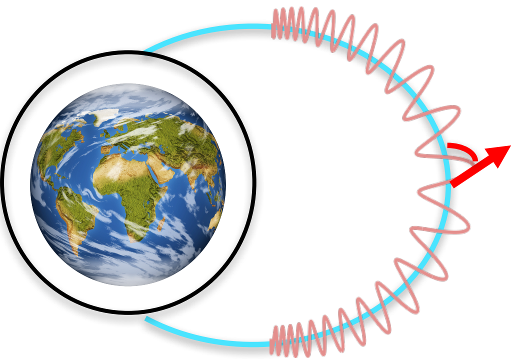
Why bounce resonance?



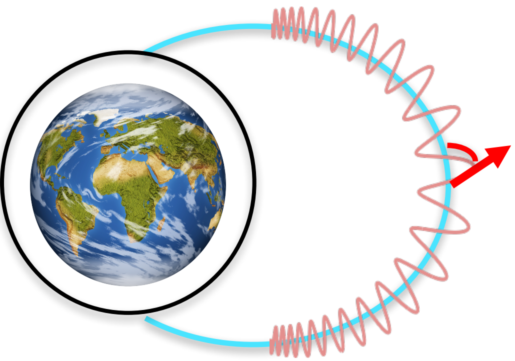
Equatorial pitch angle of 45°
Bounce motion with finite $V_{||}$.
Affected by gyro-resonance when $\omega - k_{||} v_{||} = n\Omega_e/\gamma$

Equatorial pitch angle of ~90°
$V_{||}$~0 is too small to be affected by gyro-resonances.

An effective mechanism for scattering energetic electrons.

A mechanism for removing equatorially mirroring electrons out of the equatorial plane. Coherent variation of electrons at all pitch angles has been reported [*Kanekal* et al., 2001].

Physics of bounce resonance with equatorially mirroring electrons.



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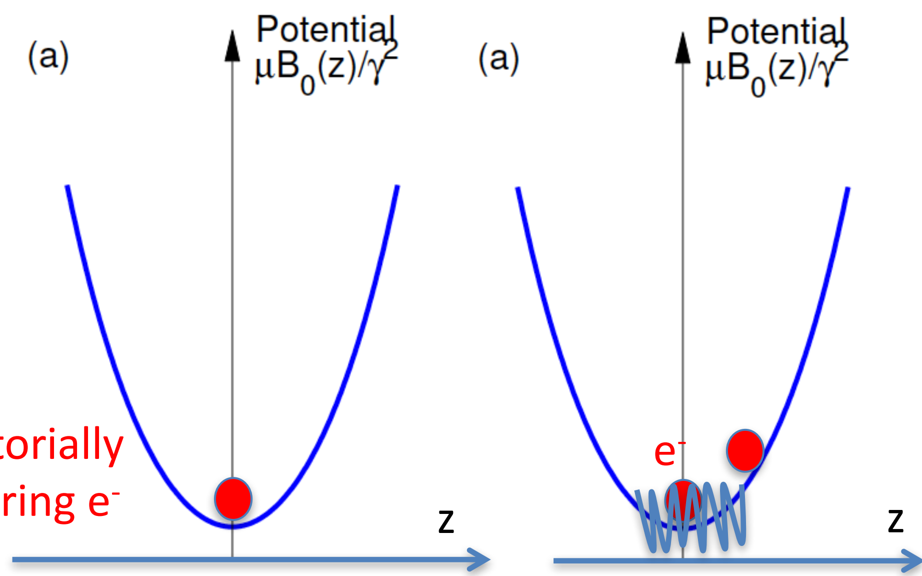
$V_{||}$~0 is too small to be affected by gyro-resonances.

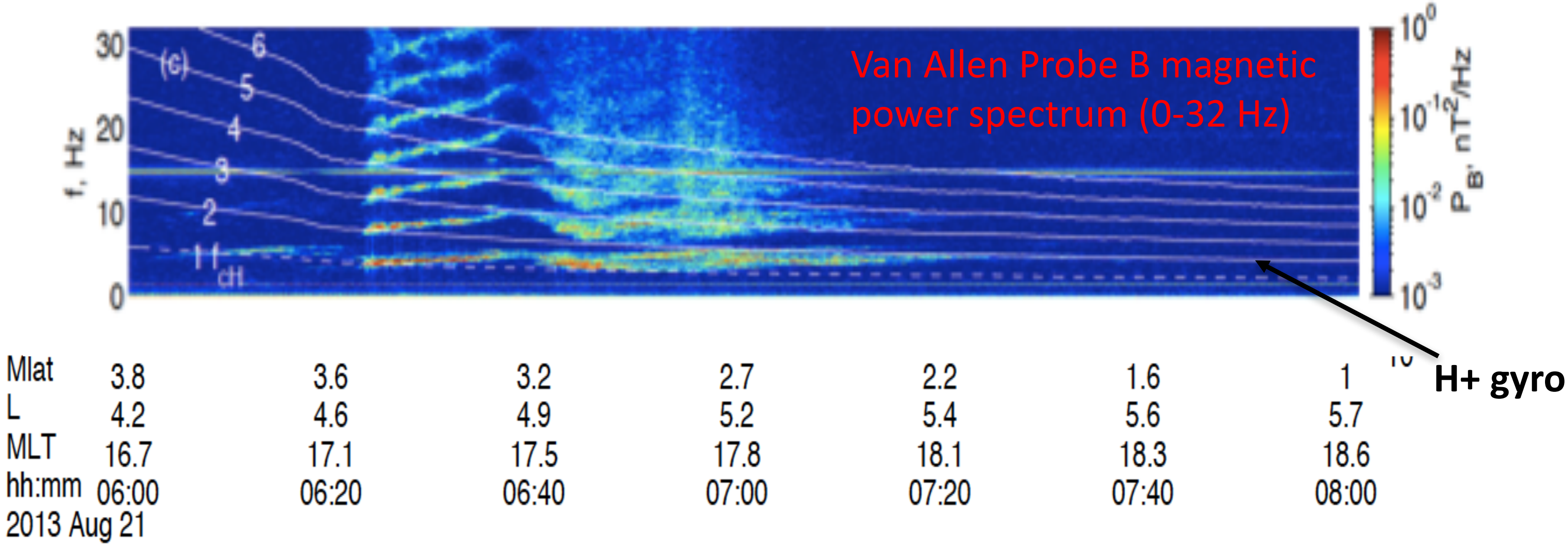
Electrons of α_{eq}=90° stay on the equator plane. They are like in the bottom of "bounce potential" well.

Any small perturbation will incur simple harmonic bounce motion of several Hz.

Equatorially mirroring e^-



Bounce resonance candidate:
Magnetosonic waves, aka, equatorial noises and ion Bernstein mode



Parallel forces available: $k_{//}Bw_{//}$, $Ew_{//,}$ Wave equatorial confinement.
Wave frequency comparable to electron bounce frequency.
Coherency longer than multiple electron bounce periods

Dynamics of equatorially trapped electrons due to bounce resonance with a coherent wave

1D gyro-averaged test particle model along a field line to study the effect of bounce resonance

Dipole — Monochromatic MS wave

$$\frac{dp_z}{dt} = -\frac{\mu}{\gamma}\frac{\partial B_0(z)}{\partial z} + \sin\phi\left(J_1(\beta)\frac{eB_x^w p_\perp}{\gamma m_e} - J_0(\beta)eE_z^w\right)g(\lambda)$$

$$\frac{dz}{dt} = \frac{p_z}{\gamma m}$$

$$\frac{d\phi}{dt} = \omega - k_z v_z$$

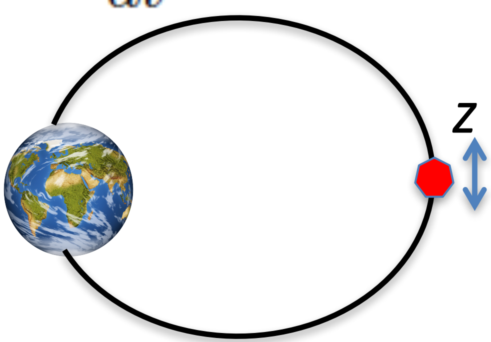
Wave magnetic mirror force

Wave $E_{//}$ force

Finite Larmor radius effect

$$\beta = \frac{k_\perp p_\perp}{eB_0}$$

Latitudinal distribution of wave amplitude



Example setup:

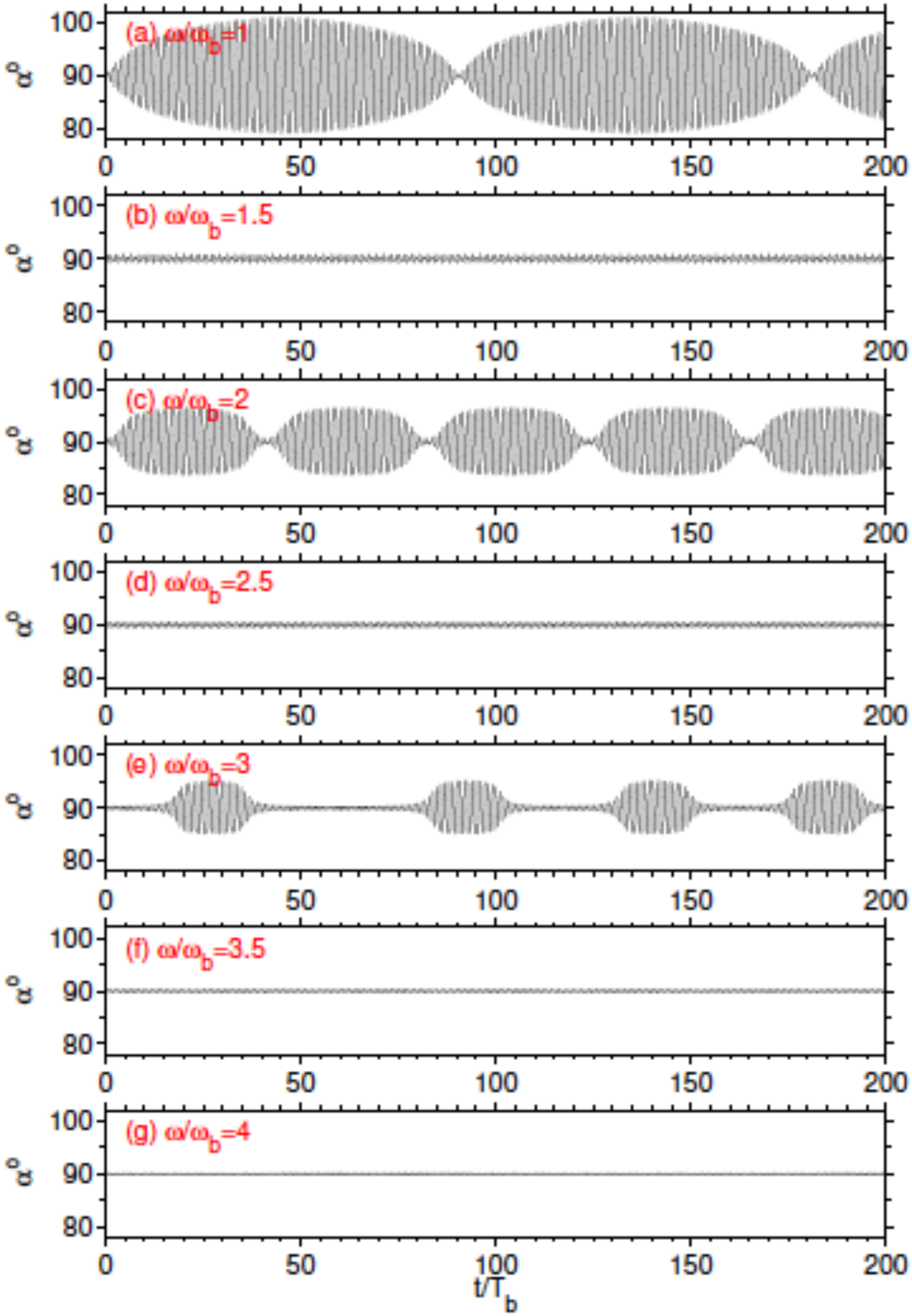
(1) A monochromatic wave with frequency *f*, wave normal angle 88°.

(2) Wave amplitude Bz = 50 pT at the equator and has Gaussian distribution over latitude λ with Δλ=3°.

(3) L=6.6 and N_e= 50 cm^{-3}

(4) A electron is launched with KE_0= 300 keV, α_0=90°, λ_0=0°.

Chen et al., 2015



Equatorial pitch angle versus time for different wave frequency.

- Bounce resonance occurs when $\omega = n\omega_b$.
- No effective bounce resonance for n>=4.
- Bounce resonance perturbs α_{eq} by a few degree, leading to finite $v_{//}$ and therefore Landau resonance (requires α_{eq} to be ~3^o away from 90^o).
- The envelope of pitch angle variations shows modulation (nonlinear tuning of ω_b).

A simplified nonlinear oscillation model where the number of parameters are reduced to 3 ($\tilde{A}$ $\tilde{k}_z$ $\tilde{\omega}$).

$$d^2\tilde{z}/d\tilde{t}^2 + \tilde{z} + \frac{39}{18}\tilde{z}^3 = -\tilde{A}\sin(\tilde{\omega}\tilde{t} - \tilde{k}_z\tilde{z} + \phi_0)g(\lambda)$$

$$\tilde{z}|_{\tilde{t}=0} = d\tilde{z}/d\tilde{t}|_{\tilde{t}=0} = 0$$

$$\tilde{A} = \frac{B_z^w k_z L R_E}{9B_0}\frac{2J_1(\beta)}{\beta} + \frac{E_z^w J_0(\beta) e L R_E \gamma}{9\mu B_0}$$ Effective amplitude

$$\tilde{k}_z = k_z L R_E$$ Normalized parallel wave number

$$\tilde{\omega} = \omega/\omega_b$$ Normalized wave frequency

$$\tilde{z} = z/(LR_E) \qquad \tilde{t} = \omega_b t$$

We find that for equatorially mirroring electrons,

(1) Bounce resonance condition $\omega=n\omega_b$. n=1,2,3.

(2) Effective higher harmonic resonance requires higher values of ($\tilde{A}$ and $\tilde{k}_z$).

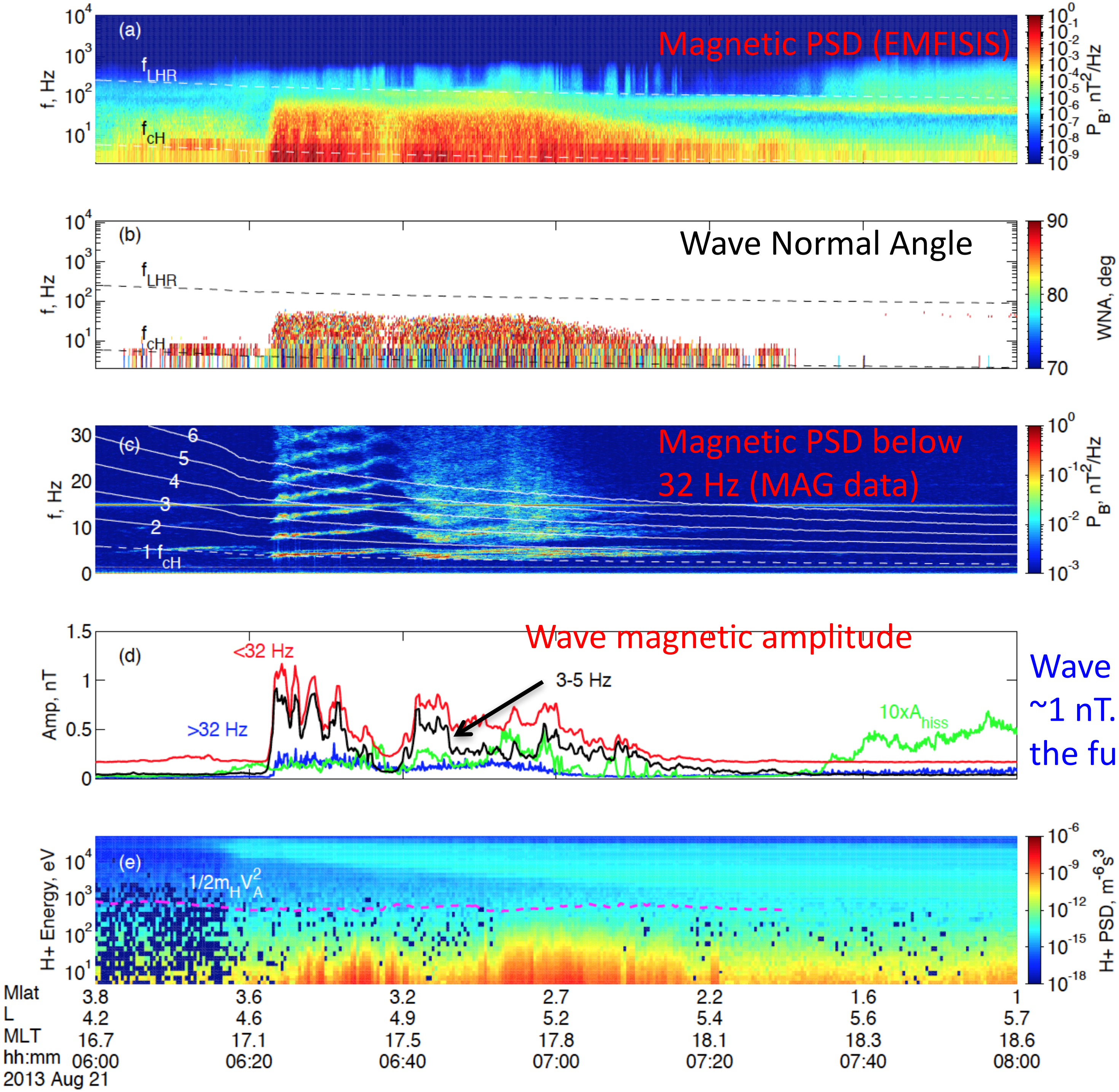
(3) Finite Larmor radius effect can not be ignored. This effect reduces effective amplitude.

(4) Linearization yields a driven Mathieu equation, which allows growing oscillation.

$$d^2\tilde{z}/d\tilde{t}^2 + \tilde{z}(1 - \tilde{A}\tilde{k}_z\cos(\tilde{\omega}\tilde{t} + \phi_0)) = -\tilde{A}\sin(\tilde{\omega}\tilde{t} + \phi_0)$$

Van Allen Probe B

L~ 4.8,
MLT ~17hr,
MLAT ~ 3^o

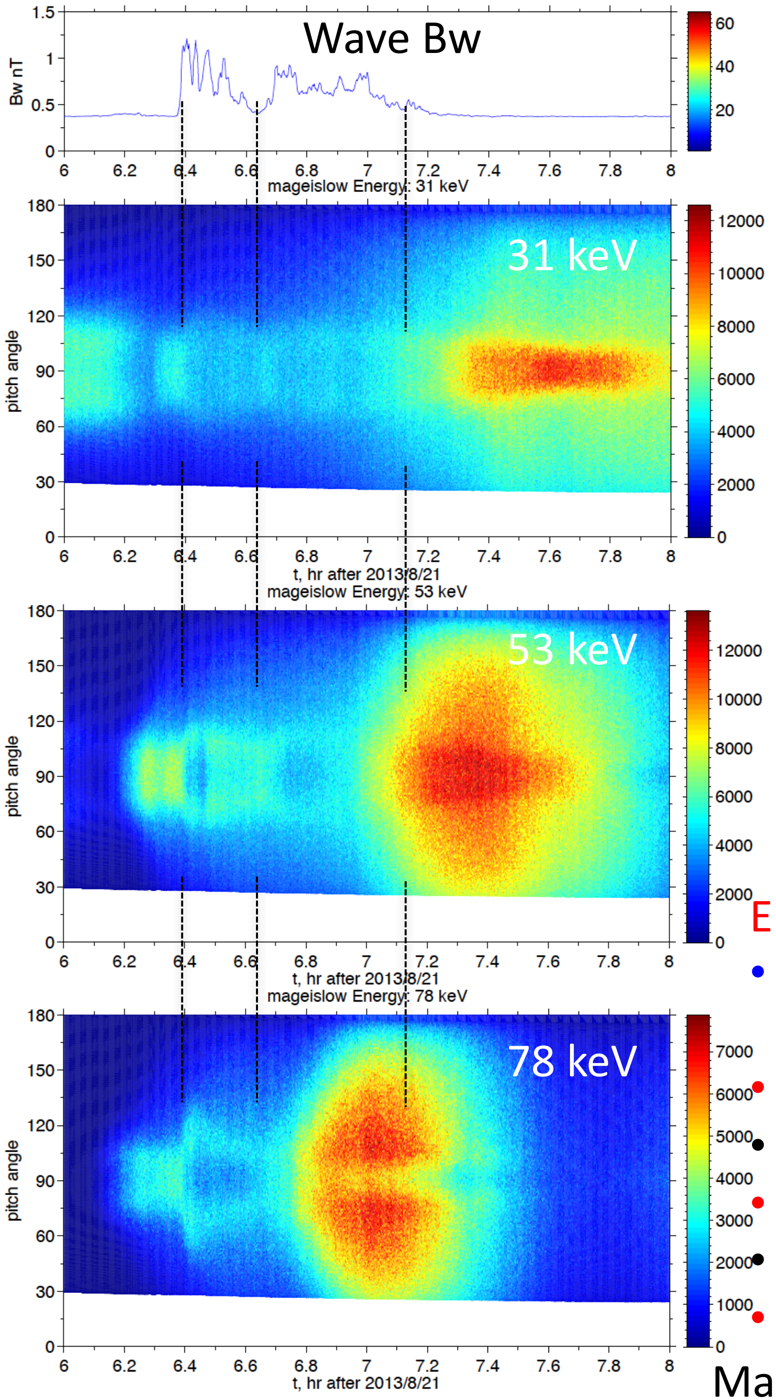


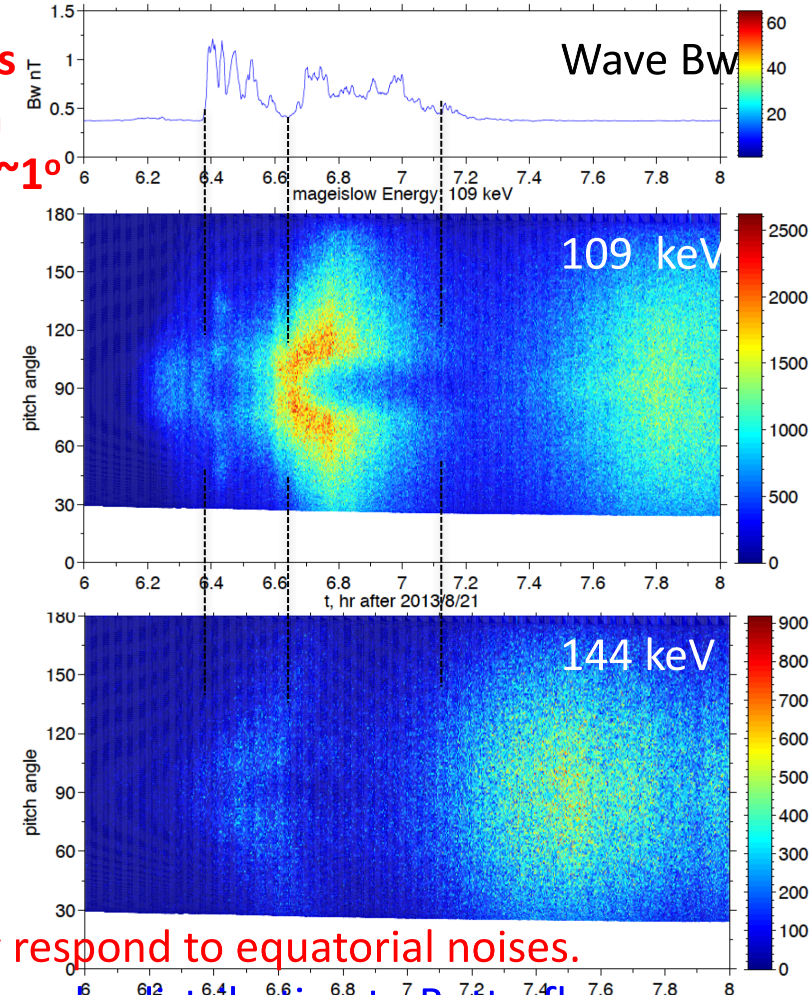
harmonic emission starting from 1st, but rising in frequency.

Wave amp. varies by up to ~1 nT. Most wave power at the fundamental (3-5 Hz).

H+ PSD at PA= 90^o
Ion ring distribution is observed during wave event period.

Van Allen Probes MagEIS electron count rate with ~1° pa resolution

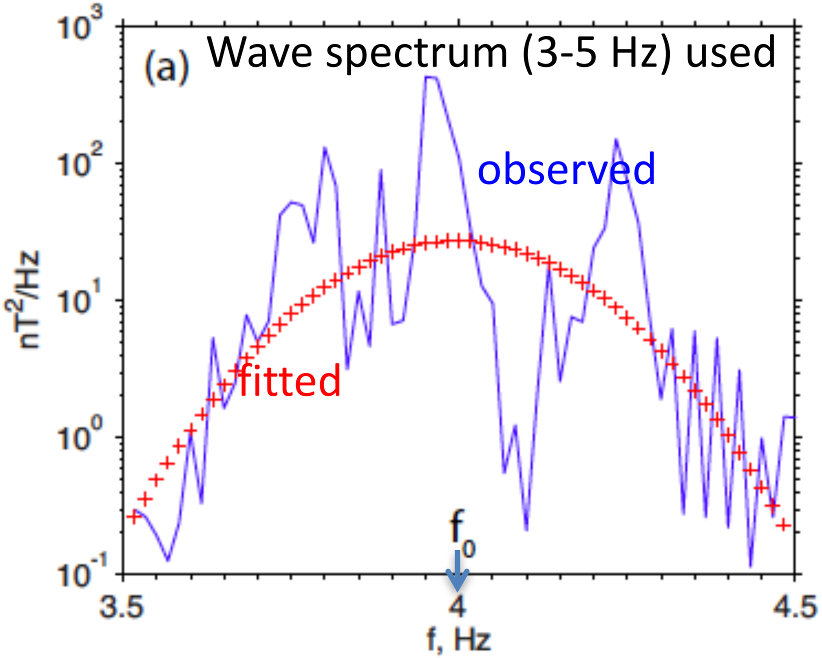




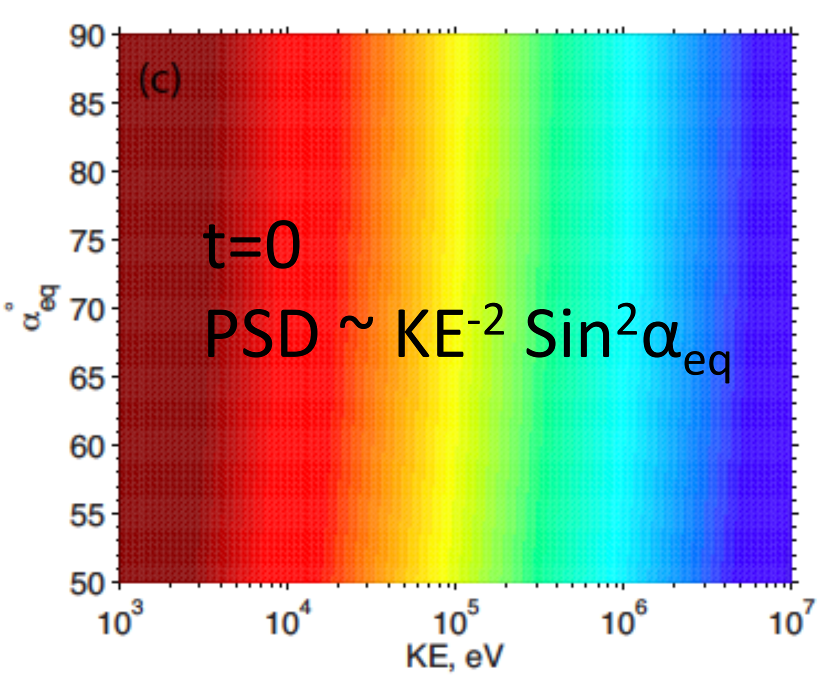
Electrons strongly respond to equatorial noises.

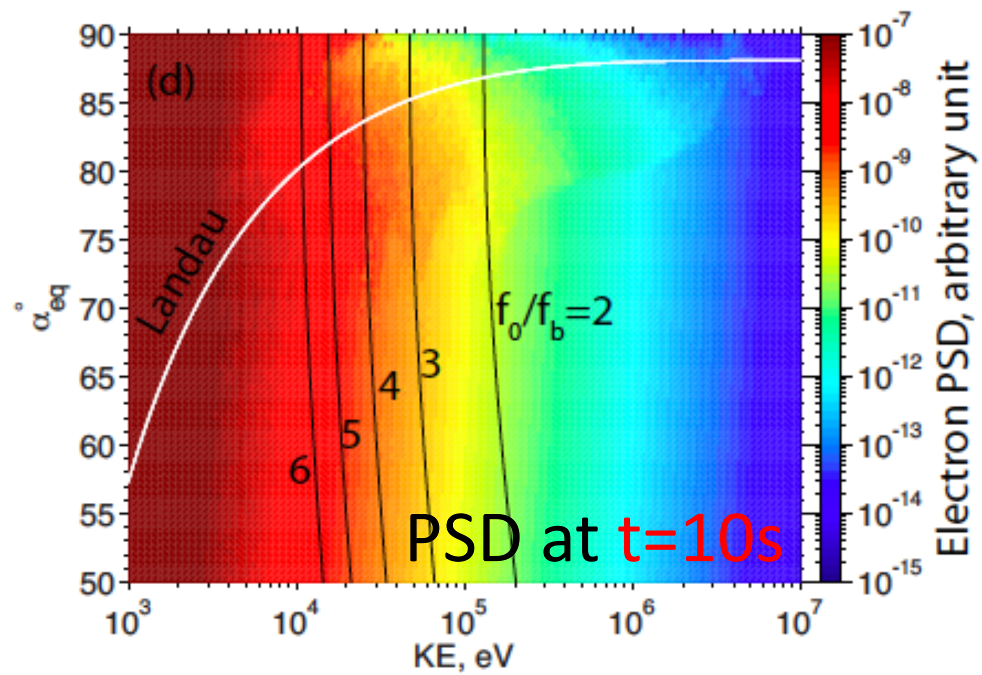
- Evolution of Pancake distribution to Butterfly distribution in high Bw.
- Modulation of pa distribution by wave intensity.
- Reduction of 90° electron count in high Bw.
- Energy dependence.
- Butterfly responses for 2, 2.3, 2.9 MeV REPT channel
- No clear responses from HOPE (<50 keV).

Maldonado et al., 2016

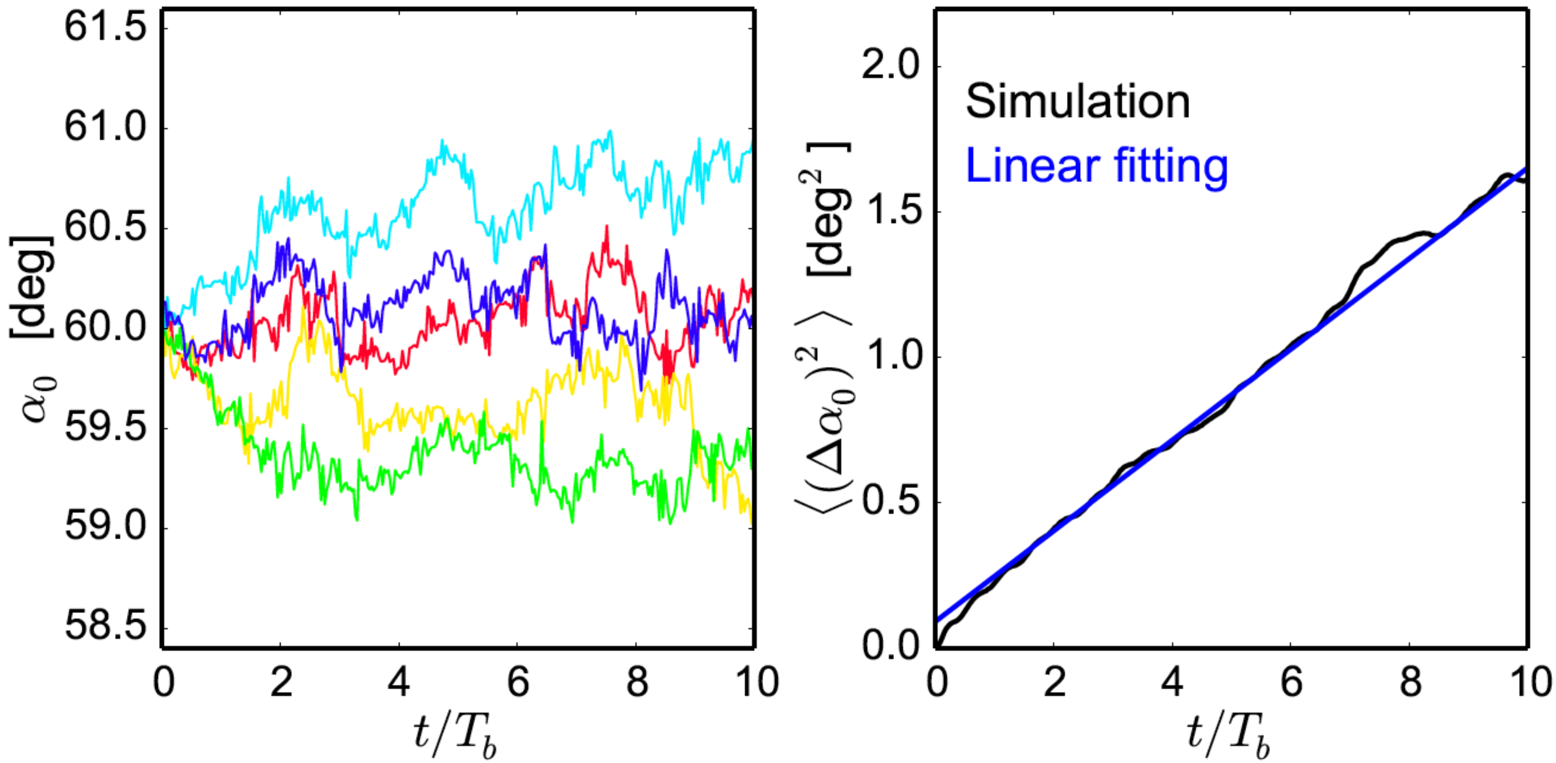


Using test particle simulation, Electron butterfly distribution is reproduced over a time scale of 10 s, because of bounce resonance, which leads to pitch angle transport from 90° to smaller values.





Diffusion due to broadband magnetosonic waves of random phases



Li, X. et al., 2015

MS wave model from Horne et al., 2007

Bounce diffusion coefficients derived.

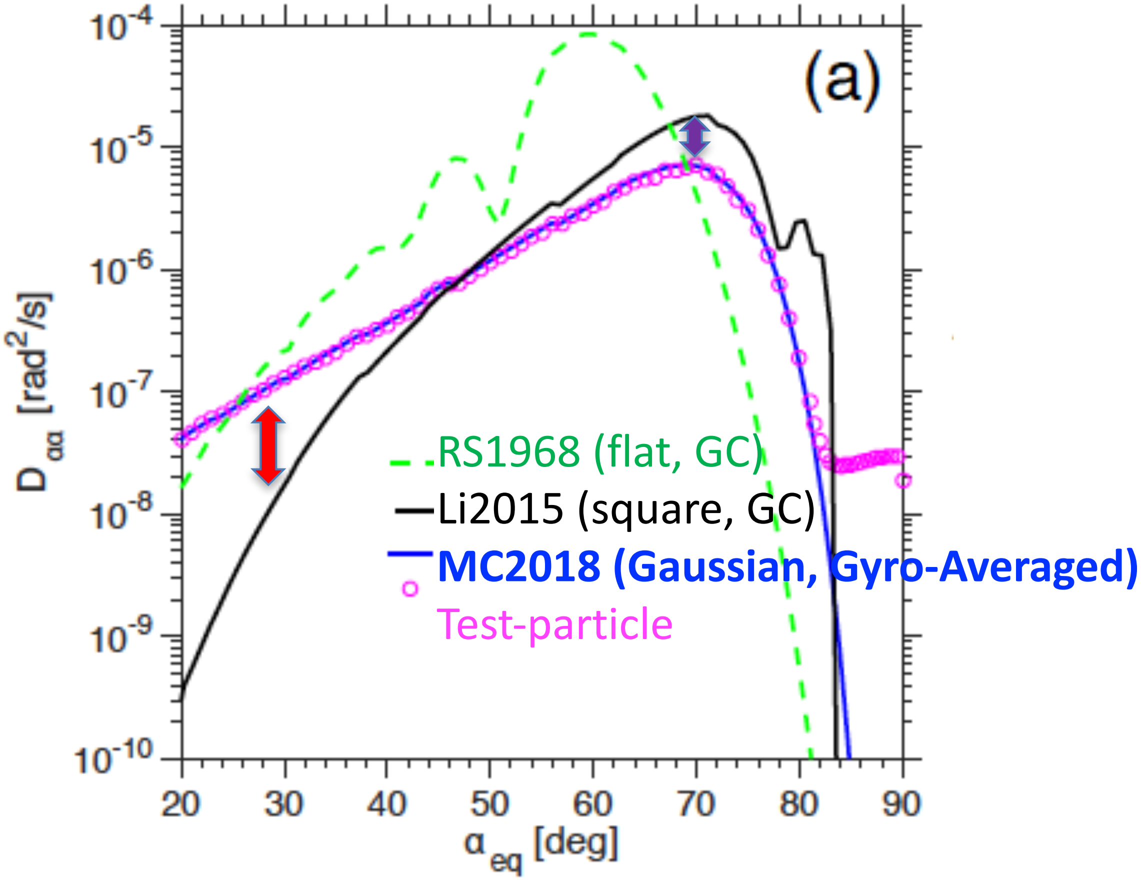
- Several formula exist [Roberts and Schulz 1968, Li et al., 2015, Shprits 2016].
- Our new formula [Maldonado and Chen 2018] consider:
  - More realistic latitudinal distribution, Gaussian instead of flat or square profile.
  - Gyro-averaged equation instead of guiding center, allowing finite Larmor radius effect and magnetic moment violation.

Quasilinear analysis

- Assume Zero-order unperturbed bounce motion.
- Derive First-order linear perturbation △a, △E.
- Perform ensemble average of $\triangle^2$ over bounce phases and wave phases and evaluate scattering rate $\triangle^2/t$ (such as D_{aa} , D_{EE})
- Obtain dependence on $k_z z_m$, $\frac{z_m}{z_w}$, $\frac{\omega}{\Omega_b}$, and $k_\perp \rho$

[Maldonado and Chen 2018]
On the diffusion rates of electron bounce resonant scattering by magnetosonic waves

Diffusion coefficient comparison with Roberts and Schulz 1968, Li et al., 2015, and test particle results



Equatorial confinement of wave power leads to less scattering at small α but more at large α.

The effect of finite Larmor radius leads to less scattering at high α.

The effect of changing magnetic moment yields more scattering at small α.

The MC2018 scattering formula is validated against test particle results except for $\alpha \sim 90^{o}$.

Summary

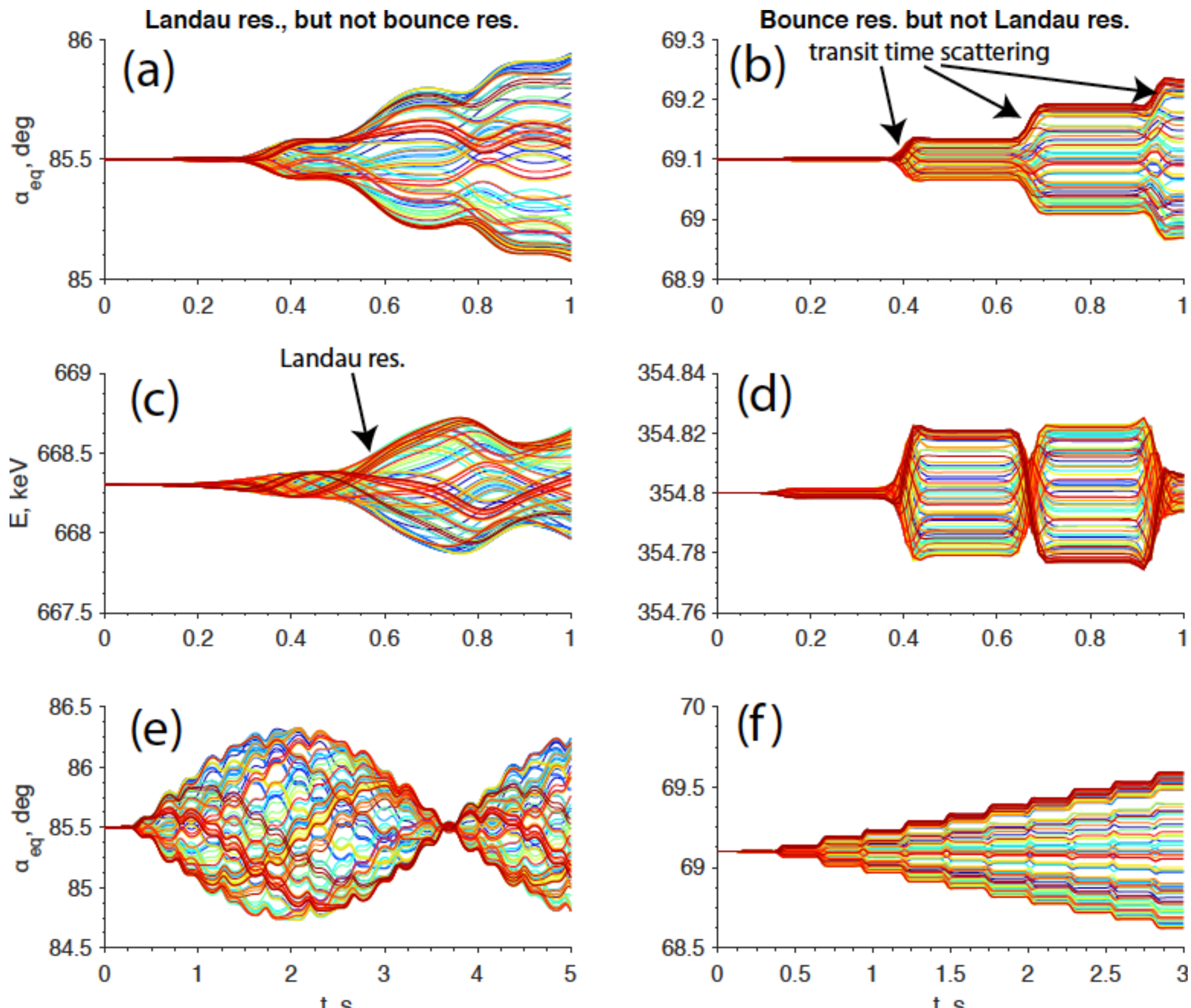
- Bounce resonance with magnetosonic wave can lead to nonlinear oscillation of equatorially trapped electrons.

- This oscillation will produce finite v_z, which could enable gyro-resonance interaction with other waves. This might explain coherent variation of electrons at all pitch angles.

- Modulation of electron pitch angle distribution by equatorial noises is presented. We propose *negative pitch angle advection through bounce resonance with equatorial noises* as mechanism responsible for rapid formation of butterfly distribution.

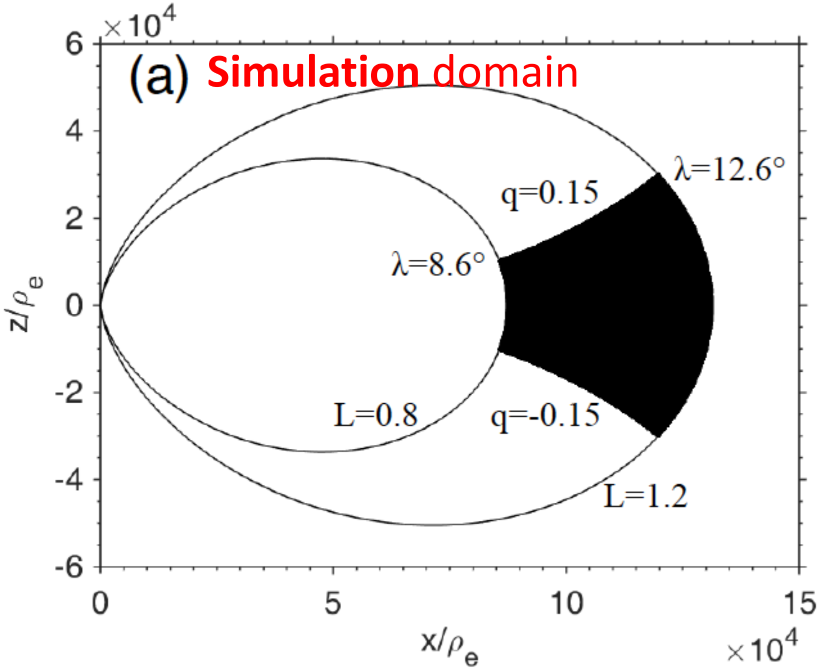
- Quasilinear diffusive theory due to bounce resonance is developed with the effect of wave power equatorial confinement, the effect of finite Larmor radius, and the effect of changing magnetic moment.

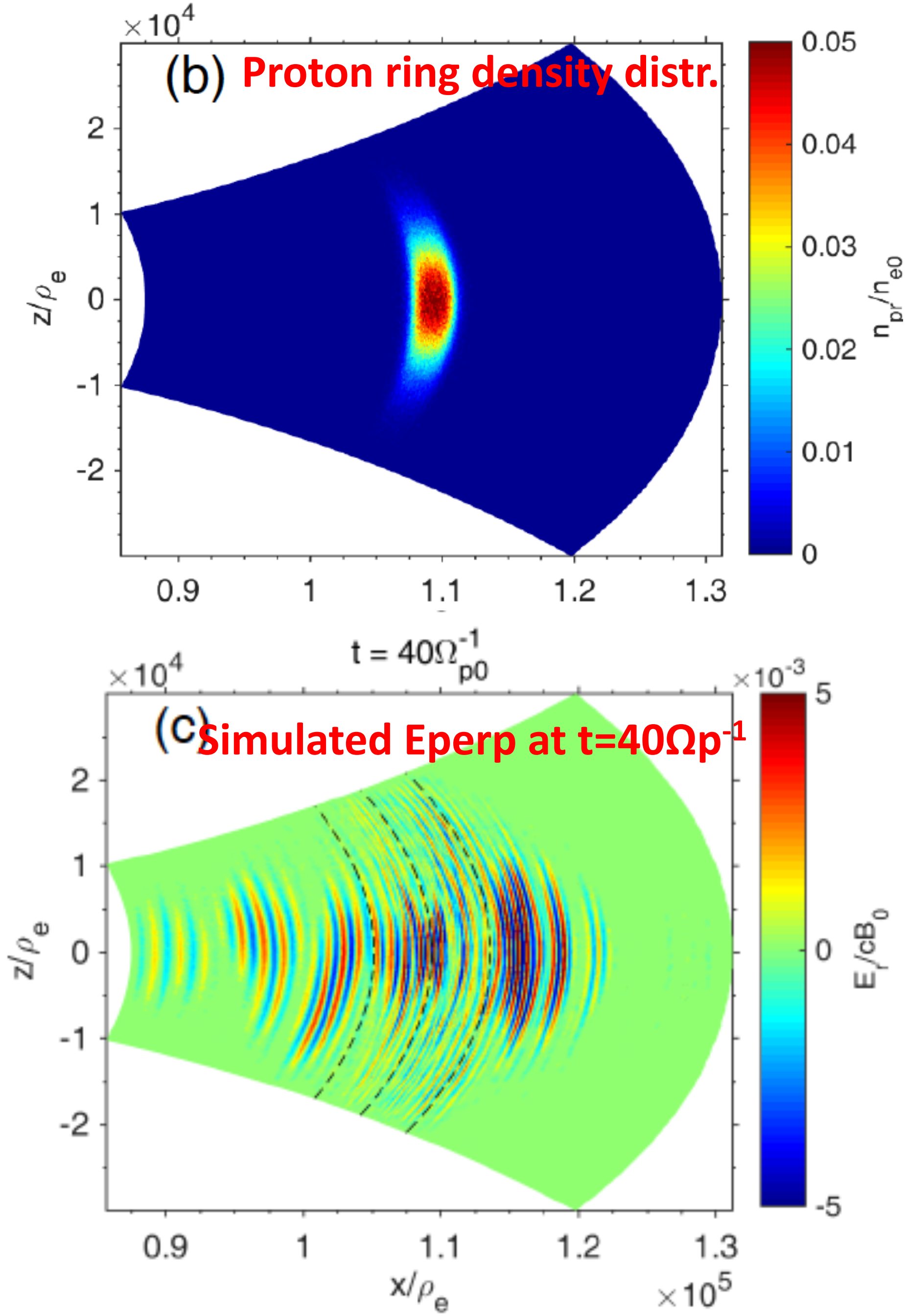
Landau resonance, nonresonance (transit time), bounce resonance when interacting with a monochromatic wave



Chen and Bortnik 2018, under review

2D PIC simulation of magnetosonic waves in a dipole magnetic field.





Waves are confined near the equator.

But it seems no phase variation along the field line. Concept of "kz"? Landau resonance may not exist?

Chen et al., 2018 under review