Kinetic Equilibrium of Dipolarization Fronts

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Generic Configuration: Plasma With Spatial Gradient

Distribution function

\[ f_0(X_g, H(x)) = \frac{N}{(\gamma_i^2)^2} Q(X_g) e^{\frac{H(x)}{kT}}, \]

Constants of motion: \( H = \frac{mv^2}{2} + e(x), \quad X_g = x + \frac{vy}{W} \)

Guiding center distribution

\[ Q(X_g) = \begin{cases} R & , X_g < X_{g1} \\ R + (S) & , X_{g1} < X_g < X_{g2} \\ S & , X_g > X_{g2} \end{cases} \]

Density: \( < f_0 > f_0(v, (x)) dv = n_0( (x)) \)

Quasi-neutrality determines the electrostatic potential

\[ n_0( (x)) = 0 \]

With \( \Psi(x) \) determined, the distribution function is fully specified

[Romero et al., GRL, 1990]
Dipolarization Fronts are Pressure Gradient Layers

In the aftermath of reconnection plasma is dragged by the magnetic field lines to create a pressure gradient layer with scale size $\sim \rho_i$ or less.

Steady state can be modeled by
- $X_{g1} = -1.0 \, \rho_i, \, X_{g2} = -1.5 \, \rho_i$: Ions
- $X_{g1} = -0.5 \, \rho_i, \, X_{g2} = -1.2 \, \rho_i$: Electrons
- $R = 1.0, \, S = 0.75$ for both ions and electrons
- $\rho_i$ evaluated in the Earthward region outside DF

Global Compression Surrogate:
- $R, \, S$, determine magnitude
- $\Delta X_g$ the pressure gradient scale size

Obtained from observation or global model

(a, b) Low-frequency magnetic field data (128 Samples/s) in GSE coordinates.
(b) Electron and proton density.
(d) Electron temperature parallel and perpendicular to the background magnetic field.
(e) Parallel and perpendicular proton temperatures
(f) Proton velocity components, in GSE coordinates.
(g) Electric field wave power spectral density.

The temperature ratio $T_e/T_i \sim 0.16$ Earthward of DF

\[ R \, S, \text{ } \rho_i \text{ evaluated in the Earthward region outside DF} \]
Generation of Ambipolar Potential and Density

Separation of electron and ion scales evident
Not possible in a MHD/Fluid model

Ambipolar Electrostatic Potential

Self-Consistent Density

Electron scale variations

$\text{Ambipolar Electrostatic Potential}$

$\text{Self-Consistent Density}$
Formation of Distinct Electron and Ion Layers

Electric field magnitude and gradient is stronger in electron layer
Kinetic property unlike in MHD/Fluid models

Perpendicular Electric Field

\[ E(x) = \frac{d}{dx} \]

Self-Consistent Flows

\[ U_\alpha = \frac{\langle \vec{v}_{f0\alpha} \rangle}{\langle f_{0\alpha} \rangle} \]

Ambipolar electric field saturates for \( L \lesssim \rho_i \)

In MHD/Fluid models
\( E \propto (\nabla P_i)/n \) blows up for \( L \to 0 \)

Separate flow layers seen in ISEE data [Parks et al. JGR, 1979]
Electric Field Gradient Modifies Particle Orbits

**Kinetic property unlike in MHD/Fluid models**

Velocity shear becomes an important factor

$$\ddot{v}_x = -\eta(x)\Omega^2 v_x + O(\varepsilon^2) \quad \eta(x) = 1 + \frac{1}{\Omega} \frac{dV_E(x)}{dx}$$

- for $\eta > 0$ orbit oscillatory but for $\eta < 0$ exponential

Affects zeroth-order plasma dynamics

- Particles can move across magnetic field lines
- Unique plasma distribution is created with temperature anisotropy in x and y directions

$$f_0(\xi, H) \approx \frac{n_0}{\sqrt{\eta(\xi)}} \left( \frac{\beta}{2\pi} \right)^{3/2} e^{-\frac{\beta}{2}(\varepsilon_x^2 + \eta(\xi)(v_y - \langle v_y \rangle)^2)} e^{-\frac{\beta}{2}v_y^2}$$

- where $\eta > 0$ and $L > \rho_e$

$$v_x = x + (v_y - V_E(x))$$

$$V_E(x) = -\frac{cE(x)}{B_0}, \quad 1/v_t^2$$

[Note: $V_E(x)$ is the potential electric field modified according to $cE(x)/B_0$.]

The ions experience strong velocity shear while electrons experience weak shear.
Non-gyrotropy and temperature anisotropy small for electrons but large for ions because velocity shear $dV_E/dx > \Omega_i$ but $< \Omega_e$
Self-Consistent Currents and Magnetic Flux Pileup

Unequal electron and ion flows lead to net perpendicular current and consequent magnetic flux pileup

Electron scale variations
Parallel Electric field and Pressure Anisotropy

The x-z component of the magnetic field rotates along the DF field lines

The $\theta$ potential $\Psi(B(s))$ changes along the field lines generating a parallel electric field

$$E_{\parallel}(s) \quad (B(s)) / s = (x / L_{\parallel}) E_x(x) \quad L_{\parallel} \sim (\ln B(s) / s)^{1}$$

Existence of parallel electric field implies non-zero off-diagonal terms of the pressure tensor for parallel force balance;

$$en(x)E_{\parallel}(x) = -(\nabla \cdot \vec{P}(x)) \cdot \hat{s} = -(\partial_x p_{xx} \hat{b}_x + \partial_x p_{xz} \hat{b}_z)$$

The parallel electric can accelerate the non-thermal particles to form parallel beams
Hierarchy of Velocity Shear Driven Waves Possible

Velocity shear, generated by global compression, is the natural source of free energy in DF

* Experiments in
- Iowa U
- WVU
- NRL
- Auburn U
- Japan
- S. Africa
- India
Velocity Gradient (not the Density Gradient) is the Source for Lower Hybrid Waves in Dipolarization Fronts

- Dispersion relation for waves with $k_{||} \sim 0$ and both density and velocity gradient is,

$$\left\{ \frac{d^2}{dx^2} - k_y^2 + G(\omega) \frac{k_y (d^2 V_E / dx^2 - \Omega_e / L_n)}{\omega - k_y V_E (x)} \right\} \varphi(x) = 0,$$

$$\frac{1}{L_n} \frac{1}{n} \frac{dn}{dx}, \quad G(\omega) = \left( \frac{2}{2 \rho_e + 2/e} \right) \left( \frac{2}{2 LH} \right), \quad 2_{LH} = \frac{2 \rho_i}{2 e + 2 \rho_i}$$

- For $V_E \to 0$ reduces to the equation for the Lower Hybrid Drift Instability (LHDI) [Mikhailvoski and Tsypin, JETP, 1963; Krall and Liewer, Phys Rev A, 1971]
- For $L_n \to \infty$ reduces to the Electron Ion Hybrid (EIH) Instability [Ganguli et al., Phys Fluids, 1988]
Conclusions

**Kinetic equilibrium model for a dipolarization front shows**
- A strong ambi-polar electric field across the magnetic field as a result of global compression
- Spatial variation in both electron and ion scales
  - Ion and electron orbits are affected differently resulting in unique particle distribution
- Spatial gradient in the electric field that causes anisotropy and non-gyrotropy in the distribution function
- MHD/fluid descriptions become inadequate for $L \lesssim 2\rho_i$

**Analytical model validated in a 1D PIC simulation**

**Magnetic field curvature leads to parallel electric field**
- Parallel electric field can accelerate non thermal particles to form parallel beams
  - Provides a non-reconnection basis for their existence
- Existence of parallel electric field implies anisotropic pressure tensor

**Plasma compression due to dipolarization of field lines generates velocity shear**
- Velocity shear generates a hierarchy of instabilities in a broad frequency and wave vector band
- In the collisionless plasma environment these waves lead to relaxation of stress in a DF
PIC Simulation of Broadband Noise in Compressed Layers

21/2 D PIC Simulation

Shear stronger in the electron layer where E X B drift is larger than the diamagnetic drift

Signatures consistent with the EIH instability

Contours of electrostatic potential

Broadband emission

Relaxation of initial flow

- $K_yL \sim O(1)$
- $L$ becomes larger with time
- This can successively trigger lower frequency waves

$L < \rho_i \Rightarrow \sim LH > \rho_i$

$L \sim \rho_i \Rightarrow \sim \rho_i$

$L > \rho_i \Rightarrow < \rho_i$

[Romero and Ganguli, Phys. Fluids, 1993; Romero and Ganguli, GRL, 1994]
1D PIC Simulation Validates the Analytical Model
Non-gyrotropy and temperature anisotropy small for electrons but large for ions because velocity shear \( \frac{dV_E}{dx} > V_i \) but \( \frac{dV_E}{dx} < V_e \).
Transverse Velocity Shear Can Drive Broadband Waves

**Weak Shear** \( L > \gamma \), \( dV_E/dx < \gamma \)

**Kelvin-Helmholtz** \( k_y V_E \ll \gamma \), \( k_y \gamma < 1 \)

\[
\left( \frac{d^2}{dx^2} + \frac{k_y V_E}{k_y V_E} \right) \phi(x) = 0
\]

\[
\frac{\partial}{\partial t} \int dx \left( \frac{1}{8\pi} \left( n_0 m_1 c E_1^2 \right)^2 + \frac{n_0 m_1}{2} I_x V_E(x) V_E'(x) \right) = 0
\]

\[
V_E^2(x) - \left\{ V_E(x + \gamma) \right\}^2 = -|x| V_E(x) V_E'(x) + O(1/L')
\]

**IEDDI** \( k_y V_E \sim n \gamma \), \( k_y \gamma \gamma \geq 1 \)

\[
\gamma \propto \left( U_{III} / U_{II} \right)
\]

**Strong Shear** \( L < \gamma \), \( dV_E/dx > \gamma \)

**Electron-Ion Hybrid (EIH)** \( k_y V_E \sim n \gamma \), \( k_y \gamma > 1 \)

\[
\left( 1 + \frac{2}{\gamma} \right) \frac{\partial}{\partial t} \int dx \left( \frac{1}{8\pi} \left( n_0 m_1 c E_1^2 \right)^2 + \frac{n_0 m_1}{2} I_x V_E(x) V_E'(x) \right) = 0
\]

\[
V_E^2(x) - \left\{ V_E(x + \gamma) \right\}^2 = -|x| V_E(x) V_E'(x) + O(1/L')
\]

**Electron IEDDI** \( k_y V_E \sim n \gamma \), \( k_y \gamma \gamma \sim 1 \)

\[
\gamma \propto \left( U_{III} / U_{II} \right)
\]