Kinetic Equilibrium of Dipolarization Fronts*

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Generic Configuration: Plasma With Spatial Gradient

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Distribution function

$$f_{oa}(X_g, H_a(x)) = \frac{N_a}{(\pi v_{ta}^2)^2} Q_a(X_g) e^{-\frac{H_a(x)}{kT_a}},$$

Constants of motion: $H_a = mv^2/2 + e\Psi(x), \quad X_g = x + v_y/\Omega$

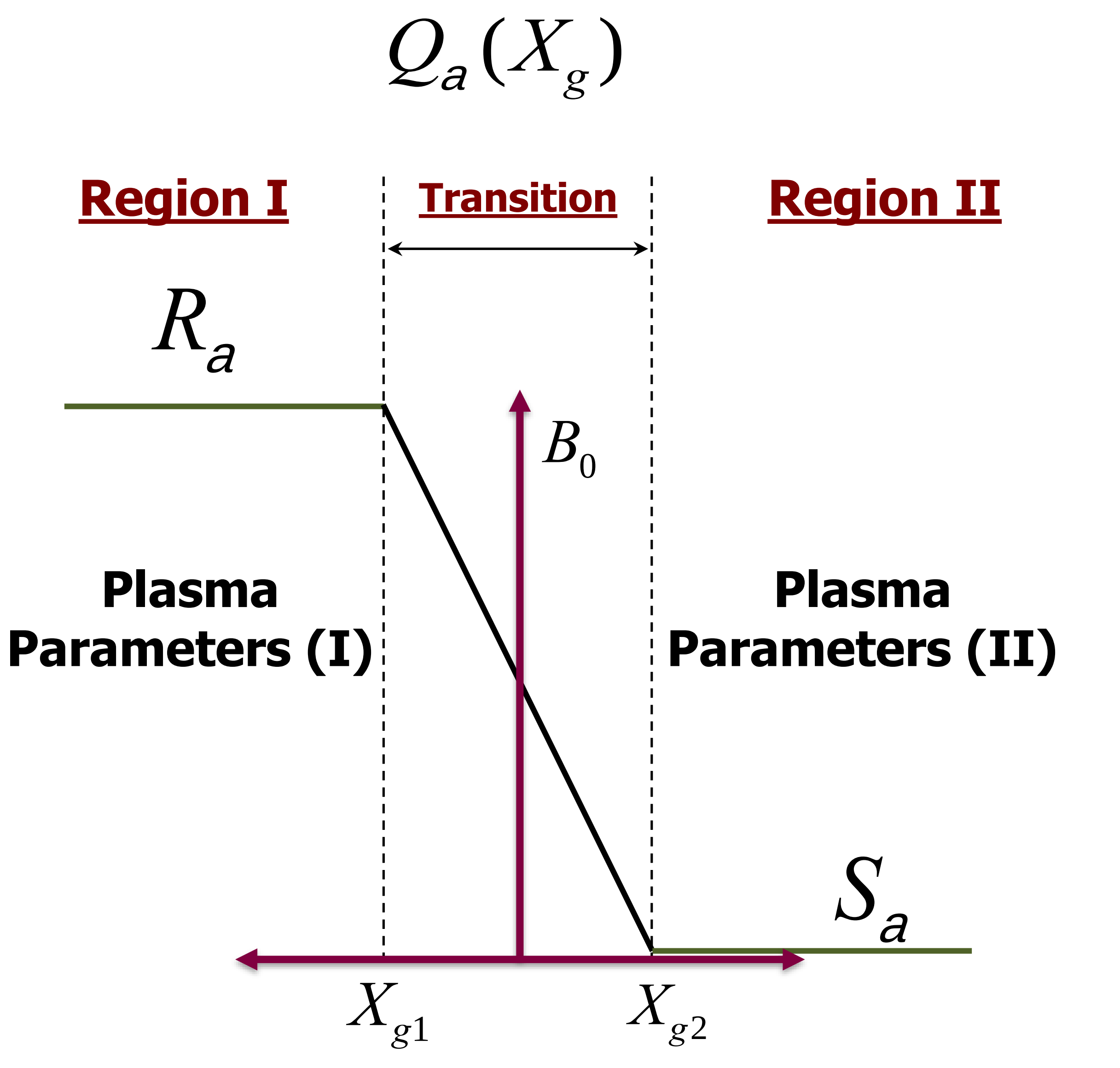
Guiding center distribution

$$Q_a(X_g) = \begin{cases} R_a & , X_g < X_{g1} \\ R_a + (S_a - R_a)\left(\dfrac{X_g - X_{g1}}{X_{g2} - X_{g1}}\right), & X_{g1} < X_g < X_{g2} \\ S_a & , X_g > X_{g2} \end{cases}$$

Density: $< f_{0a} > \equiv \int f_{0a}(v, \Psi(x))dv = n_{0a}(\Psi(x))$

Quasi-neutrality determines the electrostatic potential

$$\sum_a n_{0a}(\Psi(x)) = 0$$



[Romero et al., GRL, 1990]

With Ψ(x) determined, the distribution function is fully specified

Dipolarization Fronts are Pressure Gradient Layers

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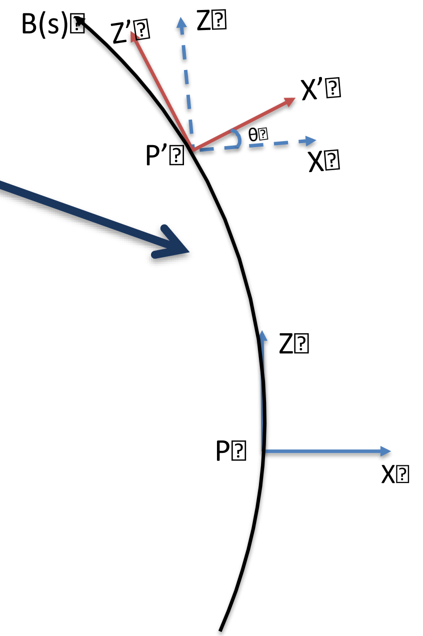
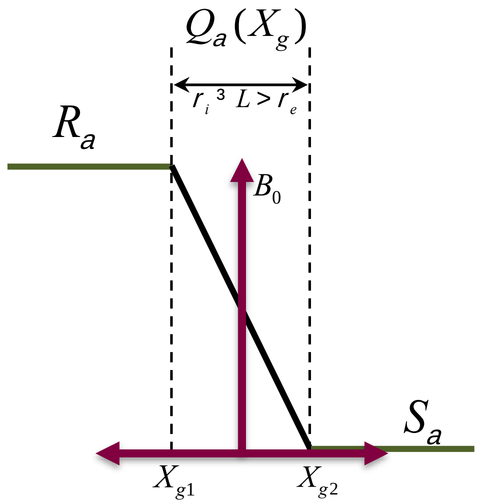
In the aftermath of reconnection plasma is dragged by the magnetic field lines to create a pressure gradient layer with scale size ~ ρ_i or less

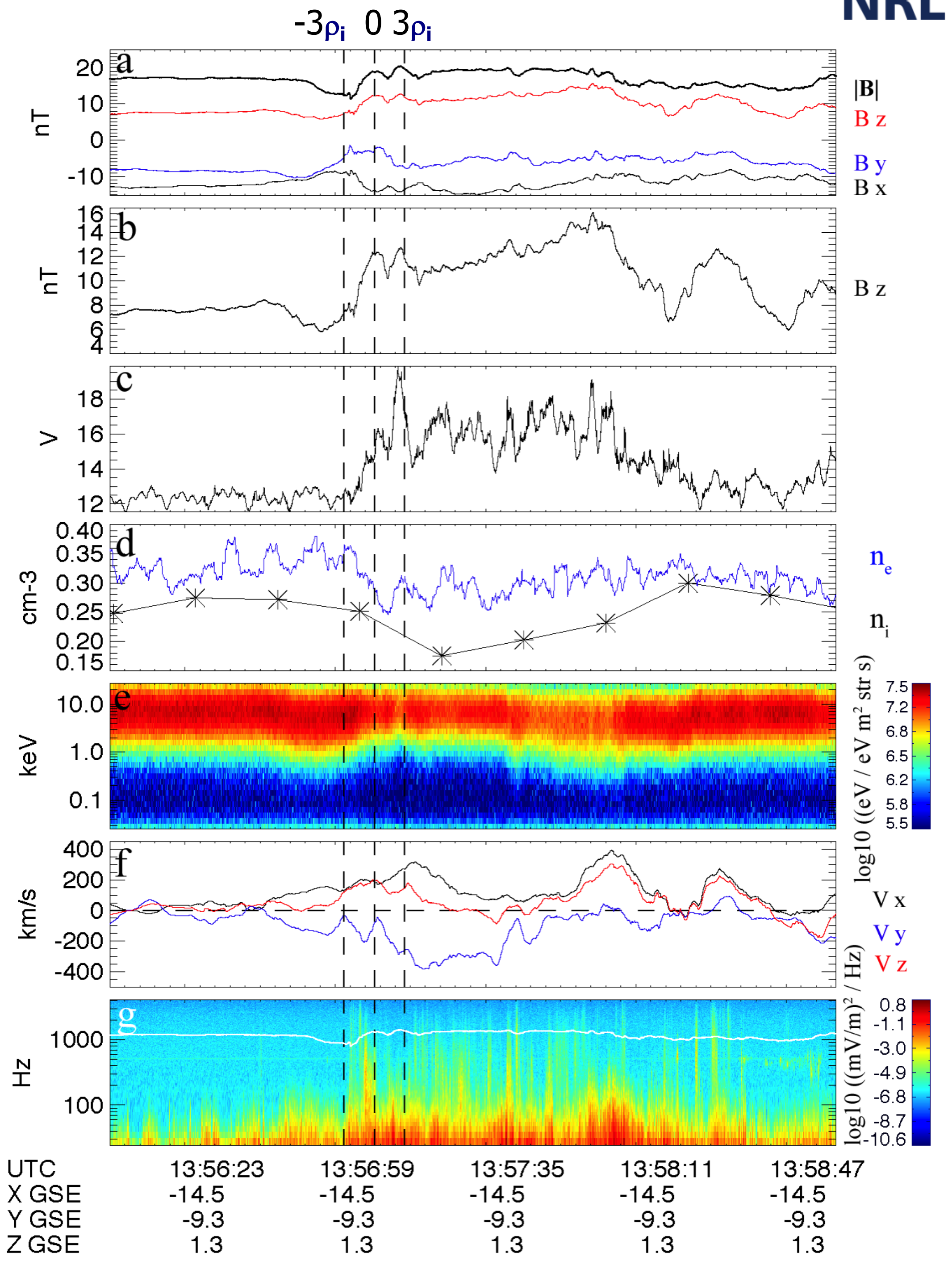
Steady state can be modeled by

- $X_{g1} = -1.0\ \rho_i$, $X_{g2} = -1{,}5\ \rho_i$: Ions
- $X_{g1} = -0.5\ \rho_i$, $X_{g2} = -1.2\ \rho_i$: Electrons
- R = 1.0, S = 0.75 for both ions and electrons
- ρ_i evaluated in the Earthward region outside DF

DF Geometry







(a, b) Low-frequency magnetic field data (128 Samples/s) in GSE coordinates.
(c) Electron and proton density.
(d) Electron temperature parallel and perpendicular to the background magnetic field.
(e) Parallel and perpendicular proton temperatures
(f) Proton velocity components, in GSE coordinates.
(g) Electric field wave power spectral density.

The temperature ratio $T_{e0}/T_{i0} \sim 0.16$ Earthward of DF

Global Compression Surrogate:

- R, S, determine magnitude
- ΔX_g the pressure gradient scale size

Obtained from observation or global model

Generation of Ambipolar Potential and Density

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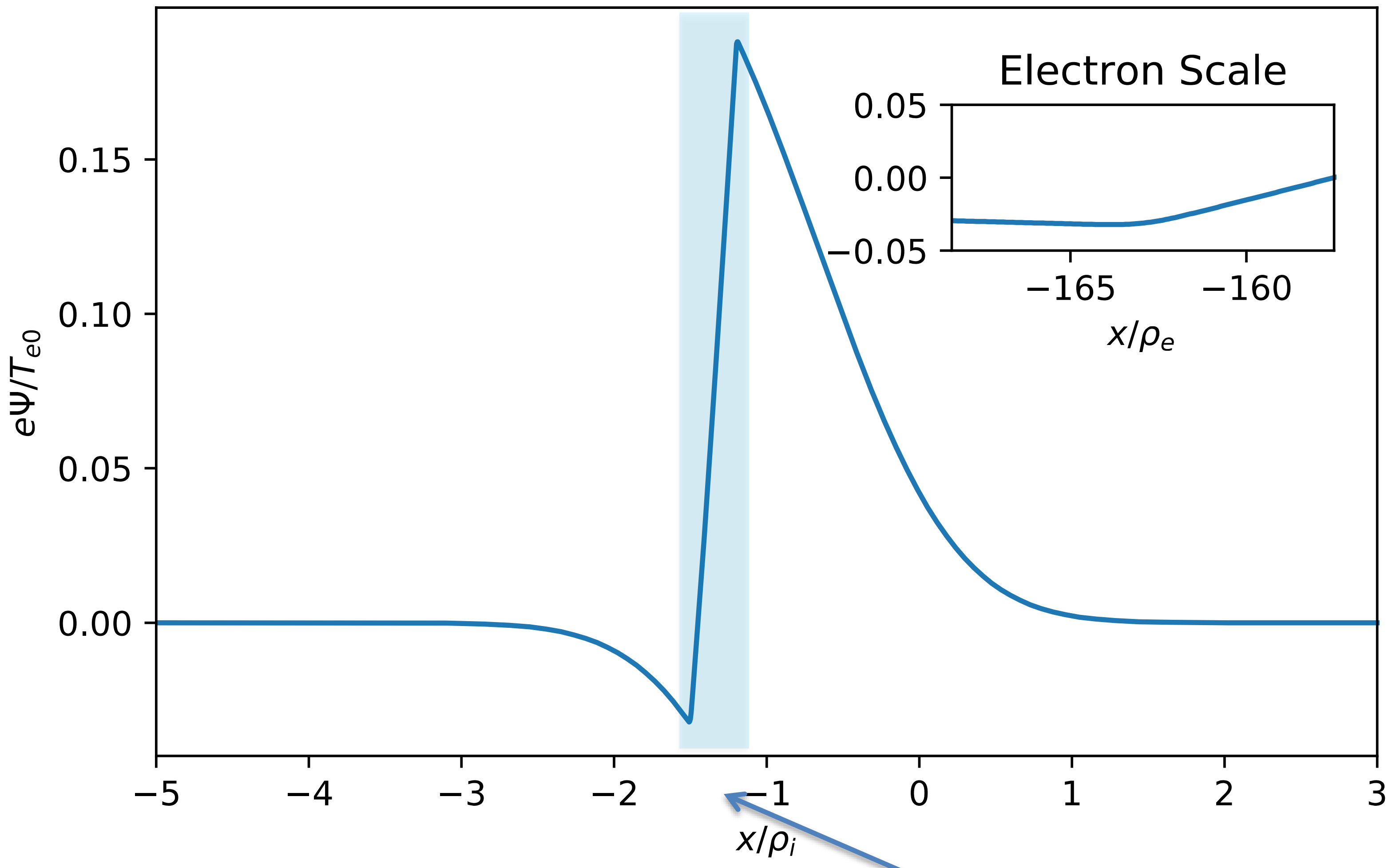
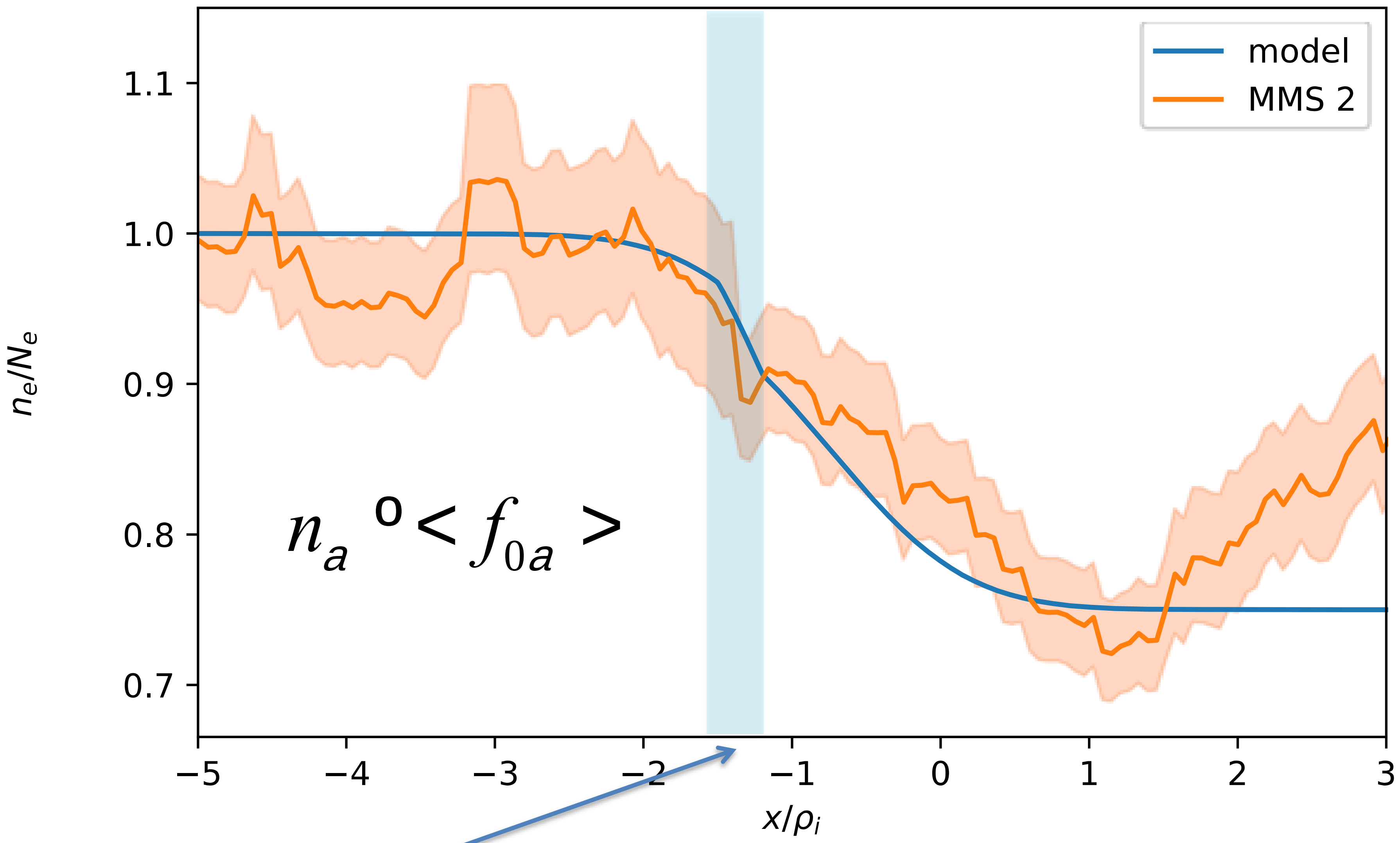
Separation of electron and ion scales evident

Not possible in a MHD/Fluid model

Ambipolar Electrostatic Potential



Self-Consistent Density



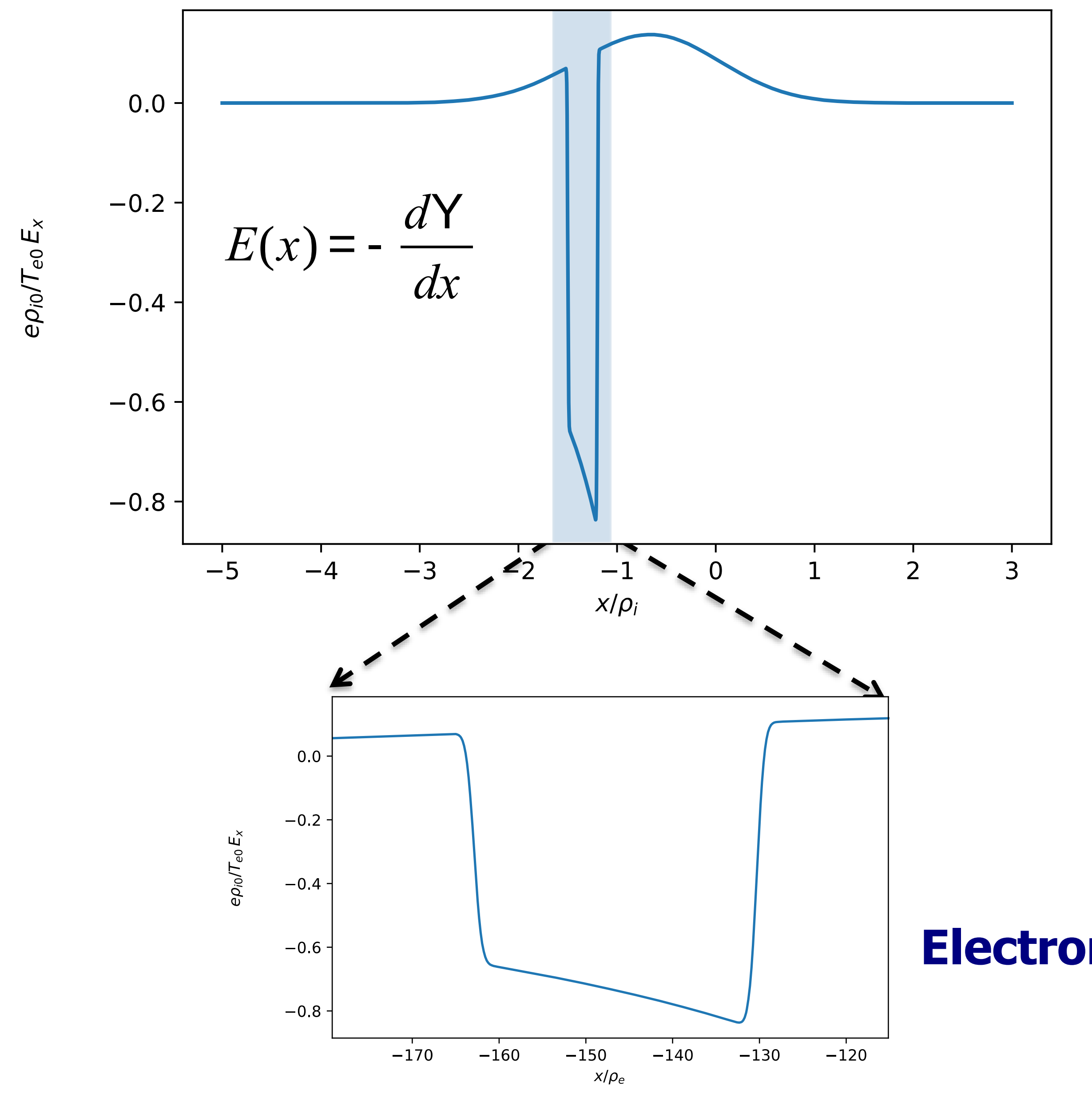
Electron scale variations

Formation of Distinct Electron and Ion Layers

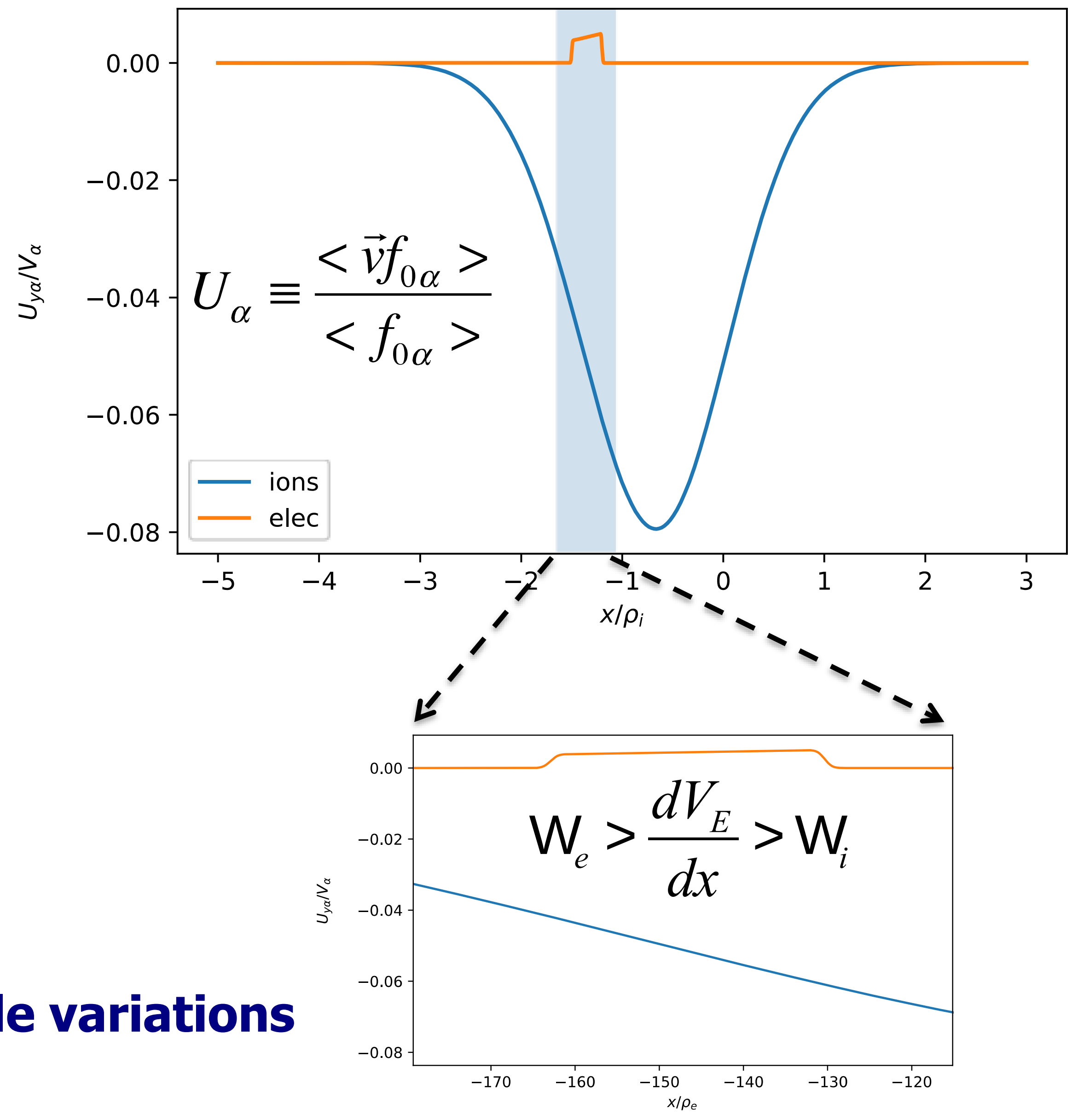
Electric field magnitude and gradient is stronger in electron layer

Kinetic property unlike in MHD/Fluid models

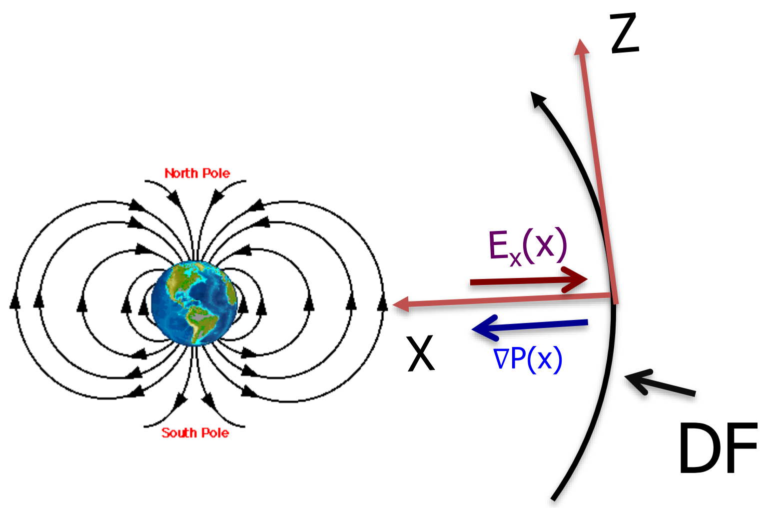
Perpendicular Electric Field



Self-Consistent Flows



Electron scale variations



Ambipolar electric field saturates for L ≲ ρ_i

In MHD/Fluid models $E \propto (\nabla P_i)/n$ blows up for L → 0

Separate flow layers seen in ISEE data [Parks et al. JGR, 1979]

Electric Field Gradient Modifies Particle Orbits

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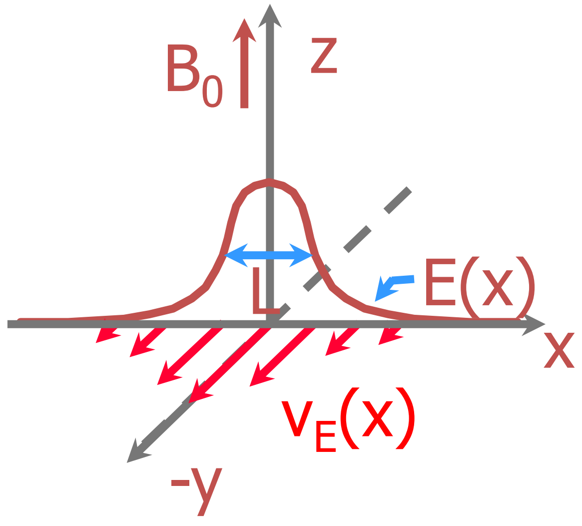
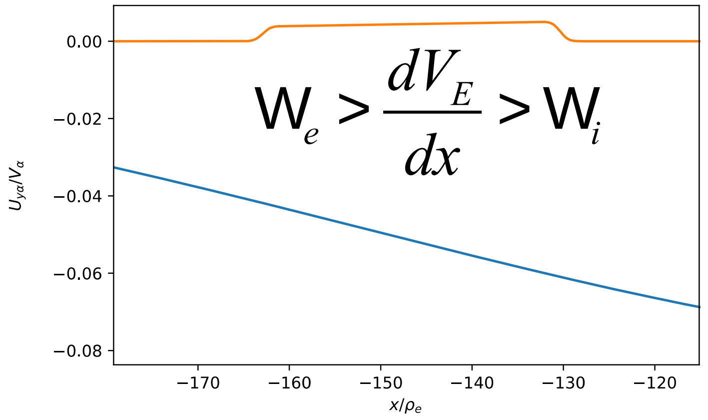
Kinetic property unlike in MHD/Fluid models

Velocity shear becomes an important factor

$$\ddot{v}_x = -\eta(x)\Omega^2 v_x + O(\varepsilon^2) \quad \eta(x) = 1 + \frac{1}{\Omega}\underbrace{\frac{dV_E(x)}{dx}}_{\text{Velocity Shear}} \qquad \Omega \to \underbrace{\sqrt{\eta(\xi)}\Omega}_{\text{Renormalized Gyro-frequency}},$$

- for $\eta > 0$ orbit oscillatory but for $\eta < 0$ exponential





Affects zeroth-order plasma dynamics

- Particles can move across magnetic field lines

$$f_0(\xi, H) \approx \frac{n_0}{\sqrt{\eta(\xi)}}\left(\frac{\beta}{2\pi}\right)^{3/2} e^{\overbrace{-(\beta/2)(v_x^2+\eta(\xi)(v_y-<v_y>)^2)}^{Tx \neq Ty}} e^{-(\beta/2)v_z^2}$$

- Unique plasma distribution is created with temperature anisotropy in x and y directions

- where $\eta > 0$ and L > ρ_e

$$X = x + (v_y - V_E(X)),$$

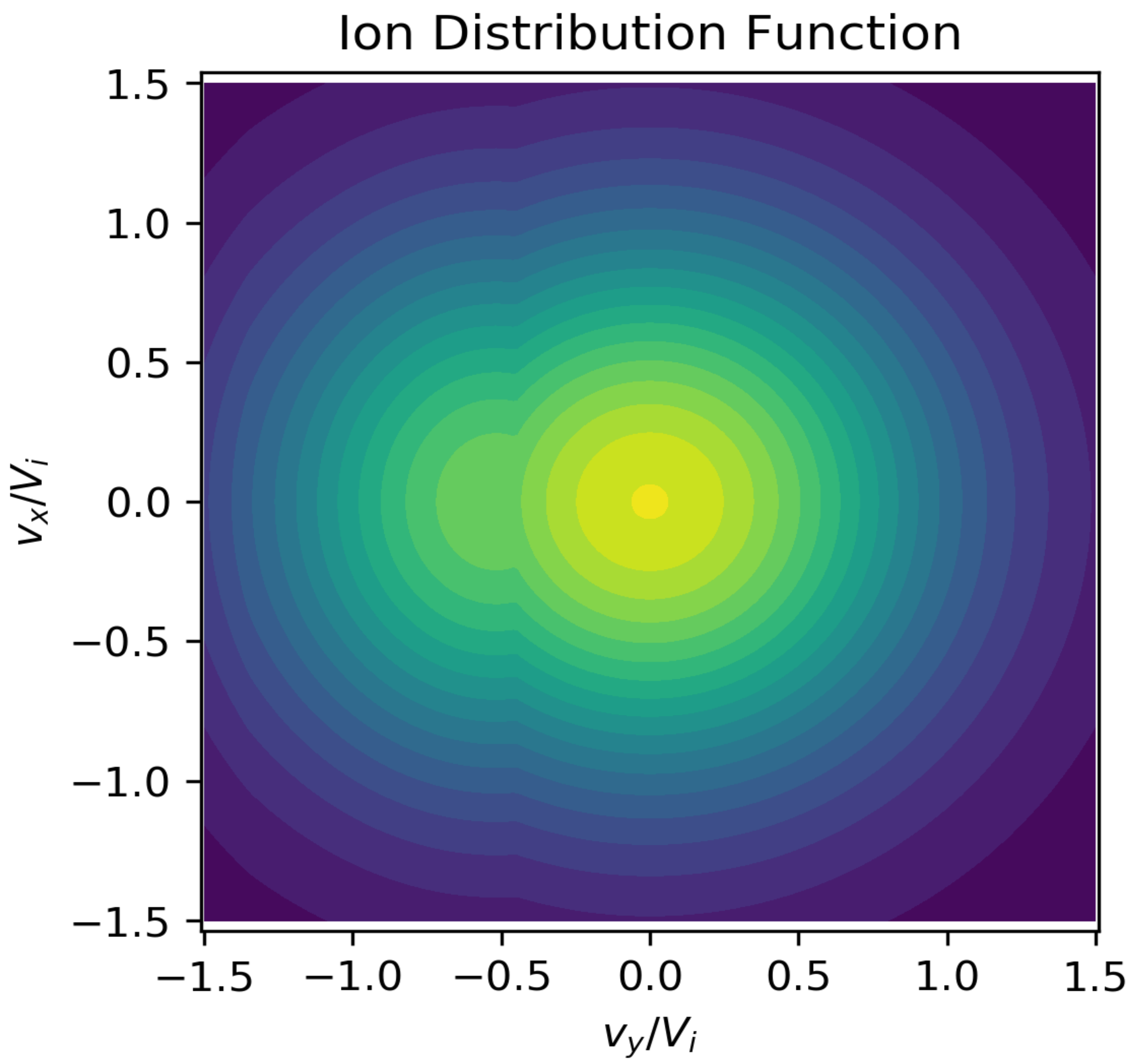
$$V_E(X) = - cE(X)/B_0, \quad b = 1/v_{th}^2$$

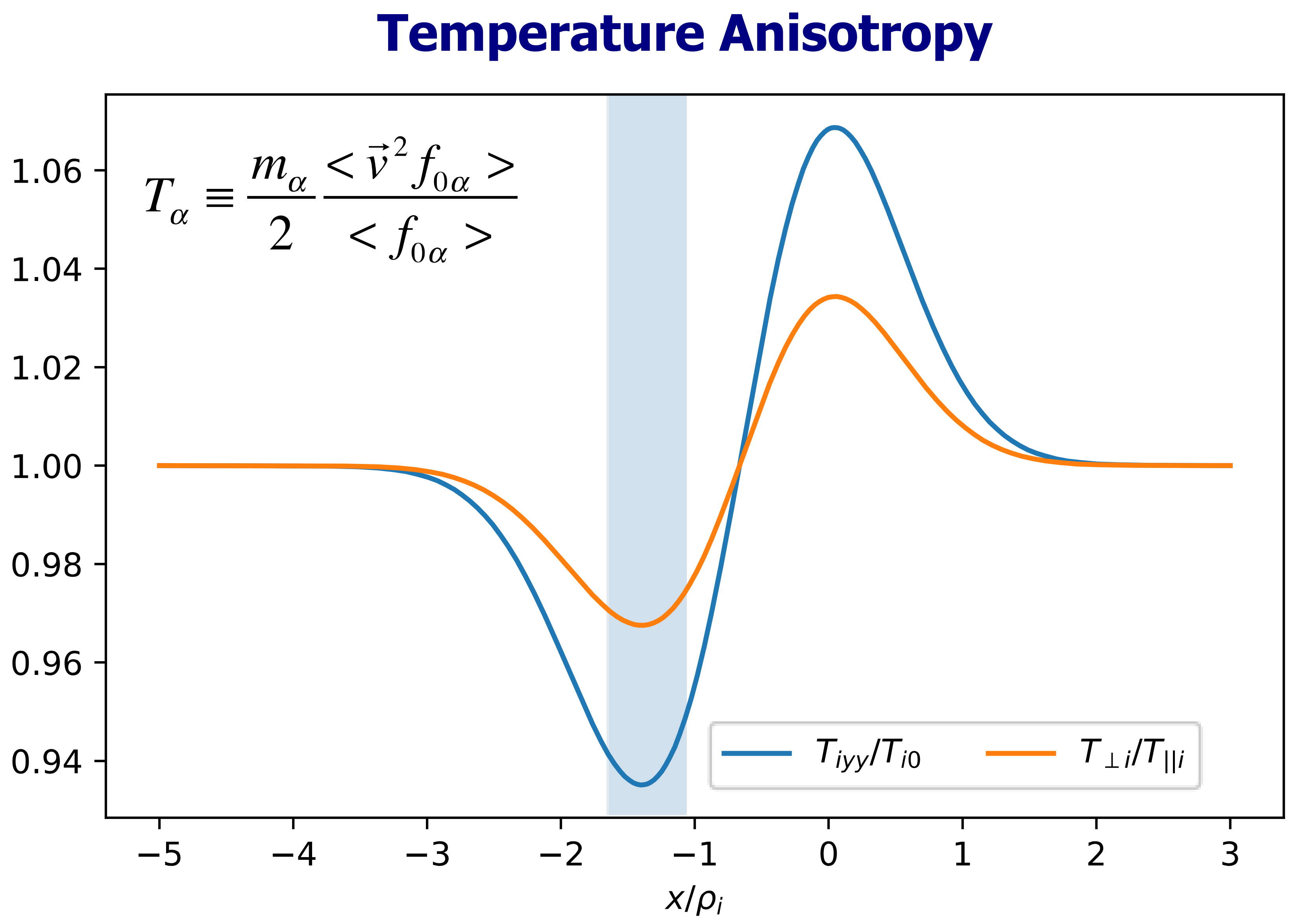
[Ganguli et al., Phys Fluids. 1988]

The ions experience strong velocity shear while electrons experience weak shear

Electric Field Gradient Breaks Gyrotropy and Introduces Temperature Anisotropy

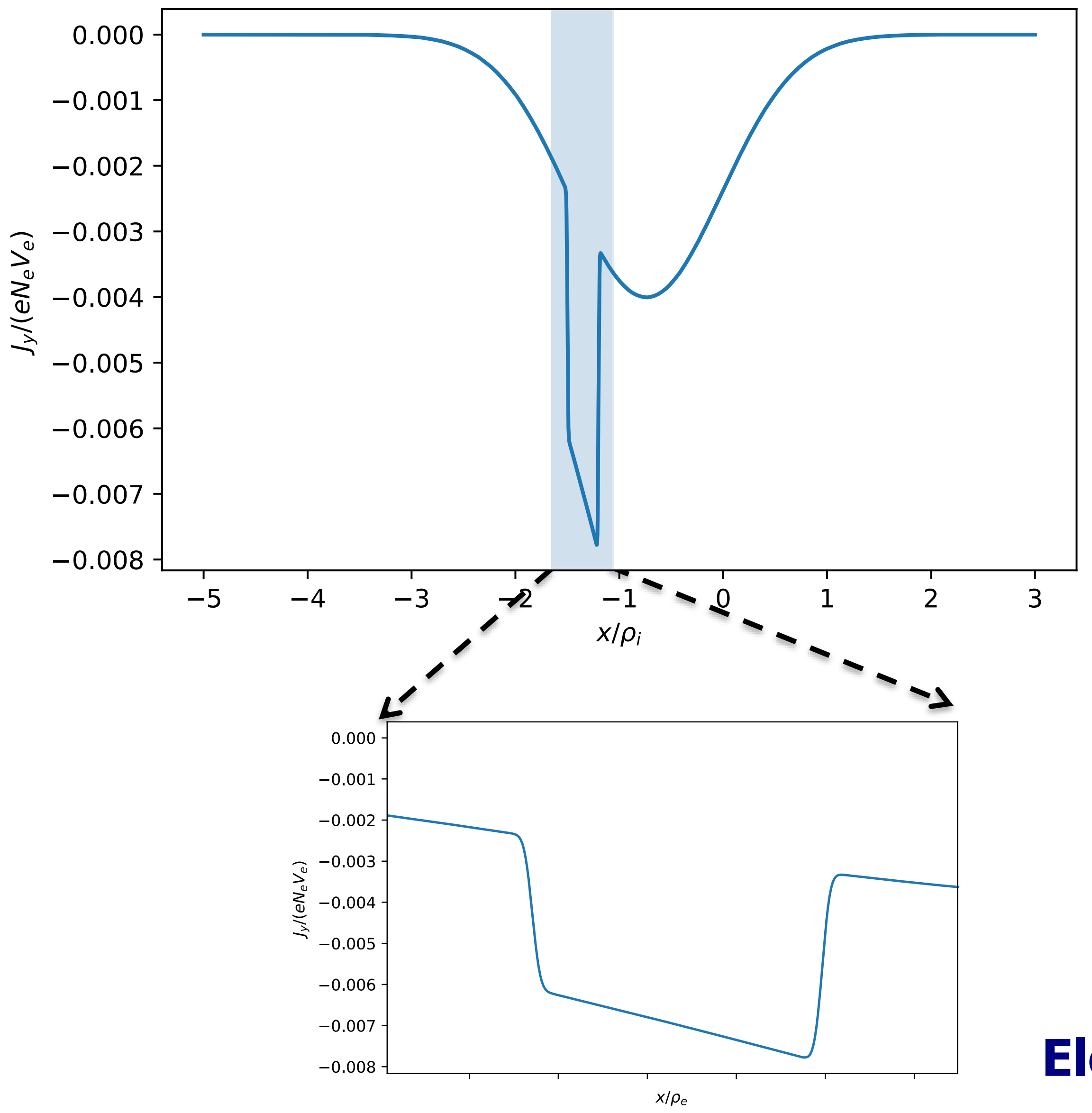
Non-gyrotropy and temperature anisotropy small for electrons but large for ions because velocity shear $dV_E/dx > \Omega_I$ but $< \Omega_e$

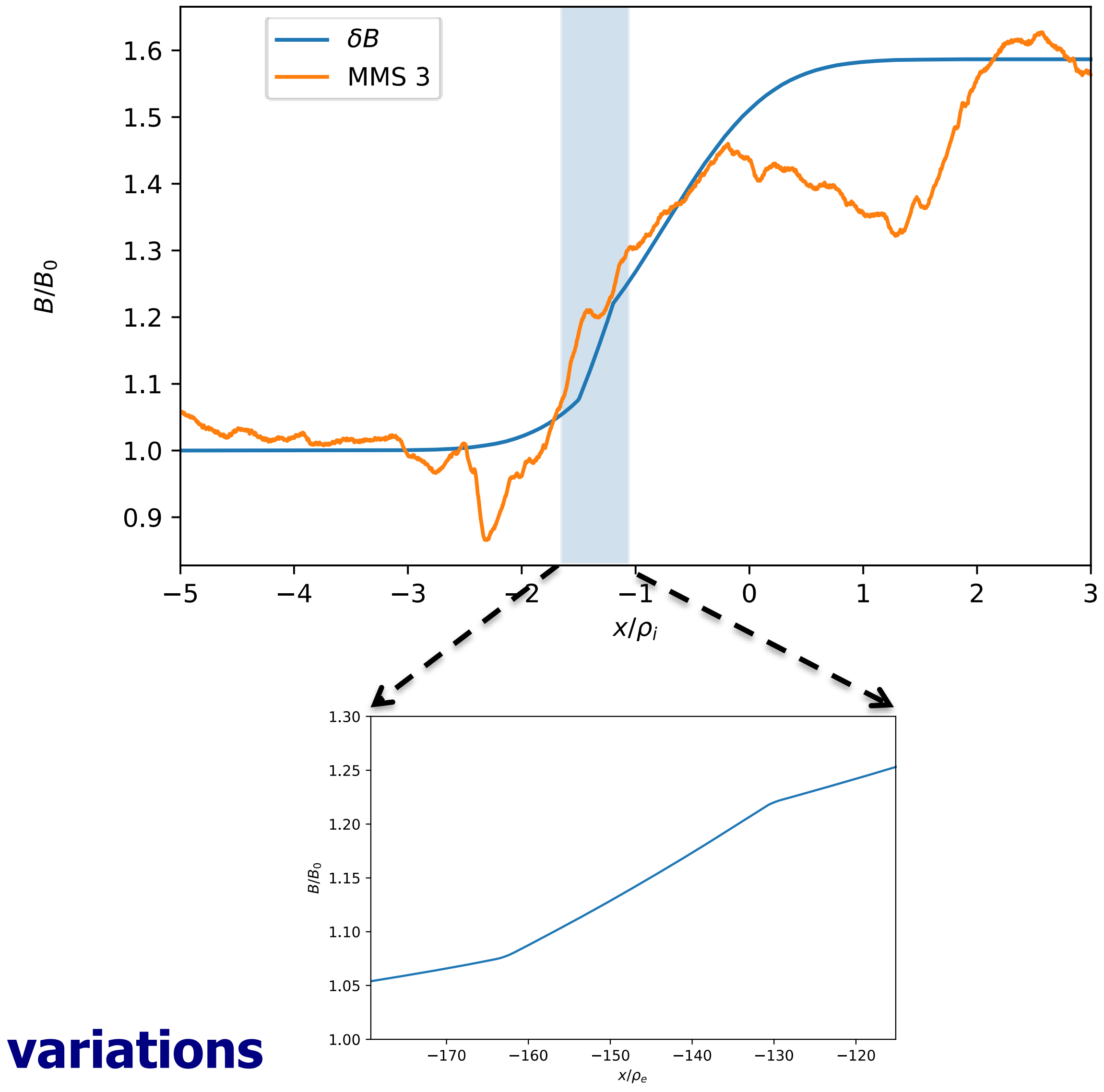




Self-Consistent Currents and Magnetic Flux Pileup

Unequal electron and Ion flows lead to net perpendicular current and consequent magnetic flux pileup





Electron scale variations

Parallel Electric field and Pressure Anisotropy

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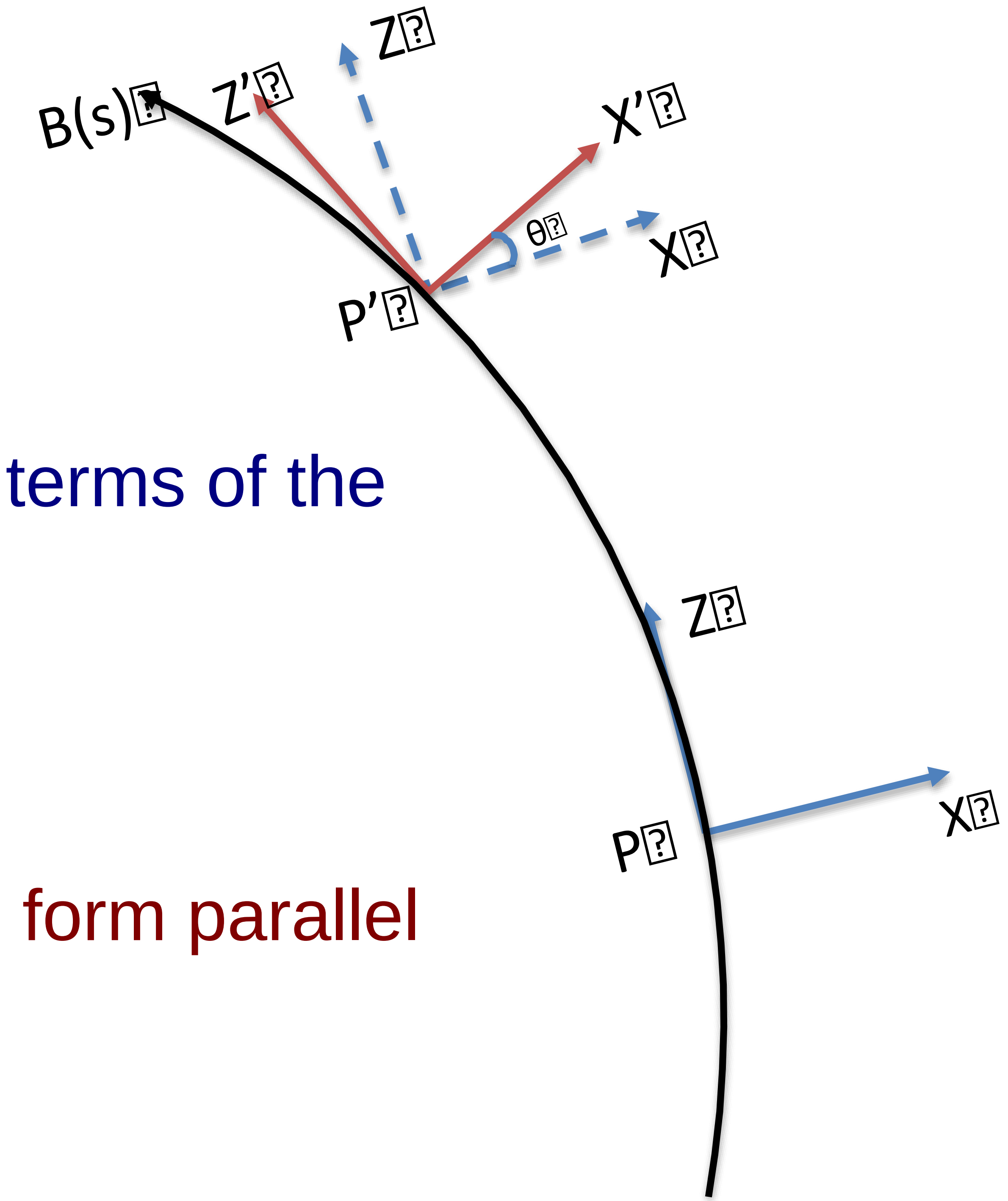
The x-z component of the magnetic field rotates along the DF field lines

The es potential Ψ(B(s)) changes along the field lines generating a parallel electric field

$$E_{\parallel}(s) \equiv -\partial \psi(B(s))/\partial s = (x/L_{\parallel})E_x(x) \qquad L_{\parallel} \sim (\partial \ln B(s)/\partial s)^{-1}$$



Existence of parallel electric field implies non-zero off-diagonal terms of the pressure tensor for parallel force balance;

$$en(x)E_{\parallel}(x) = -(\nabla \cdot \overleftrightarrow{P}(x)) \cdot \hat{s} = -(\partial_x p_{xx}\hat{b}_x + \partial_x p_{xz}\hat{b}_z)$$

The parallel electric can accelerate the non-thermal particles to form parallel beams

Hierarchy of Velocity Shear Driven Waves Possible

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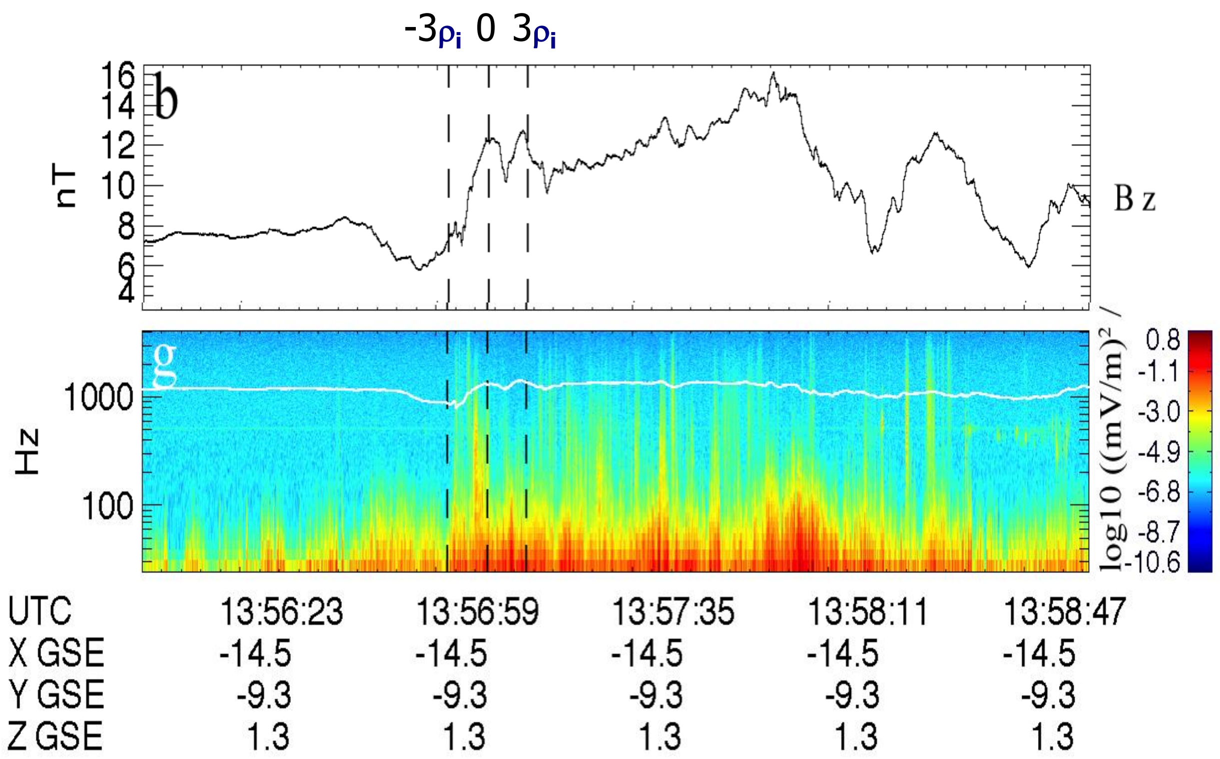
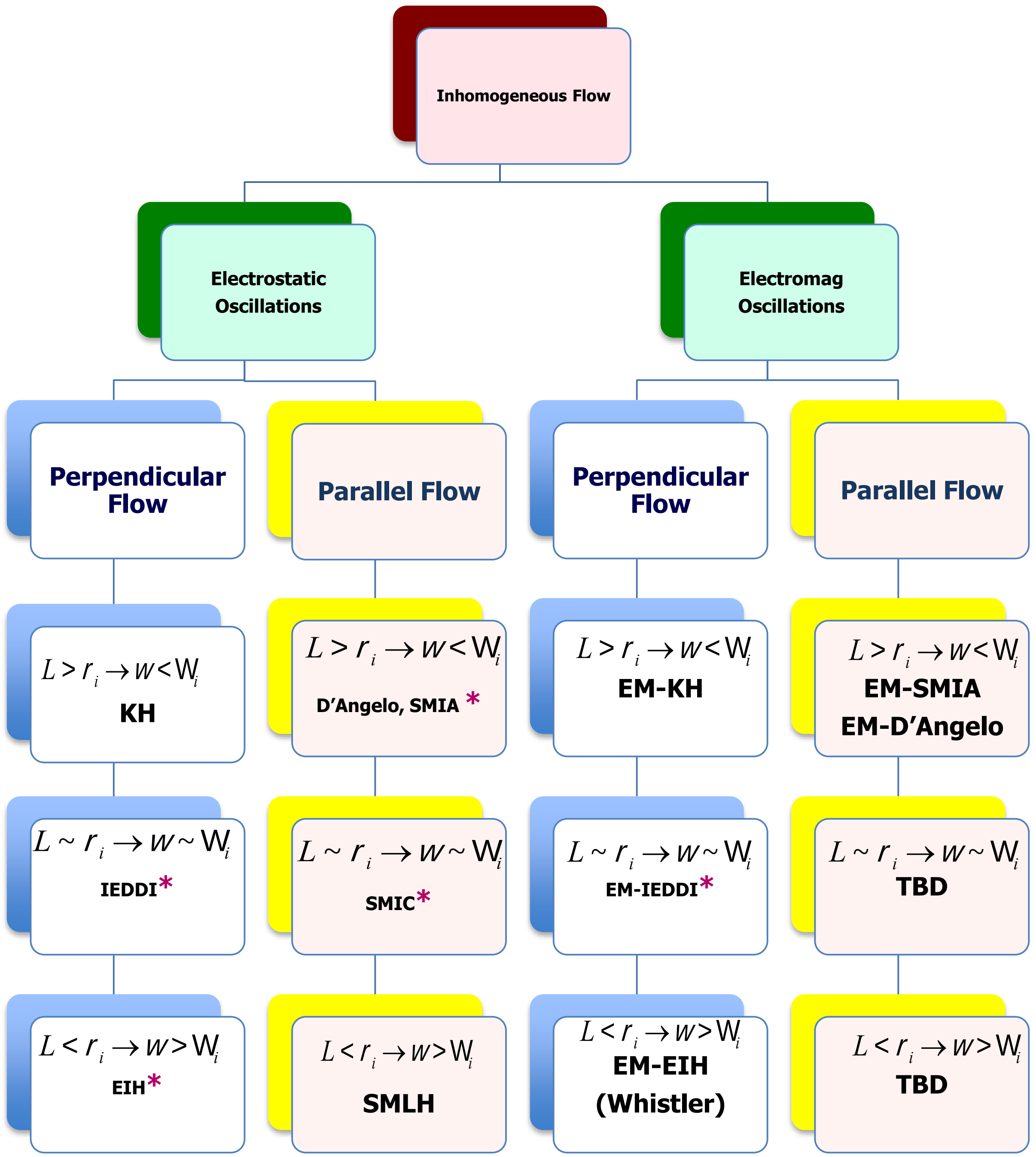
Velocity shear, generated by global compression, is the natural source of free energy in DF





*** Experiments in**

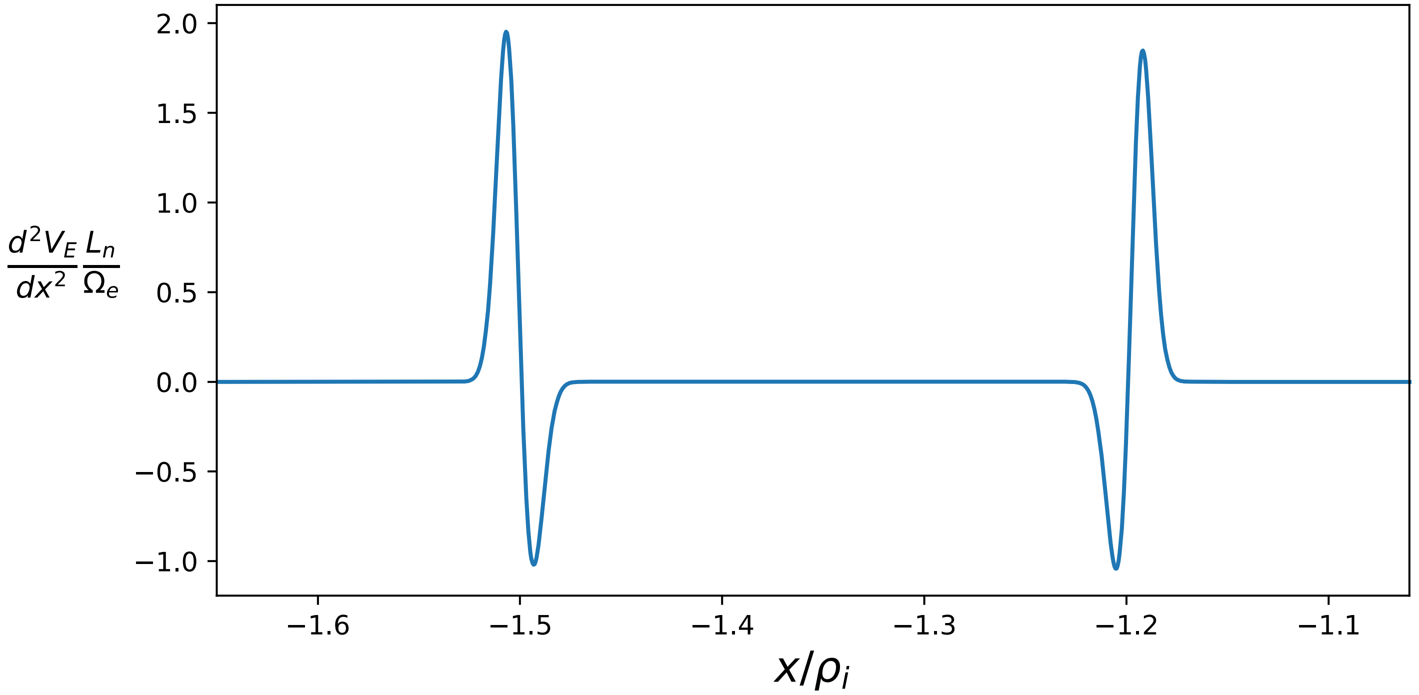
- Iowa U
- WVU
- NRL
- Auburn U
- Japan
- S. Africa
- India

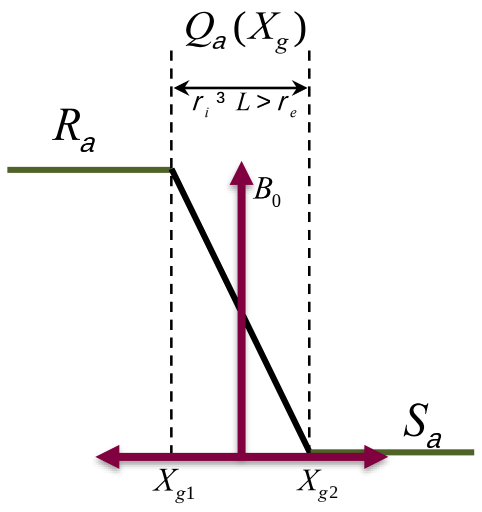
Velocity Gradient (not the Density Gradient) is the Source for Lower Hybrid Waves in Dipolarization Fronts

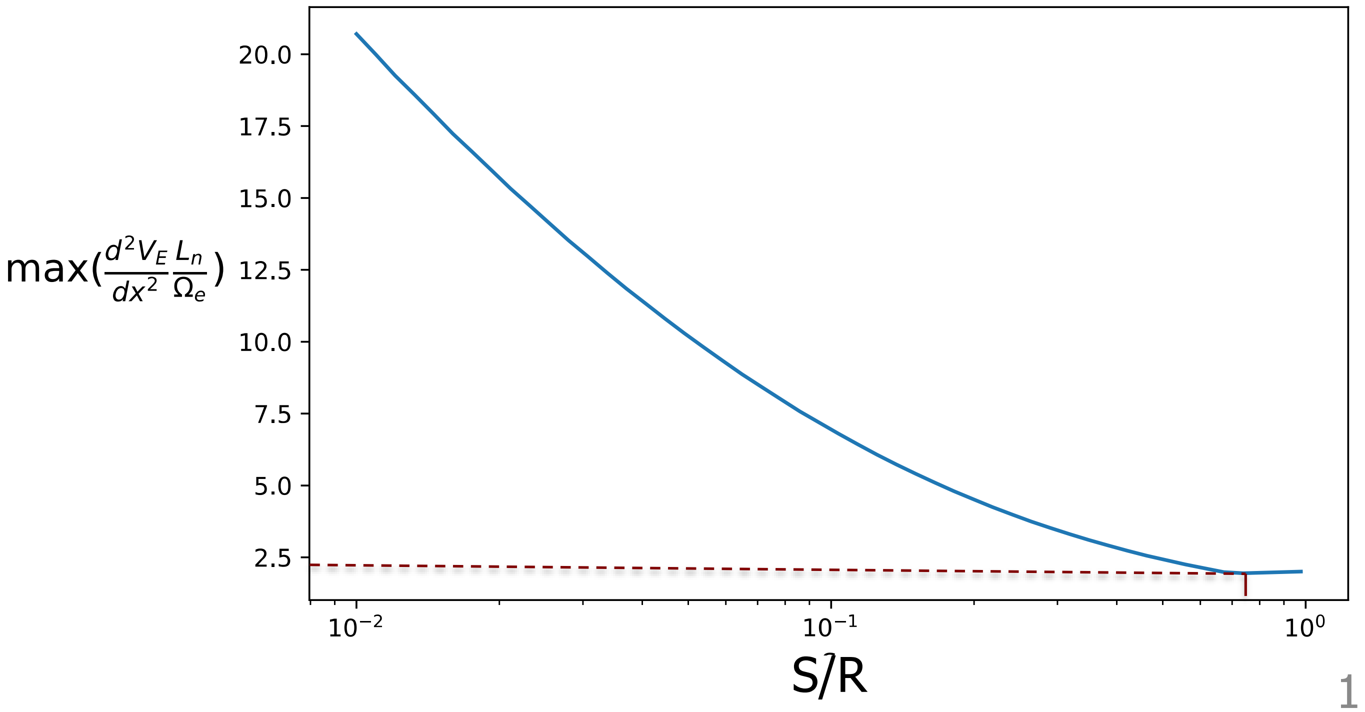
- Dispersion relation for waves with $k_{||} \sim 0$ and both density and velocity gradient is,

$$\left\{\frac{d^2}{dx^2} - k_y^2 + G(\omega)\frac{k_y(\overbrace{d^2V_E/dx^2}^{EIH} - \overbrace{\Omega_e/L_n}^{LHDI})}{\omega - k_yV_E(x)}\right\}\varphi(x) = 0, \qquad \frac{1}{L_n} \equiv \frac{1}{n}\frac{dn}{dx}, \qquad G(\omega) = \left(\frac{\omega_{pe}^2}{\omega_{pe}^2 + \Omega_e^2}\right)\left(\frac{\omega^2}{\omega^2 - \omega_{LH}^2}\right), \qquad \omega_{LH}^2 = \frac{\omega_{pi}^2\Omega_e^2}{\Omega_e^2 + \omega_{pe}^2}$$

  - For $V_E \to 0$ reduces to the equation for the Lower Hybrid Drift Instability (LHDI) [Mikhailvoski and Tsypin, *JETP*, 1963; Krall and Liewer, *Phys Rev A*, 1971]
  - For $L_n \to \infty$ reduces to the Electron Ion Hybrid (EIH) Instability [Ganguli et al., *Phys Fluids*, 1988]







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Conclusions

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Kinetic equilibrium model for a dipolarization front shows

- A strong ambi-polar electric field across the magnetic field as a result of global compression
- Spatial variation in both electron and ion scales
  - Ion and electron orbits are affected differently resulting in unique particle distribution
- Spatial gradient in the electric field that causes anisotropy and non-gyrotropy in the distribution function
- MHD/fluid descriptions become inadequate for $L \lesssim 2\rho_i$

Analytical model validated in a 1D PIC simulation

Magnetic field curvature leads to parallel electric field

- Parallel electric field can accelerate non thermal particles to form parallel beams
  - Provides a non-reconnection basis for their existence
- Existence of parallel electric field implies anisotropic pressure tensor

Plasma compression due to dipolarization of field lines generates velocity shear

- Velocity shear generates a hierarchy of instabilities in a broad frequency and wave vector band
- In the collisionless plasma environment these waves lead to relaxation of stress in a DF

PIC Simulation of Broadband Noise in Compressed Layers

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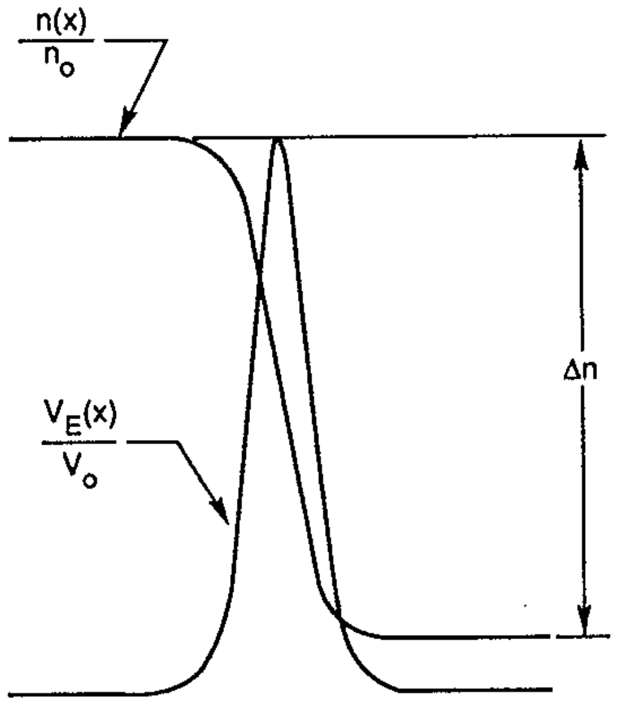
21/2 D PIC Simulation

Shear stronger in the electron layer where E X B drift is larger than the diamagnetic drift

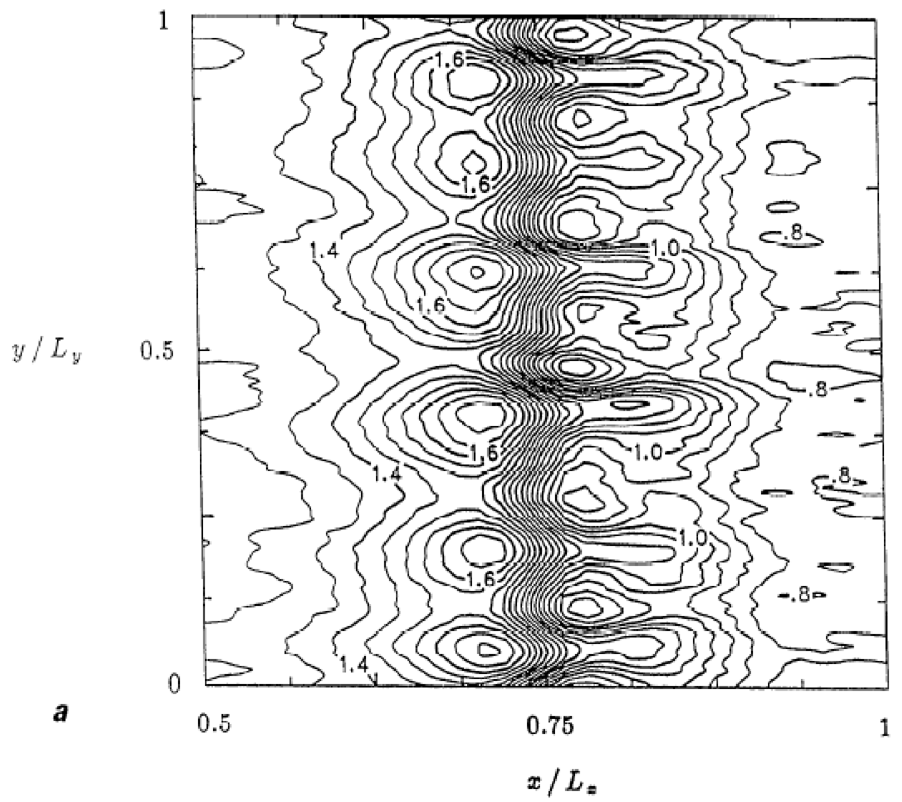
Signatures consistent with the EIH instability



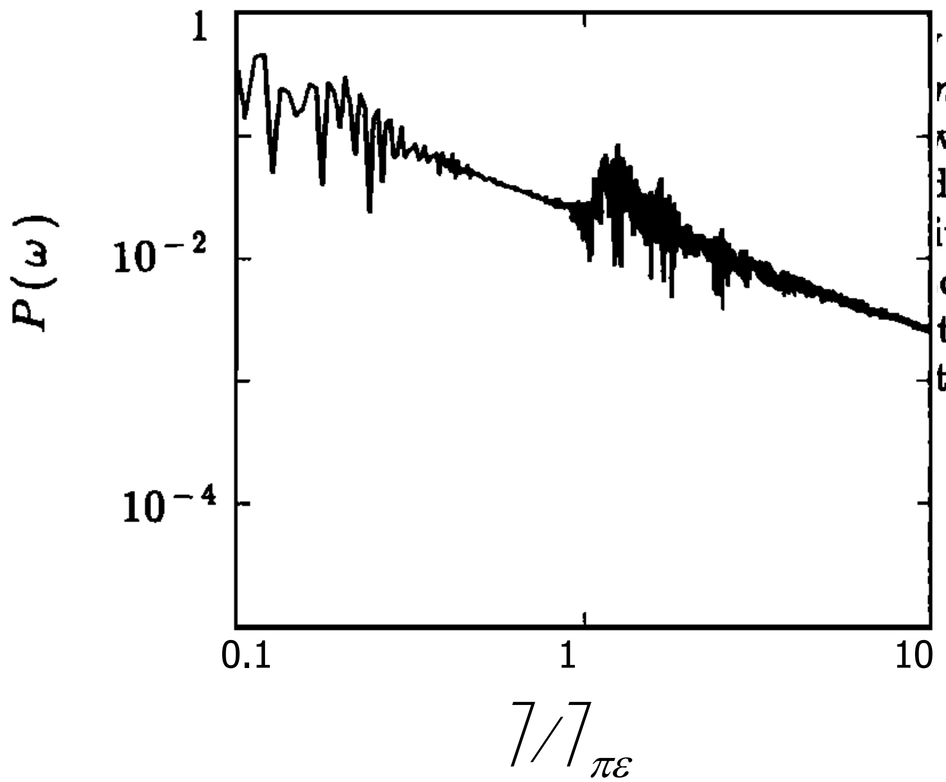
Relaxation of initial flow

- K_yL ~ O(1)
- L becomes larger with time
- This can successively trigger lower frequency waves

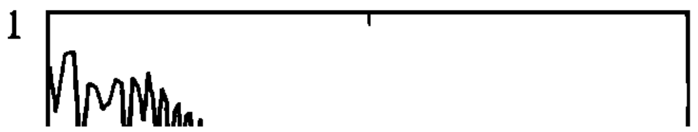
Contours of electrostatic potential



Broadband emission



the EIH-induced anomalous viscosity can be found in
mero and Ganguli, 1993. We also note that the final
profile shown in Fig. 2 extends between 1 and 2 ρ_i
l that its magnitude is larger than the ion thermal ve-
ity. This implies that the state of the system is now
ceptible to the Inhomogeneous Energy Density Driven
tability (IEDDI) [*Ganguli, et al.*, 1988a]. Further de-
tion of shear due to IEDDI waves may seed the system
the Kelvin-Helmholtz ($\omega < \Omega_i$) instability unless a



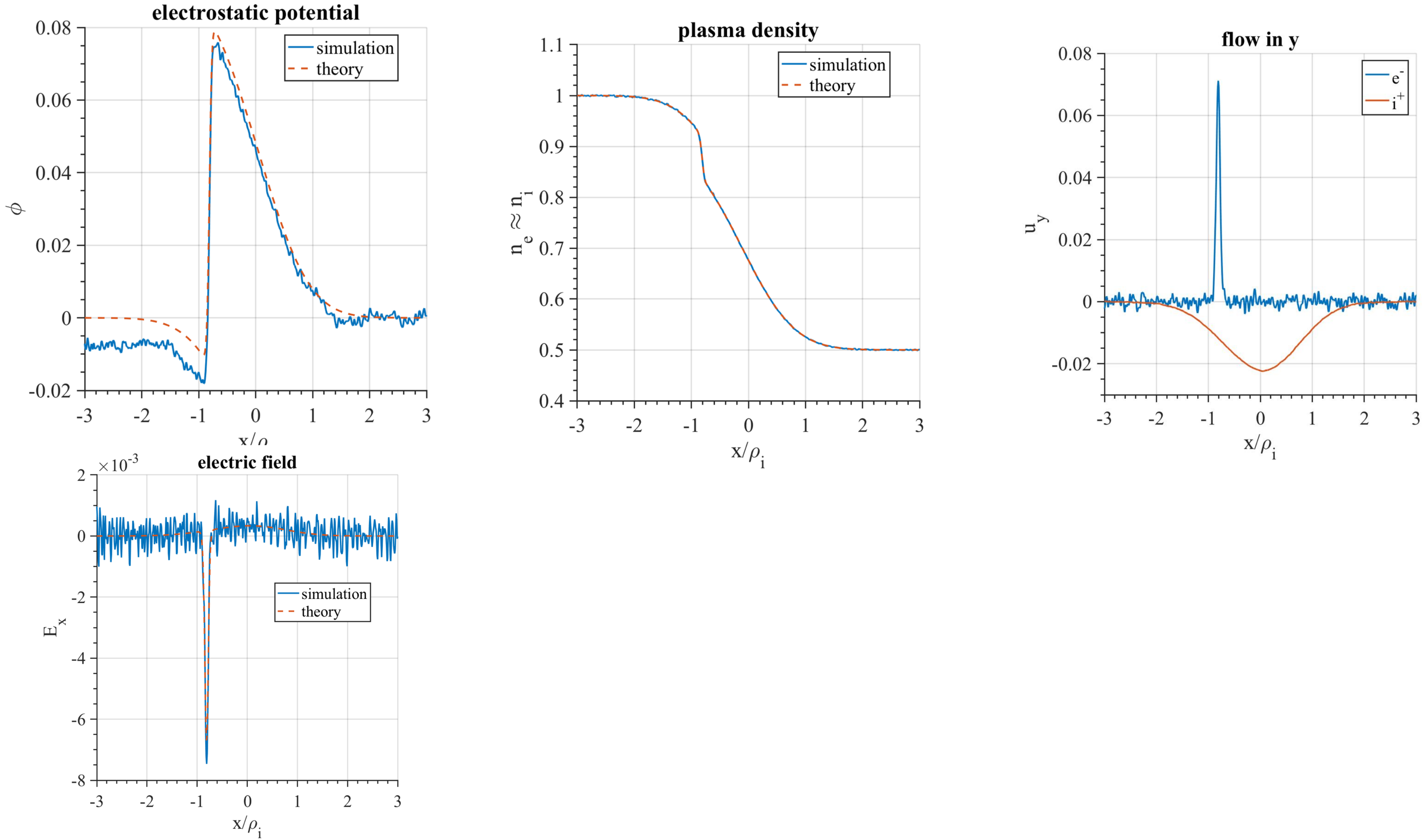
$$L < r_i \Rightarrow w \sim w_{LH} > W_i$$

$$L \sim r_i \Rightarrow w \sim W_i$$

$$L > r_i \Rightarrow w < W_i$$

[Romero and Ganguli, Phys. Fluids, 1993; Romero and Ganguli, GRL, 1994]

1D PIC Simulation Validates the Analytical Model



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Electric Field Gradient Breaks Gyrotropy and Introduces Temperature Anisotropy

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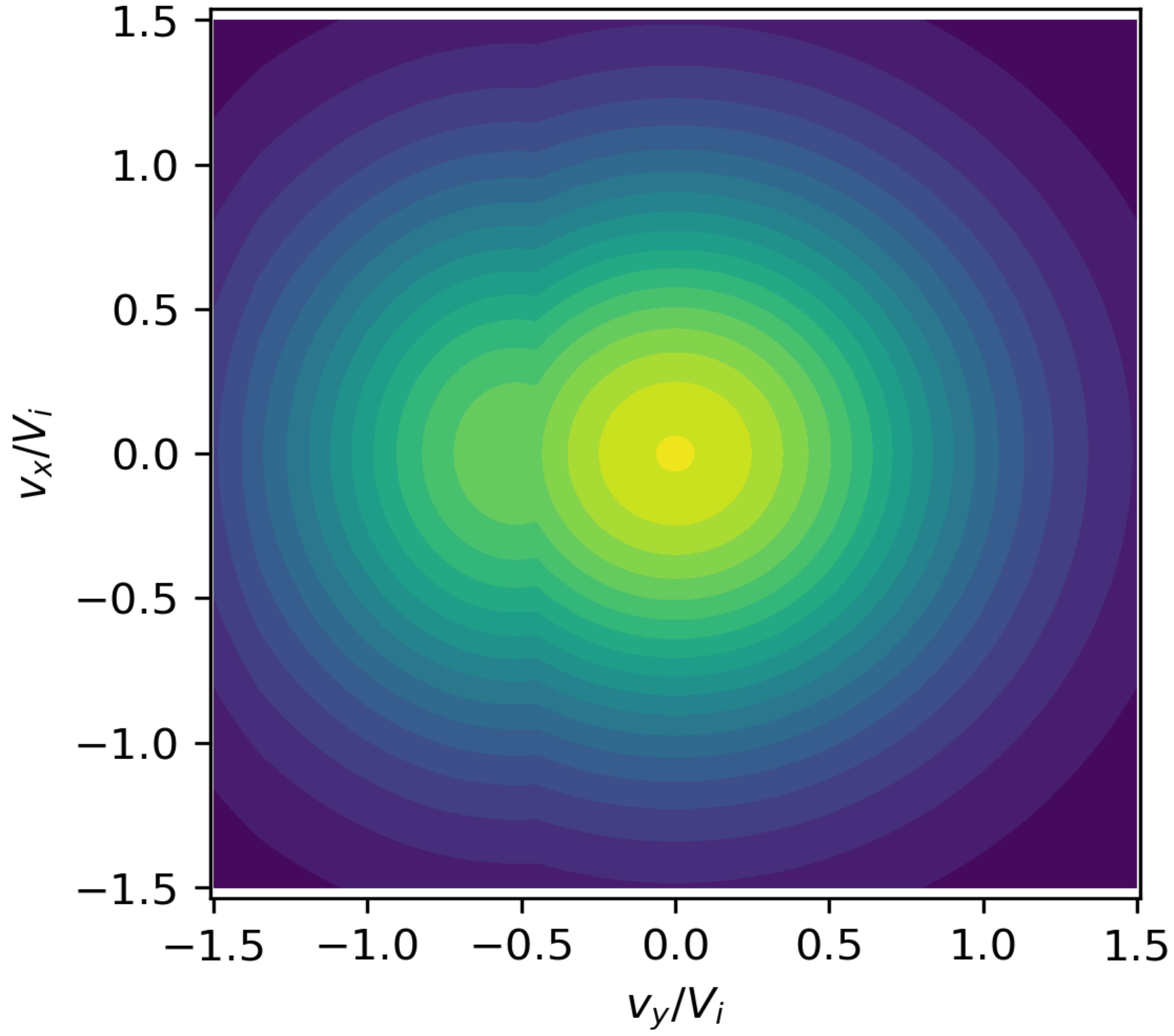
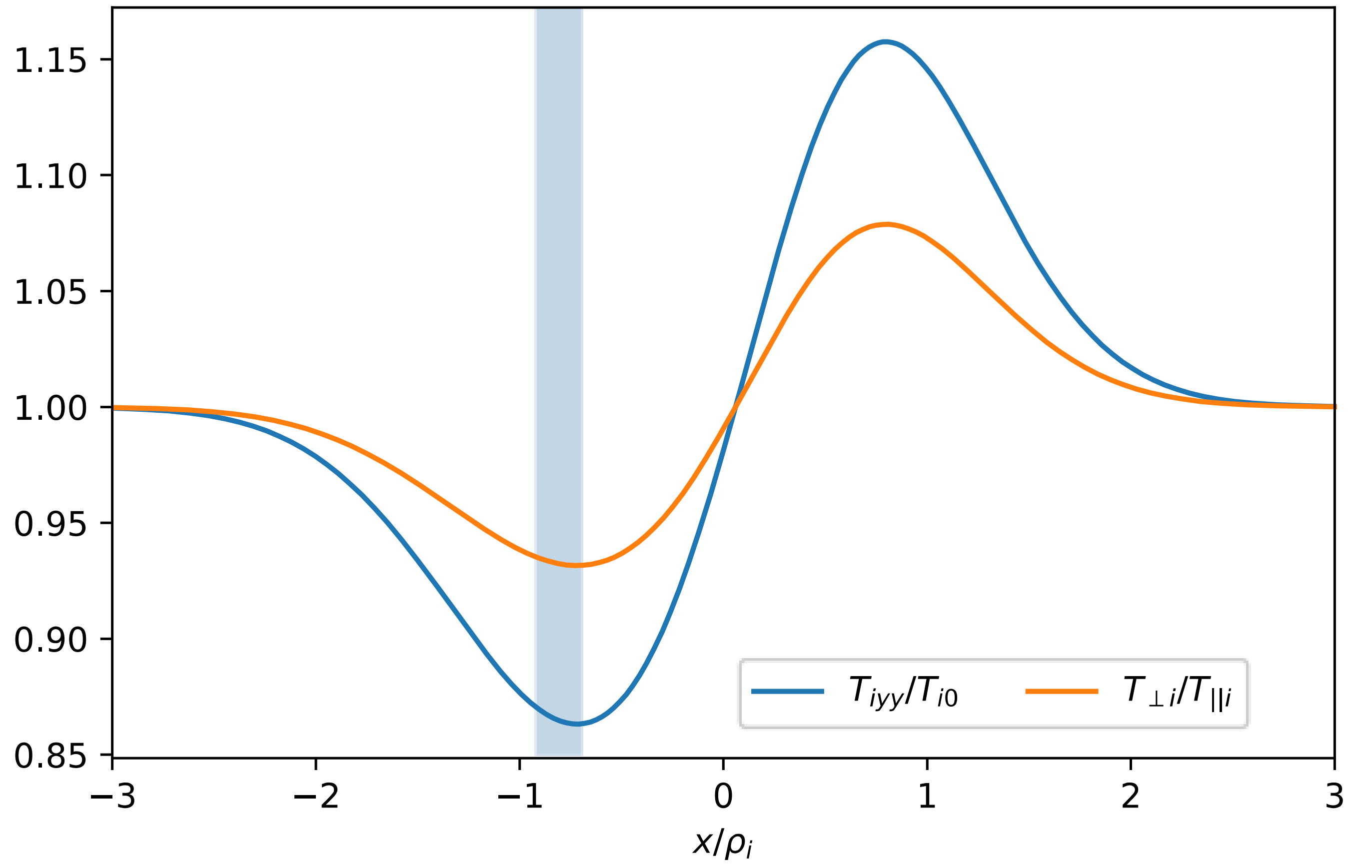
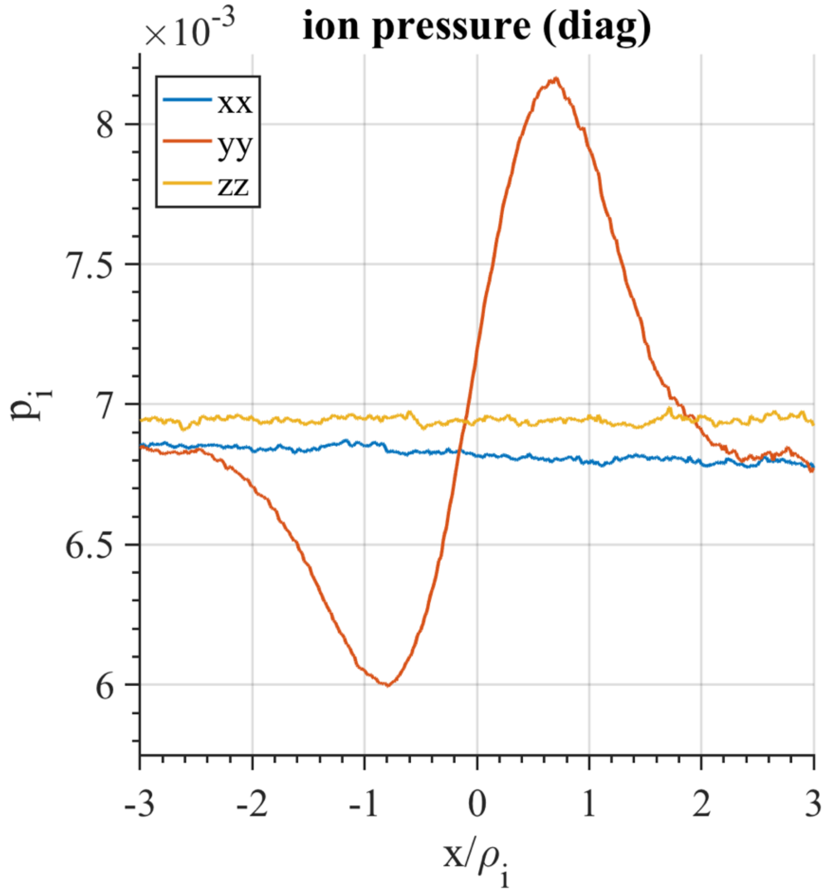
Non-gyrotropy and temperature anisotropy small for electrons but large for ions because velocity shear $dV_E/dx > \Lambda_I$ but $< \Lambda_e$



Analytical Model



1D PIC Simulation



Transverse Velocity Shear Can Drive Broadband Waves

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Weak Shear $L > \rho_i,\ dV_E/dx < \Omega_i$

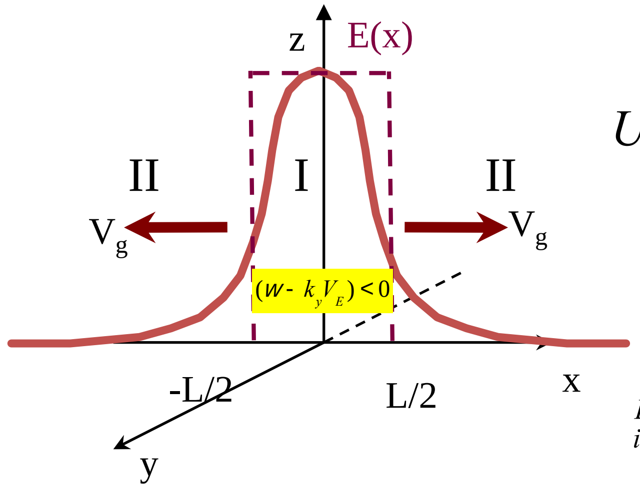
Kelvin-Helmholtz $(\omega - k_y V_E) << \Omega_i,\ k_y\rho_i < 1$

$$\left(\frac{d^2}{dx^2} - k_y^2 + \frac{k_y V_E''}{\omega - k_y V_E}\right) f_1(x) = 0$$

$$\frac{\partial}{\partial t}\int dx \left(\underbrace{\frac{|E_1|^2}{8\pi} + \frac{n_{0i}m_i}{2}\frac{|cE_1|^2}{B_0^2}}_{Wave\ Energy} + \underbrace{\frac{n_{0i}m_i}{2}|\xi_1|^2\, V_E(x)V_E''(x)}_{Energy\ Extracted\ Fron\ E_0\times B_0\ Drift} \right) = 0$$

$$\underbrace{V_E^2(x)}_{No\ Waves} - \underbrace{\left\langle V_E(x+\xi_1)\right\rangle^2}_{With\ Waves} = -|\xi_1|^2\, V_E(x)V_E''(x) + O(1/L^3)$$

IEDDI $(\omega - k_y V_E) \sim n\Omega_i,\ k_y\rho_i \geq 1$



$$U \equiv \frac{\partial \omega D}{\partial \omega} \propto \omega\underbrace{(\omega - k_y V_E)}_{Doppler-Shifted\ Frequency}$$

$$\underbrace{\gamma U_I L A_{y,z}}_{Energy\ Creation\ in\ Region-I} = -\underbrace{V_g U_{II} A_{y,z}}_{Energy\ Convection\ From\ Region-I}$$

$$\gamma \propto -\left(U_{II}/U_I\right)$$

Strong Shear $L < \rho_i,\ dV_E/dx > \Omega_i$

Electron-Ion Hybrid (EIH) $\Omega_i < \omega < \Omega_e,\ k_y\rho_i > 1$

$$\left(1 + \frac{\omega_{pe}^2}{\Omega_e^2} - \frac{\omega_{pi}^2}{\omega^2}\right)\nabla^2 f_1(x) + \left(\frac{\omega_{pe}^2}{\Omega_e^2}\right)\frac{k_y V_E''}{\omega - k_y V_E} f_1(x) = 0$$

$$\frac{\partial}{\partial t}\int dx \left(\underbrace{\frac{|E_1|^2}{8\pi} + \frac{n_{0e}m_e}{2}\frac{|cE_1|^2}{B_0^2} + \frac{n_{0i}m_i}{2}\frac{|cE_1|^2}{m_i^2\omega_r^2}}_{Wave\ Energy} + \underbrace{\frac{n_{0e}m_e}{2}|\xi_1|^2\, V_E(x)V_E''(x)}_{Energy\ Extracted\ Fron\ E_0\times B_0\ Drift} \right) = 0$$

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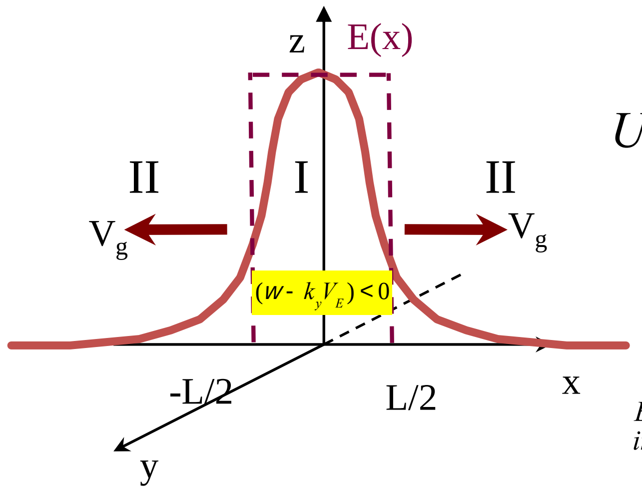
Electron IEDDI $(\omega - k_y V_E) \sim n\Omega_e,\ k_y\rho_e \sim 1$



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$$\underbrace{\gamma U_I L A_{y,z}}_{Energy\ Creation\ in\ Region-I} = -\underbrace{V_g U_{II} A_{y,z}}_{Energy\ Convection\ From\ Region-I}$$

$$\gamma \propto -\left(U_{II}/U_I\right)$$